Critical values adjusted for bandwidth snooping

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1 Looking up and computing snooping-adjusted critical values

The function `SnoopingCV` looks up appropriate critical values adjusted for bandwidth snooping, as described in Armstrong and Kolesár (2016) from a table of pre-computed critical values. If, for a given kernel and order of a local polynomial no critical value is found, the function computes an appropriate critical value using Monte Carlo simulation as explained in the next section.

```
library("BWSnooping")
SnoopingCV(bwratio = 6.2, kernel = "triangular", boundary = TRUE, order = 1,
    alpha = 0.01)
#> [1] 2.96321
```

This gives appropriate 99% critical value for a regression discontinuity design using a triangular kernel and local linear regression, with ratio of maximum to minimum bandwidths equal to 6.2. The following call gives the critical value for Nadarya-Watson (local constant) regression. Since values of `bwratio` greater than 100 are not pre-computed, it will be computed by simulation:

```
SnoopingCV(bwratio = 102, kernel = "triangular", boundary = FALSE, order = 0,
    alpha = 0.05)
#> Computing critical value by Monte Carlo simulation
#> [1] 2.59216
```

2 Calculation of critical values

This section describes the method used to tabulate the critical values based on Theorem 3.1 in Armstrong and Kolesár (2016).

Let k be a kernel (i.e. a non-negative function symmetric around zero that integrates to one) with bounded support normalized to $[-1, 1]$. We want to compute the quantiles of

$$\sup_{1\leq h\leq t} \mathbb{H}(h) =_d \sup_{1/t\leq s\leq 1} \mathbb{H}(s), \qquad \text{and} \qquad \sup_{1\leq h\leq t} |\mathbb{H}(h)| =_d \sup_{1/t\leq s\leq 1} |\mathbb{H}(s)| \tag{1}$$

where $=_d$ means "equals in distribution", and $\mathbb{H}(h)$ is a mean-zero Gaussian process with covariance function

$$\operatorname{cov}(\mathbb{H}(h), \mathbb{H}(h')) = \frac{\int_{-\infty}^{\infty} k(u/h)k(u/h')\,\mathrm{d}u}{\sqrt{\int_{-\infty}^{\infty} k(u/h)^2\,\mathrm{d}u \cdot \int_{-\infty}^{\infty} k(u/h')^2\,\mathrm{d}u}} = \frac{\int_0^{\infty} k(u/h)k(u/h')\,\mathrm{d}u}{\sqrt{hh'}\int_0^{\infty} k(u)^2\,\mathrm{d}u}.$$

Let $\{Y_i\}_{1\leq i\leq T}$ be independent samples from a standard normal distribution, where T is the number of points sampled in each simulation draw. Let

$$\hat{\mathbb{H}}(s) = \frac{1/\sqrt{T}\sum_{i=1}^{T} Y_i k\left(i/sT\right)}{\sqrt{T^{-1}\sum_{i=1}^{T} k(i/sT)^2}}.$$

Observe that $(\hat{\mathbb{H}}(s))_{s\in(0,1]}$ is a centered Gaussian process with covariance function

$$\mathbb{E}[\hat{\mathbb{H}}(s)\hat{\mathbb{H}}(s')] = \frac{1/T\sum_{i=1}^{T} k(i/sT)k(i/s'T)}{\sqrt{T^{-1}\sum_{i=1}^{T} k(i/sT)^2}\sqrt{T^{-1}\sum_{i=1}^{T} k(i/s'T)^2}}.$$

As $T\to\infty$, this process converges to $(\mathbb{H}(s))_{s\in(0,1]}$, since $\mathbb{E}[\hat{\mathbb{H}}(s)\hat{\mathbb{H}}(s')] \to \operatorname{cov}(\mathbb{H}(s),\mathbb{H}(s'))$.

We then approximate the quantiles of the two distributions in Equation (1) by empirical quantiles based on S simulation draws $\{\hat{\mathbb{H}}_m(h)\}_{m=1}^{S}$. The process $\hat{\mathbb{H}}_m(h)$ is evaluated on the log-grid

$$\exp(\log(t) : \text{stepsize} : \log(1)).$$

2.1 Example: uniform kernel

If the kernel is uniform, then $\hat{\mathbb{H}}(h)$ simplifies to

$$\hat{\mathbb{H}}(h) = \frac{\sum_{i=1}^{\lfloor hT\rfloor} Y_i}{\sqrt{\lfloor hT\rfloor}}.$$

so that $\hat{\mathbb{H}}(h)$ is mean-zero Gaussian process with covariance function $\mathbb{E}[\hat{\mathbb{H}}(h)\hat{\mathbb{H}}(h')] = \frac{\lfloor hT\rfloor\wedge\lfloor h'T\rfloor}{\sqrt{\lfloor hT\rfloor\lfloor h'T\rfloor}}$.

2.2 Note: Alternative approaches

An alternative used in an earlier version of the paper is to replace $\hat{\mathbb{H}}(h)$ with

$$\tilde{\mathbb{H}}(h) = \frac{1/\sqrt{T}\sum_{i=1}^{T} Y_i k\left(i/hT\right)}{\sqrt{h\int_0^\infty k(u)^2\,\mathrm{d}u}}$$

This method, however, has the disadvantage what for any finite T, the variance of the process $\tilde{\mathbb{H}}(h)$ is not equal to one exactly.

A third possible approach is to replace $\hat{\mathbb{H}}(h)$ with

$$\hat{\mathbb{H}}(h) = \frac{\frac{1}{\sqrt{T}}\sum_{i=1}^{T} Y_i k(X_i/h)}{\sqrt{\frac{1}{2}\int k(u/h)^2\,du}} = \frac{\frac{1}{\sqrt{T}}\sum_{i=1}^{T} Y_i k(X_i/h)}{\sqrt{\frac{h}{2}\int k(u)^2\,du}},$$

where $\{(X_i,Y_i)\}_{1\le i\le T}$ be i.i.d. with X_i independent of Y_i and $X_i\sim U[-1,1]$ and $Y_i\sim N(0,1)$. Note that $\hat{\mathbb{H}}(h)=0$ and

$$
\begin{aligned}
\operatorname{cov}(\hat{\mathbb{H}}(h), \hat{\mathbb{H}}(h')) &= \frac{EY_i^2 k(X_i/h)k(X_i/h')}{\sqrt{\frac{1}{2}\int k(u/h)^2\,du}\sqrt{\frac{1}{2}\int k(u/h')^2\,du}} \\
&= \frac{Ek(X_i/h)k(X_i/h')}{\frac{1}{2}\sqrt{\int k(u/h)^2\,du}\sqrt{\int k(u/h')^2\,du}} = \frac{\frac{1}{2}\int_{x=-1}^{1} k(x/h)k(x/h')\,dx}{\frac{1}{2}\sqrt{\int k(u/h)^2\,du}\sqrt{\int k(u/h')^2\,du}} \\
&= \frac{\int_{x=-\infty}^{\infty} k(x/h)k(x/h')\,dx}{\sqrt{\int k(u/h)^2\,du}\sqrt{\int k(u/h')^2\,du}}
\end{aligned}
$$

for $h \le 1$ (the last step follows since, for $h \le 1$, the integral is over $-h \le x \le h$ for both the last and second to last line). Thus, $\hat{\mathbb{H}}(h)$ and $\mathbb{H}(h)$ have the same covariance function and, for large enough T, have approximately the same distribution.

3 Regression near a boundary point

Since the critical values at a boundary and interior points differ, this may create a non-uniformity issue for estimation problems in which the point of interest lies near a boundary point. To deal with this case, it is possible to extend the method in Armstrong and Kolesár (2016) by modeling the point of interest as being local to boundary, as in Chapter 3.2.5 of Fan and Gijbels (1996) (see Section S2.1 in the supplement to Armstrong and Kolesár (2016)).

In particular, for local polynomial estimation of $E[Y_i \mid X_i = x_0]$ where $x_0 = c\underline{h}_n$ and the lower support point of the density of X_i is zero, the appropriate critical value can be computed using the function `TableSnoopingCVNearBd`. For convenience, the function reports a table of critical values. For example, to calculate critical values for local linear regression and bandwidth ratios $\overline{h}/\underline{h}$ equal to $t \in \{1, 2, 3\}$ and the local parameter equal to $c \in \{1, 2, 10\}$, one calls

```
nb <- TableSnoopingCVNearBd(bwratios = c(1, 2, 3), kernel = "triangular", db = c(1,
    2, 10), order = 1)
knitr::kable(nb$table.twosided, row.names = FALSE, digits = 2, caption = "Local linear regression near a
```

Table 1: Local linear regression near a boundary

t	1	2	10
1	1.97	1.96	1.96
2	2.09	2.15	2.14
3	2.13	2.21	2.24

```
knitr::kable(nb$table.onesided, row.names = FALSE, digits = 2, caption = "Local linear regression near a
```

Table 2: Local linear regression near a boundary

t	1	2	10
1	1.66	1.67	1.61
2	1.78	1.84	1.83
3	1.83	1.91	1.92

4 Tables and graphs of critical values

The tables and graphs in the paper can be reproduced using the functions `DFSnoopingCV` and `SnoopingTablesGraphs`. `DFSnoopingCV` computes a data frame of critical values, and `SnoopingTablesGraphs` reproduces tables and graphs reported in the paper:

```
## The function DFSnoopingCV may long time to compute, the package has
## results stored for DFSnoopingCV(S=60000, T=10000, ngr=1000) under the data
## frame snoopingcvs
r <- SnoopingTablesGraphs(snoopingcvs)
```

4.1 Tables for two-sided critical values

- For Nadarya-Watson regression, boundary and interior critical values coincide
- In the tables, "u" stands for uniform, "e" for Epanechnikov, and "t" for triangular kernel, so that "0.9u" corresponds to 90%-level critical value for the uniform kernel, "0.99t" corresponds to 99%-level critical value for the triangular kernel, and so on.

```
t1 <- subset(r$table.twosided, boundary == TRUE & order == 0)[, -c(1, 2)]
knitr::kable(t1, row.names = FALSE, digits = 2, caption = "Boundary Nadaraya-Watson regression")
```

Table 3: Boundary Nadaraya-Watson regression

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
1.00	1.65	1.96	2.57	1.64	1.96	2.58	1.64	1.96	2.58
1.20	1.92	2.24	2.85	1.70	2.01	2.63	1.71	2.03	2.65
1.40	2.02	2.33	2.93	1.74	2.05	2.67	1.77	2.08	2.69
1.60	2.09	2.40	2.98	1.78	2.09	2.70	1.81	2.12	2.72
1.80	2.14	2.45	3.03	1.81	2.11	2.72	1.85	2.15	2.75
2.01	2.18	2.48	3.07	1.83	2.14	2.75	1.87	2.17	2.78
3.01	2.30	2.60	3.18	1.91	2.22	2.83	1.96	2.27	2.86
4.01	2.37	2.66	3.24	1.96	2.26	2.86	2.02	2.31	2.92
5.02	2.41	2.70	3.28	2.00	2.30	2.90	2.05	2.35	2.95
6.01	2.44	2.73	3.31	2.02	2.32	2.92	2.08	2.37	2.97
7.03	2.47	2.75	3.34	2.04	2.34	2.94	2.10	2.39	2.99
8.03	2.49	2.77	3.35	2.06	2.35	2.95	2.12	2.41	3.01
9.01	2.51	2.79	3.37	2.07	2.37	2.96	2.14	2.42	3.02
10.02	2.52	2.80	3.38	2.08	2.38	2.97	2.15	2.44	3.04
20.01	2.61	2.89	3.45	2.16	2.45	3.03	2.23	2.51	3.10
50.08	2.70	2.97	3.51	2.24	2.53	3.10	2.31	2.59	3.15
100.00	2.75	3.02	3.56	2.29	2.57	3.14	2.36	2.64	3.20

```
t2 <- subset(r$table.twosided, boundary == FALSE & order == 0)[, -c(1, 2)]
knitr::kable(t2, row.names = FALSE, digits = 2, caption = "Interior Nadaraya-Watson regression")
```

Table 4: Interior Nadaraya-Watson regression

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
1.00	1.65	1.96	2.57	1.64	1.96	2.58	1.64	1.96	2.58
1.20	1.92	2.24	2.85	1.70	2.01	2.63	1.71	2.03	2.65
1.40	2.02	2.33	2.93	1.74	2.05	2.67	1.77	2.08	2.69
1.60	2.09	2.40	2.98	1.78	2.09	2.70	1.81	2.12	2.72

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
1.80	2.14	2.45	3.03	1.81	2.11	2.72	1.85	2.15	2.75
2.01	2.18	2.48	3.07	1.83	2.14	2.75	1.87	2.17	2.78
3.01	2.30	2.60	3.18	1.91	2.22	2.83	1.96	2.27	2.86
4.01	2.37	2.66	3.24	1.96	2.26	2.86	2.02	2.31	2.92
5.02	2.41	2.70	3.28	2.00	2.30	2.90	2.05	2.35	2.95
6.01	2.44	2.73	3.31	2.02	2.32	2.92	2.08	2.37	2.97
7.03	2.47	2.75	3.34	2.04	2.34	2.94	2.10	2.39	2.99
8.03	2.49	2.77	3.35	2.06	2.35	2.95	2.12	2.41	3.01
9.01	2.51	2.79	3.37	2.07	2.37	2.96	2.14	2.42	3.02
10.02	2.52	2.80	3.38	2.08	2.38	2.97	2.15	2.44	3.04
20.01	2.61	2.89	3.45	2.16	2.45	3.03	2.23	2.51	3.10
50.08	2.70	2.97	3.51	2.24	2.53	3.10	2.31	2.59	3.15
100.00	2.75	3.02	3.56	2.29	2.57	3.14	2.36	2.64	3.20

```r
t3 <- subset(r$table.twosided, boundary == TRUE & order == 1)[, -c(1, 2)]
knitr::kable(t3, row.names = FALSE, digits = 2, caption = "Boundary local linear regression")
```

Table 5: Boundary local linear regression

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
1.00	1.63	1.96	2.57	1.64	1.95	2.57	1.64	1.96	2.57
1.20	1.92	2.23	2.83	1.72	2.03	2.64	1.73	2.05	2.66
1.40	2.02	2.33	2.93	1.77	2.08	2.69	1.80	2.11	2.72
1.60	2.09	2.39	3.00	1.80	2.12	2.73	1.84	2.15	2.76
1.80	2.14	2.44	3.04	1.84	2.16	2.76	1.88	2.19	2.81
2.01	2.18	2.48	3.08	1.87	2.18	2.78	1.91	2.22	2.83
3.01	2.30	2.60	3.18	1.96	2.27	2.86	2.01	2.32	2.91
4.01	2.36	2.66	3.24	2.01	2.32	2.90	2.06	2.37	2.95
5.02	2.41	2.71	3.27	2.05	2.35	2.94	2.11	2.41	2.99
6.01	2.44	2.73	3.31	2.08	2.37	2.96	2.13	2.43	3.01
7.03	2.47	2.76	3.33	2.10	2.39	2.98	2.16	2.45	3.04
8.03	2.49	2.78	3.35	2.12	2.41	2.99	2.18	2.47	3.05
9.01	2.50	2.79	3.37	2.14	2.43	3.00	2.20	2.48	3.06
10.02	2.52	2.81	3.39	2.15	2.44	3.01	2.21	2.50	3.07
20.01	2.61	2.89	3.45	2.23	2.52	3.08	2.29	2.58	3.14
50.08	2.70	2.98	3.52	2.32	2.60	3.16	2.38	2.66	3.21
100.00	2.76	3.02	3.56	2.37	2.65	3.20	2.44	2.71	3.25

```r
t4 <- subset(r$table.twosided, boundary == FALSE & order == 1)[, -c(1, 2)]
knitr::kable(t4, row.names = FALSE, digits = 2, caption = "Interior local linear regression")
```

Table 6: Interior local linear regression

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
1.00	1.65	1.96	2.57	1.64	1.96	2.58	1.64	1.96	2.58
1.20	1.92	2.24	2.85	1.70	2.01	2.63	1.71	2.03	2.65
1.40	2.02	2.33	2.93	1.74	2.05	2.67	1.77	2.08	2.69
1.60	2.09	2.40	2.98	1.78	2.09	2.70	1.81	2.12	2.72
1.80	2.14	2.45	3.03	1.81	2.11	2.72	1.85	2.15	2.75
2.01	2.18	2.48	3.07	1.83	2.14	2.75	1.87	2.17	2.78

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
3.01	2.30	2.60	3.18	1.91	2.22	2.83	1.96	2.27	2.86
4.01	2.37	2.66	3.24	1.96	2.26	2.86	2.02	2.31	2.92
5.02	2.41	2.70	3.28	2.00	2.30	2.90	2.05	2.35	2.95
6.01	2.44	2.73	3.31	2.02	2.32	2.92	2.08	2.37	2.97
7.03	2.47	2.75	3.34	2.04	2.34	2.94	2.10	2.39	2.99
8.03	2.49	2.77	3.35	2.06	2.35	2.95	2.12	2.41	3.01
9.01	2.51	2.79	3.37	2.07	2.37	2.96	2.14	2.42	3.02
10.02	2.52	2.80	3.38	2.08	2.38	2.97	2.15	2.44	3.04
20.01	2.61	2.89	3.45	2.16	2.45	3.03	2.23	2.51	3.10
50.08	2.70	2.97	3.51	2.24	2.53	3.10	2.31	2.59	3.15
100.00	2.75	3.02	3.56	2.29	2.57	3.14	2.36	2.64	3.20

```r
t5 <- subset(r$table.twosided, boundary == TRUE & order == 2)[, -c(1, 2)]
knitr::kable(t5, row.names = FALSE, digits = 2, caption = "Boundary local quadratic regression")
```

Table 7: Boundary local quadratic regression

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
1.00	1.64	1.96	2.58	1.64	1.96	2.57	1.64	1.96	2.58
1.20	1.93	2.25	2.85	1.73	2.05	2.66	1.75	2.06	2.67
1.40	2.03	2.34	2.95	1.79	2.10	2.70	1.82	2.13	2.73
1.60	2.09	2.41	3.00	1.83	2.14	2.75	1.87	2.18	2.77
1.80	2.14	2.45	3.04	1.87	2.18	2.79	1.91	2.22	2.82
2.01	2.18	2.49	3.08	1.90	2.21	2.81	1.95	2.25	2.85
3.01	2.30	2.60	3.17	2.00	2.29	2.89	2.04	2.34	2.94
4.01	2.37	2.66	3.22	2.05	2.35	2.93	2.10	2.40	2.99
5.02	2.41	2.70	3.27	2.09	2.38	2.97	2.14	2.44	3.02
6.01	2.45	2.73	3.30	2.12	2.41	2.99	2.17	2.46	3.04
7.03	2.47	2.76	3.32	2.14	2.44	3.01	2.20	2.49	3.06
8.03	2.49	2.78	3.34	2.16	2.45	3.03	2.22	2.51	3.08
9.01	2.51	2.80	3.36	2.18	2.47	3.04	2.24	2.52	3.10
10.02	2.53	2.81	3.37	2.19	2.48	3.05	2.25	2.53	3.10
20.01	2.61	2.89	3.45	2.27	2.56	3.13	2.33	2.62	3.18
50.08	2.70	2.98	3.51	2.36	2.64	3.20	2.42	2.70	3.25
100.00	2.76	3.03	3.56	2.42	2.69	3.23	2.48	2.75	3.29

```r
t6 <- subset(r$table.twosided, boundary == FALSE & order == 2)[, -c(1, 2)]
knitr::kable(t6, row.names = FALSE, digits = 2, caption = "Interior local quadratic regression")
```

Table 8: Interior local quadratic regression

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
1.00	1.64	1.95	2.56	1.64	1.95	2.57	1.64	1.95	2.58
1.20	1.92	2.23	2.83	1.72	2.03	2.65	1.73	2.04	2.65
1.40	2.02	2.33	2.93	1.77	2.09	2.70	1.80	2.11	2.72
1.60	2.09	2.40	2.99	1.81	2.13	2.74	1.84	2.15	2.77
1.80	2.14	2.44	3.05	1.85	2.16	2.77	1.88	2.20	2.79
2.01	2.18	2.48	3.08	1.88	2.19	2.79	1.92	2.23	2.82
3.01	2.30	2.60	3.19	1.96	2.27	2.88	2.02	2.32	2.92
4.01	2.36	2.66	3.26	2.02	2.32	2.92	2.07	2.38	2.97

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
5.02	2.41	2.70	3.29	2.06	2.36	2.95	2.11	2.41	3.00
6.01	2.44	2.73	3.31	2.09	2.39	2.97	2.14	2.44	3.03
7.03	2.47	2.76	3.34	2.11	2.41	2.99	2.17	2.47	3.05
8.03	2.49	2.77	3.36	2.12	2.43	3.01	2.19	2.48	3.06
9.01	2.51	2.79	3.37	2.14	2.44	3.02	2.20	2.50	3.08
10.02	2.52	2.81	3.38	2.15	2.45	3.03	2.22	2.51	3.10
20.01	2.60	2.88	3.44	2.24	2.53	3.09	2.30	2.59	3.16
50.08	2.70	2.97	3.52	2.33	2.60	3.16	2.39	2.67	3.22
100.00	2.76	3.02	3.56	2.38	2.66	3.21	2.45	2.72	3.27

4.2 Tables for one-sided critical values

```r
o1 <- subset(r$table.onesided, boundary == TRUE & order == 0)[, -c(1, 2)]
knitr::kable(o1, row.names = FALSE, digits = 2, caption = "Boundary Nadaraya-Watson regression")
```

Table 9: Boundary Nadaraya-Watson regression

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
1.00	1.29	1.66	2.33	1.29	1.66	2.34	1.29	1.66	2.34
1.20	1.57	1.94	2.64	1.35	1.72	2.39	1.36	1.73	2.41
1.40	1.67	2.04	2.73	1.39	1.76	2.44	1.42	1.79	2.45
1.60	1.75	2.11	2.79	1.42	1.80	2.47	1.46	1.83	2.50
1.80	1.80	2.15	2.83	1.46	1.83	2.49	1.49	1.86	2.53
2.01	1.84	2.19	2.85	1.48	1.85	2.52	1.52	1.89	2.55
3.01	1.97	2.31	2.97	1.56	1.93	2.58	1.62	1.98	2.64
4.01	2.04	2.38	3.02	1.61	1.97	2.63	1.68	2.03	2.68
5.02	2.09	2.42	3.05	1.65	2.01	2.66	1.71	2.07	2.71
6.01	2.12	2.45	3.08	1.68	2.03	2.68	1.74	2.09	2.74
7.03	2.15	2.48	3.10	1.71	2.05	2.70	1.77	2.11	2.76
8.03	2.17	2.50	3.12	1.72	2.07	2.72	1.79	2.13	2.77
9.01	2.19	2.52	3.14	1.74	2.09	2.73	1.80	2.14	2.79
10.02	2.21	2.53	3.16	1.76	2.10	2.74	1.82	2.16	2.81
20.01	2.29	2.62	3.23	1.83	2.17	2.80	1.91	2.24	2.87
50.08	2.40	2.71	3.31	1.92	2.25	2.87	2.00	2.32	2.94
100.00	2.46	2.77	3.36	1.98	2.30	2.92	2.06	2.37	2.99

```r
o2 <- subset(r$table.onesided, boundary == FALSE & order == 0)[, -c(1, 2)]
knitr::kable(o2, row.names = FALSE, digits = 2, caption = "Interior Nadaraya-Watson regression")
```

Table 10: Interior Nadaraya-Watson regression

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
1.00	1.29	1.66	2.33	1.29	1.66	2.34	1.29	1.66	2.34
1.20	1.57	1.94	2.64	1.35	1.72	2.39	1.36	1.73	2.41
1.40	1.67	2.04	2.73	1.39	1.76	2.44	1.42	1.79	2.45
1.60	1.75	2.11	2.79	1.42	1.80	2.47	1.46	1.83	2.50
1.80	1.80	2.15	2.83	1.46	1.83	2.49	1.49	1.86	2.53
2.01	1.84	2.19	2.85	1.48	1.85	2.52	1.52	1.89	2.55
3.01	1.97	2.31	2.97	1.56	1.93	2.58	1.62	1.98	2.64

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
4.01	2.04	2.38	3.02	1.61	1.97	2.63	1.68	2.03	2.68
5.02	2.09	2.42	3.05	1.65	2.01	2.66	1.71	2.07	2.71
6.01	2.12	2.45	3.08	1.68	2.03	2.68	1.74	2.09	2.74
7.03	2.15	2.48	3.10	1.71	2.05	2.70	1.77	2.11	2.76
8.03	2.17	2.50	3.12	1.72	2.07	2.72	1.79	2.13	2.77
9.01	2.19	2.52	3.14	1.74	2.09	2.73	1.80	2.14	2.79
10.02	2.21	2.53	3.16	1.76	2.10	2.74	1.82	2.16	2.81
20.01	2.29	2.62	3.23	1.83	2.17	2.80	1.91	2.24	2.87
50.08	2.40	2.71	3.31	1.92	2.25	2.87	2.00	2.32	2.94
100.00	2.46	2.77	3.36	1.98	2.30	2.92	2.06	2.37	2.99

```r
o3 <- subset(r$table.onesided, boundary == TRUE & order == 1)[, -c(1, 2)]
knitr::kable(o3, row.names = FALSE, digits = 2, caption = "Boundary local linear regression")
```

Table 11: Boundary local linear regression

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
1.00	1.28	1.64	2.33	1.29	1.65	2.33	1.28	1.65	2.33
1.20	1.57	1.93	2.59	1.36	1.72	2.40	1.38	1.74	2.42
1.40	1.67	2.03	2.69	1.41	1.78	2.46	1.44	1.80	2.47
1.60	1.74	2.10	2.76	1.46	1.81	2.49	1.49	1.85	2.52
1.80	1.80	2.15	2.81	1.49	1.84	2.52	1.53	1.88	2.55
2.01	1.84	2.19	2.84	1.52	1.87	2.55	1.56	1.91	2.58
3.01	1.96	2.30	2.95	1.62	1.96	2.62	1.67	2.01	2.67
4.01	2.03	2.36	3.02	1.67	2.01	2.67	1.73	2.06	2.72
5.02	2.08	2.41	3.04	1.71	2.05	2.70	1.77	2.11	2.76
6.01	2.12	2.44	3.07	1.74	2.08	2.72	1.80	2.13	2.77
7.03	2.14	2.47	3.09	1.76	2.10	2.74	1.83	2.16	2.80
8.03	2.17	2.49	3.11	1.79	2.12	2.75	1.85	2.18	2.81
9.01	2.18	2.51	3.12	1.80	2.14	2.77	1.87	2.20	2.82
10.02	2.20	2.52	3.13	1.82	2.15	2.79	1.88	2.21	2.84
20.01	2.29	2.61	3.22	1.90	2.23	2.86	1.97	2.29	2.91
50.08	2.39	2.70	3.30	1.99	2.31	2.92	2.06	2.38	2.99
100.00	2.45	2.76	3.35	2.05	2.37	2.96	2.12	2.44	3.03

```r
o4 <- subset(r$table.onesided, boundary == FALSE & order == 1)[, -c(1, 2)]
knitr::kable(o4, row.names = FALSE, digits = 2, caption = "Interior local linear regression")
```

Table 12: Interior local linear regression

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
1.00	1.29	1.66	2.33	1.29	1.66	2.34	1.29	1.66	2.34
1.20	1.57	1.94	2.64	1.35	1.72	2.39	1.36	1.73	2.41
1.40	1.67	2.04	2.73	1.39	1.76	2.44	1.42	1.79	2.45
1.60	1.75	2.11	2.79	1.42	1.80	2.47	1.46	1.83	2.50
1.80	1.80	2.15	2.83	1.46	1.83	2.49	1.49	1.86	2.53
2.01	1.84	2.19	2.85	1.48	1.85	2.52	1.52	1.89	2.55
3.01	1.97	2.31	2.97	1.56	1.93	2.58	1.62	1.98	2.64
4.01	2.04	2.38	3.02	1.61	1.97	2.63	1.68	2.03	2.68
5.02	2.09	2.42	3.05	1.65	2.01	2.66	1.71	2.07	2.71

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
6.01	2.12	2.45	3.08	1.68	2.03	2.68	1.74	2.09	2.74
7.03	2.15	2.48	3.10	1.71	2.05	2.70	1.77	2.11	2.76
8.03	2.17	2.50	3.12	1.72	2.07	2.72	1.79	2.13	2.77
9.01	2.19	2.52	3.14	1.74	2.09	2.73	1.80	2.14	2.79
10.02	2.21	2.53	3.16	1.76	2.10	2.74	1.82	2.16	2.81
20.01	2.29	2.62	3.23	1.83	2.17	2.80	1.91	2.24	2.87
50.08	2.40	2.71	3.31	1.92	2.25	2.87	2.00	2.32	2.94
100.00	2.46	2.77	3.36	1.98	2.30	2.92	2.06	2.37	2.99

```r
o5 <- subset(r$table.onesided, boundary == TRUE & order == 2)[, -c(1, 2)]
knitr::kable(o5, row.names = FALSE, digits = 2, caption = "Boundary local quadratic regression")
```

Table 13: Boundary local quadratic regression

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
1.00	1.28	1.65	2.33	1.28	1.64	2.32	1.28	1.64	2.32
1.20	1.57	1.93	2.61	1.37	1.73	2.40	1.39	1.75	2.41
1.40	1.67	2.03	2.69	1.43	1.78	2.45	1.46	1.82	2.47
1.60	1.74	2.09	2.74	1.48	1.83	2.50	1.51	1.86	2.53
1.80	1.79	2.14	2.79	1.52	1.87	2.53	1.56	1.91	2.57
2.01	1.84	2.19	2.83	1.55	1.90	2.56	1.59	1.95	2.60
3.01	1.96	2.30	2.95	1.65	1.99	2.65	1.70	2.04	2.69
4.01	2.03	2.36	3.00	1.71	2.05	2.71	1.76	2.09	2.75
5.02	2.08	2.41	3.03	1.75	2.09	2.74	1.81	2.14	2.79
6.01	2.11	2.45	3.06	1.78	2.12	2.77	1.84	2.17	2.82
7.03	2.14	2.47	3.08	1.81	2.14	2.79	1.87	2.20	2.84
8.03	2.17	2.49	3.11	1.83	2.16	2.81	1.89	2.22	2.86
9.01	2.19	2.51	3.13	1.84	2.18	2.82	1.91	2.24	2.87
10.02	2.20	2.53	3.14	1.86	2.19	2.83	1.92	2.25	2.88
20.01	2.30	2.61	3.21	1.95	2.27	2.90	2.01	2.33	2.95
50.08	2.40	2.71	3.29	2.04	2.36	2.97	2.11	2.42	3.03
100.00	2.46	2.76	3.34	2.10	2.41	3.02	2.17	2.47	3.08

```r
o6 <- subset(r$table.onesided, boundary == FALSE & order == 2)[, -c(1, 2)]
knitr::kable(o6, row.names = FALSE, digits = 2, caption = "Interior local quadratic regression")
```

Table 14: Interior local quadratic regression

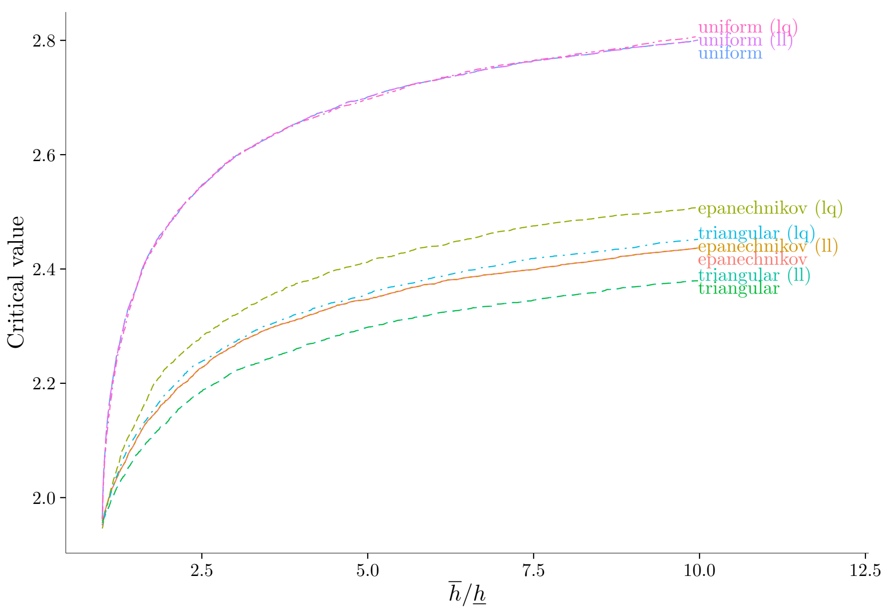
t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
1.00	1.29	1.65	2.32	1.29	1.65	2.33	1.28	1.65	2.32
1.20	1.57	1.93	2.59	1.36	1.72	2.41	1.38	1.74	2.42
1.40	1.67	2.03	2.70	1.42	1.78	2.47	1.45	1.81	2.48
1.60	1.74	2.10	2.77	1.46	1.82	2.49	1.50	1.85	2.53
1.80	1.80	2.15	2.81	1.49	1.86	2.52	1.54	1.89	2.56
2.01	1.84	2.19	2.84	1.52	1.88	2.55	1.57	1.93	2.59
3.01	1.96	2.31	2.95	1.62	1.97	2.63	1.68	2.02	2.68
4.01	2.03	2.37	3.01	1.68	2.02	2.67	1.74	2.08	2.73
5.02	2.08	2.41	3.05	1.72	2.06	2.72	1.78	2.12	2.77
6.01	2.12	2.44	3.08	1.75	2.09	2.74	1.81	2.15	2.79
7.03	2.14	2.47	3.10	1.77	2.11	2.77	1.84	2.17	2.82

t	0.9u	0.95u	0.99u	0.9t	0.95t	0.99t	0.9e	0.95e	0.99e
8.03	2.17	2.49	3.12	1.79	2.13	2.78	1.86	2.19	2.83
9.01	2.19	2.51	3.13	1.81	2.14	2.79	1.88	2.21	2.84
10.02	2.21	2.52	3.14	1.83	2.16	2.80	1.89	2.23	2.85
20.01	2.30	2.61	3.23	1.91	2.24	2.87	1.98	2.31	2.93
50.08	2.40	2.70	3.31	2.00	2.33	2.95	2.07	2.40	3.01
100.00	2.46	2.76	3.35	2.06	2.38	2.98	2.13	2.45	3.05

4.3 Graphs

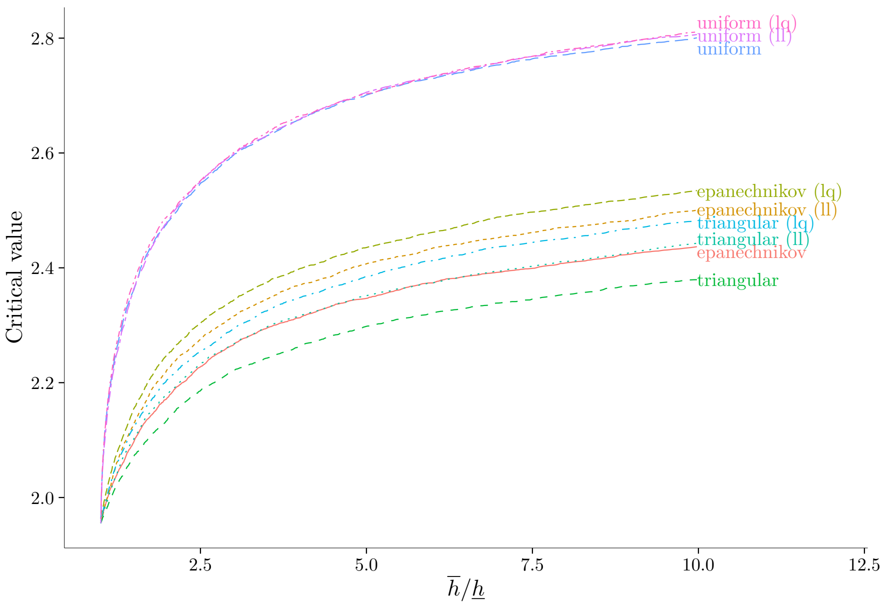
Critical values for local constant and local linear regression in the interior:

```
r$cv.interior
```



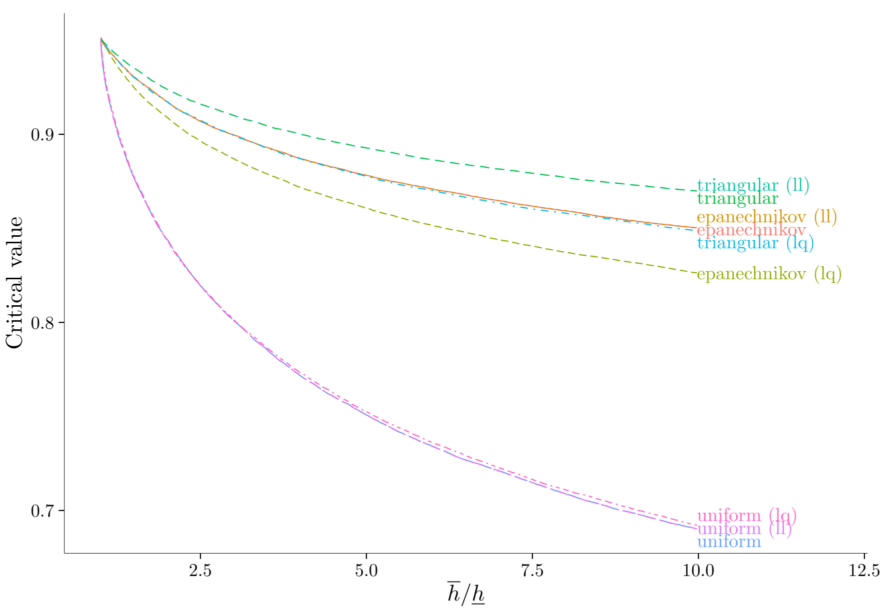
Critical values for local constant and local linear regression at the boundary:

```
r$cv.boundary
```



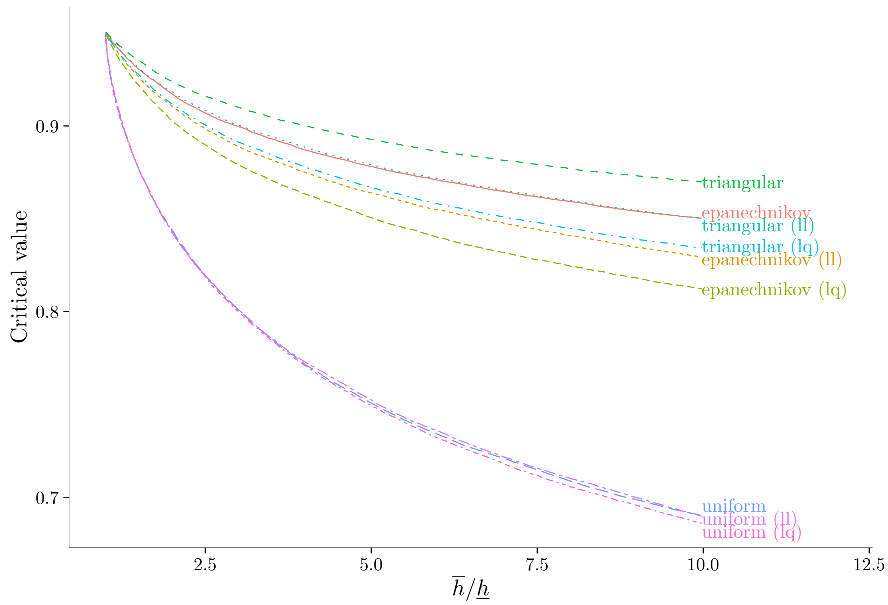
Coverage of unadjusted CIs for local constant and local linear regression in the interior:

```
r$cov.interior
```



Coverage of unadjusted CIs for local constant and local linear regression at the boundary:

```
r$cov.boundary
```

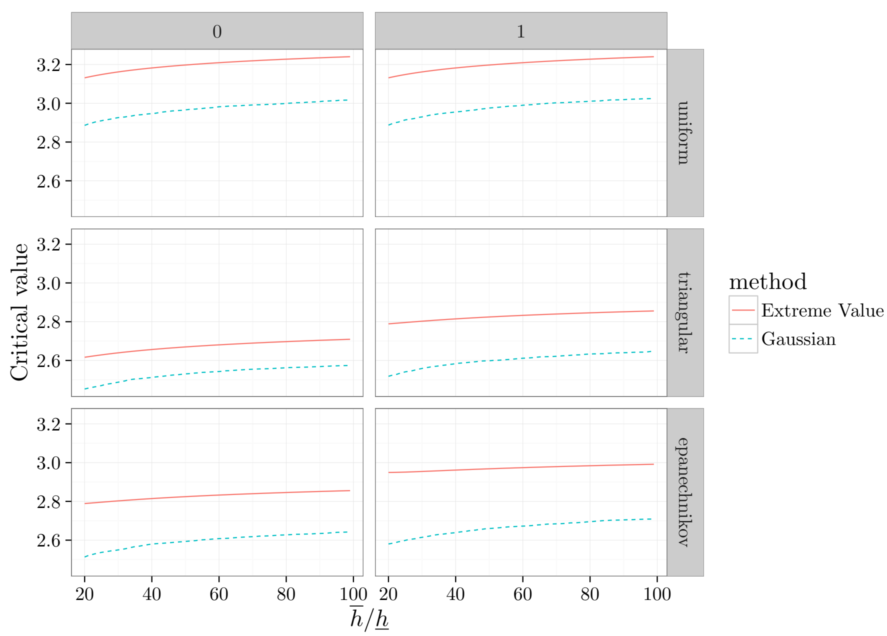


5 Critical values based on extreme value approximation

The package also provides a function `EVSnoopingCV` that calculates critical values based on a further extreme value approximation to the Gaussian process. Its use is not recommended, however. The following figure compares these critical values with those based directly on the Gaussian process.

Order "0" corresponds to Nadaraya-Watson or local linear regression in the interior, and order "1" to local linear regression at a boundary.

```
p <- PlotEVSnoopingCV()
p + ggplot2::ylab("Critical value") + ggplot2::xlab("$\\overline{h}/\\underline{h}$")
```



References

Armstrong, Timothy B., and Michal Kolesár. 2016. "A Simple Adjustment for Bandwidth Snooping."

Fan, Jianqing, and Irene Gijbels. 1996. *Local Polynomial Modelling and Its Applications*. Chapman & Hall/CRC.