

Primordial 4D Torus Hypothesis

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This hypothesis arises from the association of the primordial seed (Φ) with the exotic state described below, in previous studies conducted in [The Atlas](#), and from its consequent Singularity Void, a structure we implemented to handle exceptions where time did not yet exist within our implementation of Einstein's block universe.

Abstract

This document presents a mathematical formalization of the cosmological hypothesis proposing a closed 4D toroidal geometric structure as a pre-Big Bang state. The corresponding metrics are developed, collapse mechanisms are analyzed, and connections with known physics and falsifiable predictions are established.

1. Mathematical Formalization

1.1 Geometry of the Closed 4D Torus

The proposed structure combines elements of a **4-torus** (T^4) with non-orientable topology similar to the Klein bottle. The most appropriate metric is a generalization of the Clifford torus embedded in curved spacetime.

Generalized Clifford Torus Metric

For a 3-torus embedded in 4D with curvature, the base metric is:

$$ds^2 = -N(r)^2 dt^2 + a(r)^2 d\theta_1^2 + b(r)^2 d\theta_2^2 + c(r)^2 d\theta_3^2 + f(r)^2 dr^2$$

where:

- $\theta_1, \theta_2, \theta_3 \in [0, 2\pi]$ are periodic angular coordinates
- $N(r)$ is the lapse function
- $a(r), b(r), c(r)$ are the curvature radii in each toroidal direction
- r is the radial coordinate toward the central "throat"

Temporal Closure Condition (Closed Loop)

For time to form a closed loop (without temporal arrow), we need **closed timelike curves (CTCs)**. This is achieved with:

$$ds^2 = -N(r)^2(dt + \omega(r)d\phi)^2 + \dots$$

where $\omega(r)$ is a frame-dragging function. When:

$$g_{tt} = -N^2 + \omega^2 g_{\phi\phi} \rightarrow 0$$

CTCs form. The generalized ergosphere boundary defines the temporal loop region.

1.2 Non-Trivial Topology: "Top-Bottom" Connection

The connection between the upper and lower parts of the torus (like a Klein bottle) is described by a **non-orientable manifold**.

Fundamental Group

For a generalized 4D Klein bottle K^4 :

$$\pi_1(K^4) = \langle a, b \mid aba^{-1}b = 1 \rangle$$

This implies that a closed path can reverse orientation, crucial for connecting "top" with "bottom" with parity inversion.

Point Identification

Mathematically, if we use coordinates (t, x, y, z) , the non-orientable identification is:

$$(t, x, y, z) \sim (-t, -x, y, z + L)$$

where L is the period in the z direction. This identification:

- Reverses time ($t \rightarrow -t$)
- Reverses one spatial coordinate ($x \rightarrow -x$)
- Generates the bidirectional "hourglass" topology

1.3 Double Cone Geometry (Hourglass)

The geometry of two cones facing each other at their vertices is described by a modified **Milne double cone** metric:

$$ds^2 = -dt^2 + t^2 [d\chi^2 + \sinh^2\chi (d\theta^2 + \sin^2\theta \, d\phi^2)]$$

For the symmetric bidirectional version, we extend to $t \in \mathbb{R}$ (not just $t > 0$):

$$ds^2 = -dt^2 + |t|^2 [d\chi^2 + \sinh^2\chi \, d\Omega^2]$$

At $t = 0$ (the hourglass vertex), we have a **conical singularity** where both cones converge.

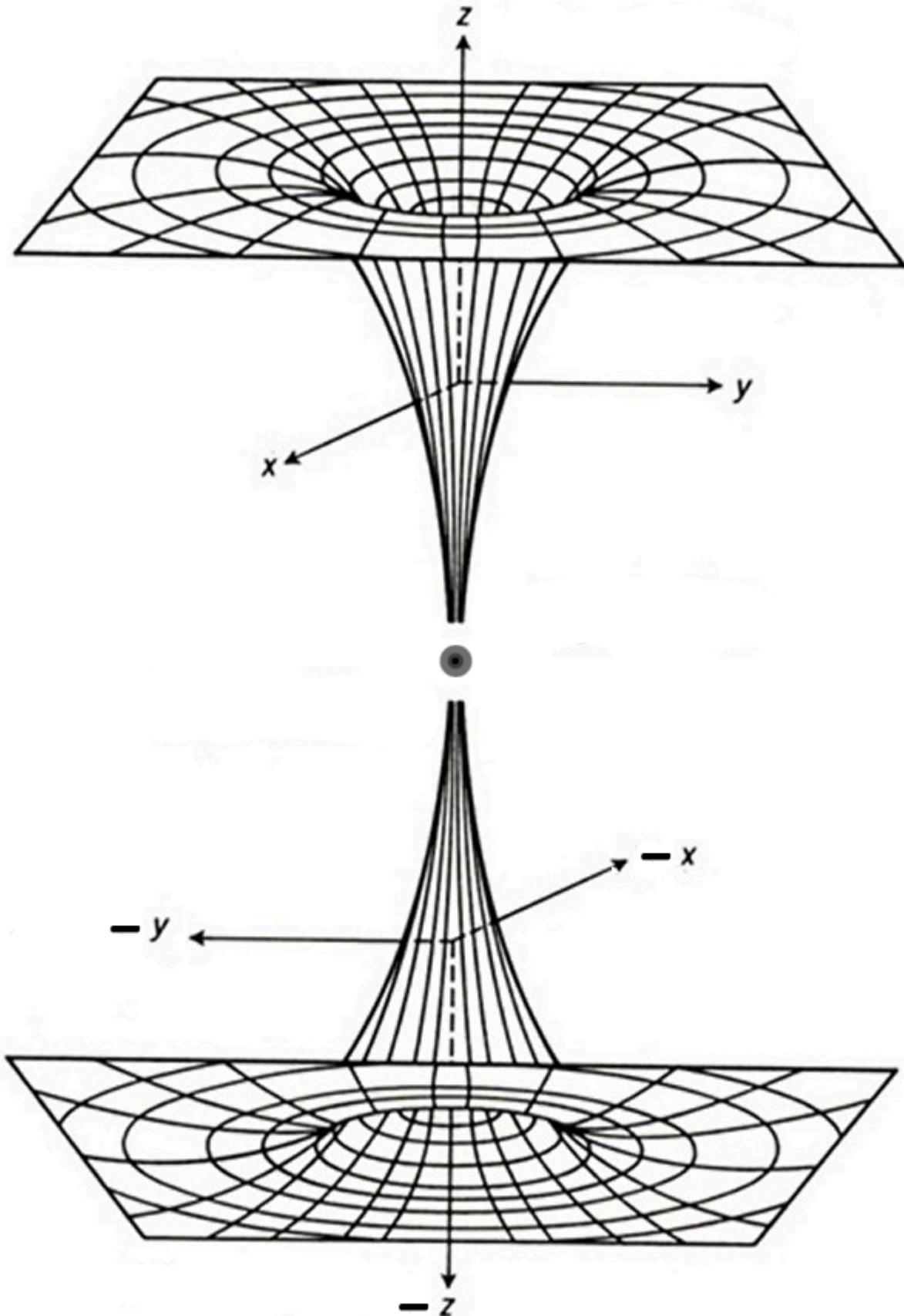


Figure 1: Double cone (hourglass) structure of the primordial state

1.4 Stress-Energy Tensor of the Central Exotic State

The "exotic state" in the torus throat requires a stress-energy tensor with extreme properties.

General Form

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} + \pi_{\mu\nu} + q_\mu u_\nu + u_\mu q_\nu$$

where:

- ρ = energy density (extreme, possibly trans-Planckian)
- p = pressure
- $\pi_{\mu\nu}$ = anisotropic stress tensor
- q_μ = heat flux

Conditions for the Exotic State

To sustain the closed toroidal geometry with CTCs, we need **exotic energy** that violates energy conditions:

Null Energy Condition (NEC) Violation:

$$T_{\mu\nu} k^\mu k^\nu < 0 \quad \text{for some null vector } k^\mu$$

This is equivalent to:

$$\rho + p < 0$$

Exotic Equation of State:

$$w = p/\rho < -1 \quad (\text{phantom energy})$$

Energy Density at the Throat

At the central vertex of the torus, density diverges:

$$\rho(r) = \rho_P \cdot (l_P/r)^\alpha$$

where:

- $\rho_P = c^5/(\hbar G^2) \approx 5.16 \times 10^{96} \text{ kg/m}^3$ (Planck density)
 - $l_P = \sqrt{(\hbar G/c^3)} \approx 1.62 \times 10^{-35} \text{ m}$ (Planck length)
 - $\alpha \geq 4$ for sufficiently strong singularity
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2. Collapse Mechanism

2.1 Closed Loop Stability

The temporally closed toroidal structure is inherently **metastable**. Stability conditions are derived from the second variation of the Einstein-Hilbert action.

Stability Condition

For a static geometry with CTCs, stability requires:

$$\delta^2 S = \int d^4x \sqrt{-g} [\delta g^{\mu\nu} G_{\mu\nu} + \text{quantum corrections}] > 0$$

The problem is that **quantum loop corrections** inevitably destabilize the configuration.

2.2 Stability Breaking Mechanisms

A) Quantum Instability (Most Promising)

The **generalized Hawking effect** in geometries with CTCs generates radiation that drains energy:

$$T_H = \hbar \kappa / (2\pi k_B c)$$

where κ is the surface gravity of the "temporal ergosphere". The accumulated energy loss:

$$dM/dt = -\sigma A \cdot T_H^4$$

eventually destroys the geometry after a characteristic time:

$$\tau_{\text{decay}} \sim M^3 / (\hbar c^4) \times f(\text{topology})$$

B) Quantum Vacuum Fluctuation

Vacuum fluctuations in geometries with CTCs grow exponentially (Cauchy instability):

$$\langle \phi^2 \rangle \sim \exp(t/\tau_{\text{CTC}})$$

where τ_{CTC} is the period of the closed timelike curve.

C) Entropic Limit ("n Cycles" Model)

If energy circulates in a loop n times, the entanglement entropy between degrees of freedom grows:

$$S_{\text{ent}}(n) = S_0 \cdot \log(n) + \text{corrections}$$

When S_{ent} reaches the Bekenstein-Hawking limit:

$$S_{\text{max}} = A/(4l_P^2)$$

the system must "discharge" its information, triggering collapse.

Critical number of cycles:

$$n_{\text{critical}} \sim \exp(A/4l_P^2) \sim \exp(10^{123})$$

(extremely large but finite number)

2.3 Bidirectional Unfolding Modeling

The bidirectional collapse is modeled through a **topological phase transition**.

Dynamic Metric Ansatz

$$ds^2 = -dt^2 + a(t)^2 d\Sigma_+^2 + b(t)^2 d\Sigma_-^2$$

where $d\Sigma_+^2$ and $d\Sigma_-^2$ are the metrics of the two emerging "planes".

Evolution Equations

Modifying the Friedmann equations for two sectors:

$$(\dot{a}/a)^2 = (8\pi G/3)\rho_+ - k_+/a^2 + \Lambda_+/3$$

$$(\dot{b}/b)^2 = (8\pi G/3)\rho_- - k_-/b^2 + \Lambda_-/3$$

with total conservation condition:

$$\rho_+ a^3 + \rho_- b^3 = \rho_{\text{total}} = \text{constant}$$

Bifurcation Condition

Unfolding occurs when the metric signature at the throat changes:

$$\det(g_{\mu\nu}) \rightarrow 0 \quad \text{at } r = r_{\text{critical}}$$

This corresponds to a **signature change surface**, studied in quantum cosmology.

Master Collapse Equation

The collapse dynamics follows:

$$\partial^2 \Phi / \partial t^2 - c^2 \nabla^2 \Phi + V'(\Phi) = 0$$

where Φ is a scalar field parameterizing the topological transition, with potential:

$$V(\Phi) = \lambda(\Phi^2 - v^2)^2$$

The minimum $\Phi = \pm v$ corresponds to the two separated universes.

2.4 The "Size" Problem in Closed 4D

A natural objection to the model is: what is the torus size R ? The critical cycle number depends on this parameter:

For Planck scale ($R \sim l_P$): $n \sim e^1 \approx 2.7$ (almost immediate collapse)
For nuclear scale ($R \sim 10^{-15} \text{ m}$): $n \sim \exp(10^{40})$ (enormous number)

However, this question reveals a deeper conceptual problem.

The Conceptual Problem

In 3D, "size" means length, area, volume, quantities measurable with an external ruler. In 4D with closed time (CTCs), the concept becomes complicated because:

1. **There is no "moment" to measure**, if time is a loop, there is no $t=0$ where "the torus measures X " can be defined
2. **The metric is inside, not outside**, the metric defines distances, but is part of the torus itself
3. **There is no exterior**, there is no external space from which to observe the structure

It is analogous to asking "what is north of the North Pole?". The question assumes a frame of reference that does not apply.

Three Possible Resolutions

Option A: Intrinsic Invariant (4-Volume)

In differential geometry, quantities exist that require no external reference:

$$V_4 = \int d^4x \sqrt{|g|}$$

For a 4-torus with radii R_1, R_2, R_3, R_4 :

$$V_4 = (2\pi)^4 \times R_1 \times R_2 \times R_3 \times R_4$$

The "size" would be $V_4^{(1/4)}$, an effective average radius.

Option B: Energetic Definition

If the torus contains all the energy of the observable universe:

$$E_{\text{total}} = M_{\text{universe}} \times c^2 \approx 10^{53} \text{ kg} \times c^2 \approx 10^{70} \text{ J}$$

By the Schwarzschild mass-radius relation:

$$R_{\text{characteristic}} = 2GM/c^2 \approx 10^{26} \text{ m}$$

Notably, this coincides with the order of magnitude of the current observable universe radius.

Option C: Size as Emergent Property (Preferred Resolution)

The most elegant answer: the question "how big is it?" makes no sense before unfolding.

"Size" only exists after linear time emerges. Before collapse, the torus possesses **topology** and **curvature**, but no measurable extension in the conventional sense. Size is an emergent property of unfolding, not an initial condition.

Implication: Topological vs. Geometric Collapse

If we adopt Option C, the critical cycle number does not depend on a size R , but on intrinsic topological properties:

$$n_{\text{critical}} \sim \exp(S_{\text{topological}})$$

where $S_{\text{topological}}$ is calculated from discrete invariants:

Invariant	Symbol	Description
Betti Number	b_k	Counts "holes" in each dimension
Euler Characteristic	χ	Global topological invariant

Invariant	Symbol	Description
Pontryagin Class	p_k	Measures manifold "torsion"
Orientability Index	σ	+1 orientable, -1 non-orientable

For a non-orientable Klein-type 4-torus:

$$\chi(K^4) = 0$$
$$b_0 = 1, \quad b_1 = 3, \quad b_2 = 4, \quad b_3 = 3, \quad b_4 = 1$$

The topological entropy would be:

$$S_{\text{topological}} = \sum_k b_k \times \log(\dim H_k) + \text{torsion corrections}$$

Profound Consequence

This reformulation implies that **topology determines the torus's fate, not geometry**.

- Topological properties are **discrete** (integers)
- There is no free parameter R to specify
- Collapse is inevitable once topology is fixed
- The "when" of collapse makes no sense, only the "if"

This is analogous to nuclear decay: the half-life is determined by the nucleus structure, not by an external "size". The torus "decays" topologically, not geometrically.

3. Connection with Known Physics

3.1 Schwarzschild and Kerr Solutions

Maximal Schwarzschild Extension

The maximally extended Schwarzschild metric (Kruskal-Szekeres coordinates) already contains the "hourglass" structure:

$$ds^2 = (32M^3/r) e^{-r/2M} (-dT^2 + dX^2) + r^2 d\Omega^2$$

The Penrose diagram shows:

- Region I: Exterior universe (our universe)

- Region II: Black hole interior
- Region III: Parallel universe (the "other plane"?)
- Region IV: White hole interior

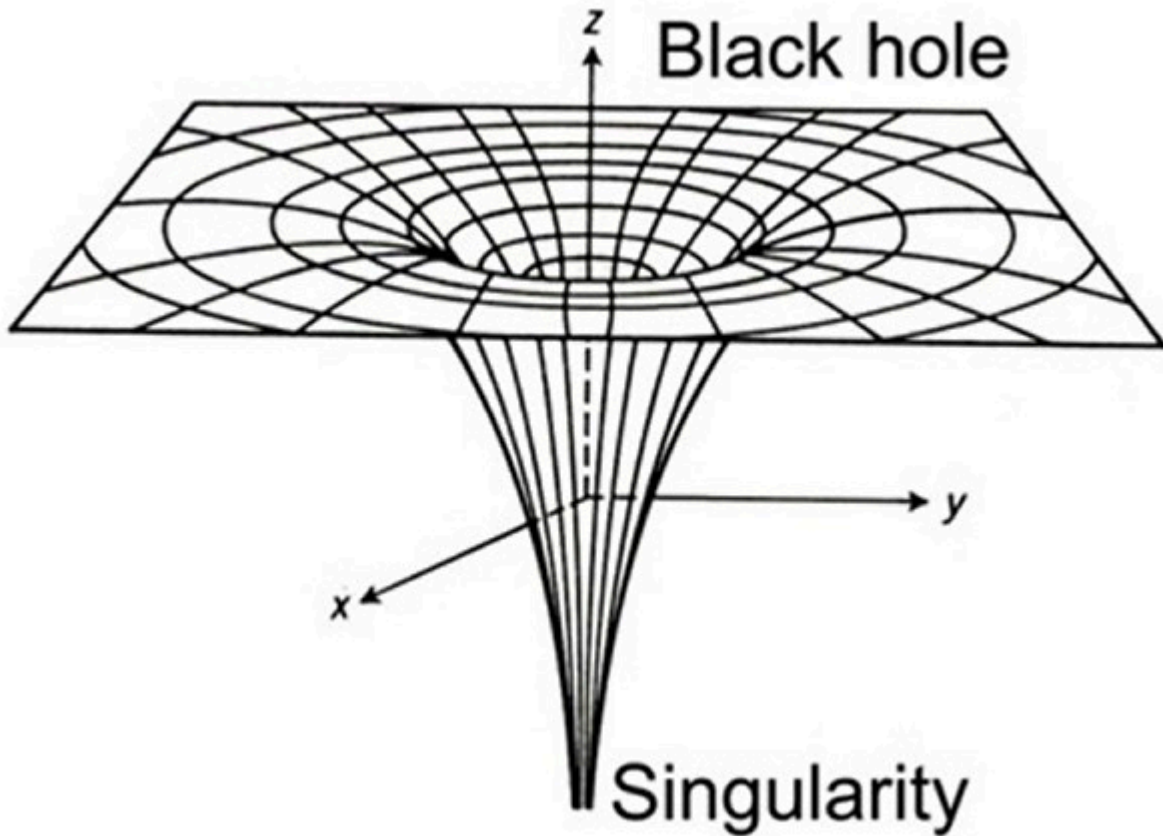


Figure 2: Representation of a black hole as a residual "scar" of primordial geometry

Connection with the hypothesis: Black holes as "scars" would correspond to points where the Region I ↔ Region III connection persists residually.

Einstein-Rosen Bridges (Wormholes)

The proposed original geometry is essentially a **primordial wormhole** connecting the two planes:

$$ds^2 = -dt^2 + dr^2 + (b_0^2 + r^2)(d\theta^2 + \sin^2\theta d\phi^2)$$

where b_0 is the throat radius.

Kerr Solution and Rotation

For rotating black holes (Kerr), the singularity is a **ring**, not a point:

$$\Sigma = r^2 + a^2 \cos^2\theta = 0 \rightarrow r = 0, \theta = \pi/2$$

Passing through the ring connects to regions with CTCs, analogous to the primordial temporal loop.

3.2 Hartle-Hawking Model (No-Boundary Proposal)

The Hartle-Hawking proposal suggests the universe has no initial temporal boundary. The wave function of the universe:

$$\Psi[h_{ij}, \varphi] = \int Dg D\varphi \exp(-I_E[g, \varphi])$$

where I_E is the Euclidean action and the integral is over compact geometries.

Connection: The primordial torus could be the Euclidean geometry dominating the path integral. The torus \rightarrow two universes transition would be the "nucleation" point where Euclidean time becomes Lorentzian.

Gravitational Instanton

The central "exotic state" would be analogous to a **Hawking-Turok instanton**:

$$ds^2 = d\tau^2 + a(\tau)^2 d\Omega_3^2$$

with $a(\tau)$ passing through a minimum (torus throat).

3.3 Loop Quantum Cosmology (LQC)

In LQC, the Big Bang singularity is resolved through a **quantum bounce** when density reaches the critical value:

$$\rho_{\text{critical}} = \rho_P \cdot (\sqrt{3}) / (32\pi^2 \gamma^3) \approx 0.41 \rho_P$$

where $\gamma \approx 0.2375$ is the Immirzi parameter.

Connection: The torus collapse would correspond to the moment where:

$$\rho \rightarrow \rho_{\text{critical}}$$

and quantum gravity corrections (holonomies) produce the bounce/unfolding.

Modified Friedmann Equation in LQC

$$H^2 = (8\pi G/3) \rho (1 - \rho/\rho_{\text{critical}})$$

When $\rho \rightarrow \rho_{\text{critical}}$, $H \rightarrow 0$, indicating the transition point.

3.4 Penrose's Conformal Cyclic Cosmology (CCC)

In Penrose's model, each "aeon" ends in a massless radiation-dominated state, conformally equivalent to the beginning of the next aeon.

Connection: The temporally closed toroidal structure could represent the **connection between aeons** in a modified CCC version:

$$g_{\mu\nu}^{\{(n+1)\}} = \Omega^2 \, g_{\mu\nu}^{\{(n)\}}$$

where Ω is the conformal factor connecting the end of one aeon with the beginning of the next.

3.5 Planck State and Quantum Gravity

Quantum Gravity Condensate

The "exotic state" could be a **graviton condensate** in the non-perturbative regime:

$$|\Psi_{\text{exotic}}\rangle = \exp\left(\int d^3k \, \alpha(k) \, a^\dagger_k\right) |0\rangle$$

where $\alpha(k)$ has support at trans-Planckian scales.

Wheeler Foam

At Planck scale, spacetime is a "foam" of fluctuating topologies:

$$\langle g_{\mu\nu}(x) g_{\rho\sigma}(x') \rangle \sim l_P^4 \, \delta^4(x-x')$$

The primordial torus would be a coherent configuration emerging from this foam.

4. Falsifiable Predictions

4.1 CMB Signatures

A) Antipodal Correlations

If the universe has residual toroidal topology, correlations should exist at antipodal points of the CMB:

$$C(\theta) = \langle T(\hat{n}) T(-\hat{n}) \rangle \neq 0 \text{ for } \theta = \pi$$

Specific prediction: Excess correlation in the two-point function at $\theta = 180^\circ \pm 2^\circ$.

Current status: There are marginal hints of anomalous correlations at large scales ($l < 5$), but not conclusive.

B) Large-Scale Suppressed Modes

A compact initial topology would suppress low- l modes:

$$C_{l^{\text{observed}}}/C_{l^{\Lambda\text{CDM}}} < 1 \quad \text{for } l \leq l_{\text{cutoff}}$$

Prediction: $l_{\text{cutoff}} \sim 2\text{-}4$, consistent with the observed quadrupole ($l=2$) suppression.

C) Concentric Circle Patterns

Following the Penrose-Gurzadyan analysis, search for anomalous temperature rings:

$$\delta T/T \sim 10^{-5} \text{ in circles of angular radius } \theta_n$$

Prediction: The circles would correspond to intersections of our bubble with the "other plane".

4.2 Black Hole Anomalies

A) Asymmetry in Black Hole Mergers

If black holes are "scars" of primordial geometry, they could have a preferred orientation:

$$\langle L \cdot \hat{n} \rangle \neq 0$$

where L is angular momentum and \hat{n} is the direction toward the "other plane".

Prediction: Statistical correlation in spin directions of black holes detected by LIGO/Virgo.

B) Anomalous Emission Near the Horizon

The event horizon could show additional thermal emission due to residual connection:

$$T_{\text{effective}} = T_{\text{Hawking}} \times (1 + \epsilon_{\text{connection}})$$

where $\epsilon_{\text{connection}} \sim l_P/r_s$ is a small correction.

C) Gravitational Wave Echoes

If internal structure exists (connection to the other plane), gravitational waves from mergers would show **echoes**:

$$h(t) = h_{\text{main}}(t) + \sum_n \alpha^n h_{\text{echo}}(t - n\Delta t)$$

with $\Delta t \sim r_s \log(r_s/l_P) / c$

Current status: There are controversial claims of echo detection in LIGO data.

4.3 Matter-Antimatter Asymmetry

If the unfolding was perfectly symmetric, one plane should have matter and the other antimatter:

Prediction: Our universe has an excess of matter because antimatter went to the other plane.

Test: The observed baryonic asymmetry ($\eta \sim 10^{-10}$) should be exactly compensated by antimatter in the other plane. This would eliminate the need for dynamic baryogenesis.

4.4 Specific Experiments

A) Event Horizon Telescope (EHT)

Search for deviations from the Kerr-predicted shadow:

$$r_{\text{shadow}} = r_{\text{Kerr}} \times (1 + \delta_{\text{topological}})$$

Required precision: $\delta < 0.01$, achievable with next-generation EHT.

B) Primordial Gravitational Waves

The primordial gravitational wave spectrum would have a low-frequency cutoff:

$$\Omega_{\text{GW}}(f) \rightarrow 0 \quad \text{for } f < f_{\text{cutoff}} \sim 10^{-18} \text{ Hz}$$

corresponding to the original torus size.

Test: LISA and pulsar timing array detectors (NANOGrav, EPTA).

C) Primordial Nucleosynthesis

If particle exchange between planes occurred near the Big Bang:

$$Y_p = 0.247 + \delta Y_{\text{exchange}}$$

Prediction: Small deviation in primordial helium abundance.

5. Constructive Criticism

5.1 Most Promising Aspects

A) Geometric Elegance

The proposal unifies several concepts:

- Big Bang origin
- Nature of black holes
- Black hole/white hole asymmetry
- Possible resolution of the arrow of time problem

B) Connection with Established Physics

The mathematical structure naturally relates to:

- Maximally extended Schwarzschild solutions
- Einstein-Rosen bridges
- Bounce cosmology (LQC)
- No-boundary proposal (Hartle-Hawking)

C) Falsifiable Predictions

The model makes specific predictions that can be tested:

- Antipodal correlations in CMB
- Gravitational wave echoes
- Asymmetry in black hole orientations

D) Problem Resolution

Potentially explains:

- Why we don't observe white holes
- Matter/antimatter asymmetry
- The origin of the arrow of time

5.2 Problematic Aspects

A) Energy Condition Violation

Maintaining the closed toroidal geometry requires exotic energy that violates NEC/WECC. Although this is quantum mechanically possible (Casimir effect), sustaining it at cosmological scale is problematic.

Quantification of the problem:

$$\int (\rho + p) dV < 0$$

requires trans-Planckian amounts of exotic energy.

B) Chronological Instability

Geometries with CTCs suffer from:

1. **Consistency paradoxes** (although resolvable with Novikov interpretation)
2. **Quantum divergences** in the renormalized stress-energy tensor
3. **Cauchy instability** that would destroy the configuration

C) Lack of Dynamic Mechanism

The hypothesis describes the initial state but does not explain:

- Why that specific geometry?
- What determines the parameters (torus size, etc.)?
- Is the configuration a dynamic attractor?

D) Quantum-Gravitational Connection

In the trans-Planckian regime, classical general relativity does not apply. A complete quantum gravity theory is needed to:

- Describe the "exotic state"
- Model the transition/collapse
- Calculate precise predictions

5.3 Modifications for Greater Robustness

A) Incorporate Loop Quantum Gravity

Use the LQC formalism to regularize singularities:

$$H^2 = (8\pi G/3)\rho(1 - \rho/\rho_c) + \text{holonomy corrections}$$

This would provide a natural bounce mechanism.

B) Field Model

Introduce a scalar field ϕ parameterizing the transition:

$$S = \int d^4x \sqrt{-g} [R/16\pi G - \frac{1}{2}(\partial\phi)^2 - V(\phi)]$$

with $V(\phi)$ having minima corresponding to the two planes.

C) Connection with String Theory

The non-trivial topology could emerge naturally from:

- Calabi-Yau compactifications
- Conifold transitions
- Colliding branes (ekpyrotic model)

D) Thermodynamic Development

Formulate a thermodynamics of the primordial state:

$$dS_{\text{universe}} = dS_{\text{plane}+} + dS_{\text{plane}-} + dS_{\text{connection}}$$

Collapse would be triggered when $dS_{\text{total}} = 0$ becomes unstable.

E) Selection Principle

Add a criterion explaining why this configuration:

- Weak anthropic principle
- Euclidean action minimization
- Dynamic attractor in configuration space

6. Required Future Theoretical Framework

6.1 Aspects Requiring New Physics

1. Non-Perturbative Quantum Gravity

- To describe the trans-Planckian regime
- Candidates: LQG, string theory, dynamical triangulations

2. Thermodynamics of Spacetimes with CTCs

- Generalization of the second law
- Entropy definition for temporal loops

3. Cosmology of Topological Transitions

- Nucleation rates of new topologies
- Signature change dynamics

6.2 Required Experimental Developments

1. **High-precision CMB** (PICO, LiteBIRD)
 2. **Low-frequency gravitational waves** (LISA, DECi-hertz)
 3. **High-resolution black hole imaging** (EHT ngVLA)
 4. **Primordial gravitational wave detection** (CMB-S4)
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7. Conclusions

The primordial 4D torus hypothesis presents a speculative but internally consistent cosmological proposal that:

1. **Can be mathematically formalized** using differential geometry and topology
2. **Connects with established physics** (Schwarzschild, Hartle-Hawking, LQC, CCC)
3. **Generates falsifiable predictions** that are specific and observationally accessible
4. **Requires additional development** in quantum gravity to be fully rigorous

The elegance of unifying the Big Bang origin, the nature of black holes, and the black/white asymmetry under a single geometric framework is notable. The greatest challenges are the need for exotic energy and the lack of an underlying quantum gravity theory.

Overall assessment: The hypothesis deserves additional theoretical development and belongs to the category of "informed speculation" along with models such as the ekpyrotic universe, brane cosmology, or the inflationary multiverse.

Appendix A: Key Equations Summary

Primordial Torus Metric

$$ds^2 = -N^2(dt + \omega d\phi)^2 + f^2 dr^2 + a^2 d\theta_1^2 + b^2 d\theta_2^2 + c^2 d\theta_3^2$$

Temporal Closure Condition

$$g_{tt} + \omega^2 g_{\phi\phi} = 0$$

Exotic Stress-Energy Tensor

$$T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu), \text{ with } \rho + p < 0$$

Collapse Equation

$$\partial^2 \Phi / \partial t^2 = c^2 \nabla^2 \Phi - \lambda(\Phi^2 - v^2)\Phi$$

CMB Prediction

$$C(\pi) / C(0) > [C(\pi) / C(0)]_{\Lambda\text{CDM}}$$

Relevant Theoretical References

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