Primordial 4D Torus Hypothesis: A Pre-Big Bang Cosmological Model

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Abstract

This document presents a mathematical formalization of the cosmological hypothesis proposing a closed 4D toroidal geometric structure as a pre-Big Bang state. The corresponding metrics are developed, collapse mechanisms are analyzed, and connections with known physics and falsifiable predictions are established.

This hypothesis arises from the association of the primordial seed (Φ) with the exotic state described below, in previous studies conducted in The Atlas [8], and from its consequent Singularity Void, a structure we implemented to handle exceptions where time did not yet exist within our implementation of Einstein's block universe.

1 Mathematical Formalization

1.1 Primordial Energy Symmetry Postulate (G_0)

Before describing the geometric structure, we establish the fundamental energy constraint of the primordial state.

1.1.1 The G_0 Equation

The primordial state satisfies exact energy cancellation enforced by non-orientable topology:

$$G_0 = \lim_{n \to n_c} \sum_{k=1}^{n} \underbrace{[(+\epsilon_k) + (-\epsilon_k)]}_{\text{topological pair}} = 0 \tag{1}$$

where:

- ϵ_k represents the k-th quantum fluctuation (energy quanta)
- The brackets $[\cdot]$ denote **simultaneous creation** imposed by the Klein-type topological identification (see Eq. 19)
- $n_c \sim \exp(S_{BH})$ is the critical cycle number before collapse

1.1.2 Physical Meaning of the Brackets

The bracket notation is not merely symbolic. It encodes a fundamental physical constraint arising from the non-orientable topology.

Without brackets (naive summation):

$$\sum_{k=1}^{\infty}(+\epsilon_k) + \sum_{k=1}^{\infty}(-\epsilon_k) = \infty - \infty \quad \text{(undefined)} \tag{2}$$

This leads to the well-known divergence catastrophe, requiring regularization techniques such as zeta function regularization ($\zeta(-1) = -1/12$) used in string theory and Casimir effect calculations.

With brackets (topologically enforced pairing):

$$\sum_{k=1}^{n} [(+\epsilon_k) + (-\epsilon_k)] = \sum_{k=1}^{n}[0] = 0 \quad \text{(exact)} \tag{3}$$

The brackets impose that the creation of $+\epsilon_k$ and $-\epsilon_k$ is a **single, inseparable event**. This is not a computational choice but a geometric necessity: the non-orientable identification (Eq. 19) guarantees that for every point with coordinates $(+t, +x)$, there exists its topological "mirror" at $(-t, -x)$. The cancellation is **geometric**, not merely algebraic.

This mechanism is analogous to supersymmetry, where boson-fermion pairing prevents ultraviolet divergences from appearing in the first place, rather than requiring post-hoc renormalization.

1.1.3 Energy-Entropy Asymmetry: The Collapse Trigger

While the G_0 equation guarantees perfect energy cancellation, entropy behaves fundamentally differently. This asymmetry is the key to understanding why the primordial state collapses.

	Behavior	T-sym
Energy	$\sum E = 0$	Yes
Entropy	$S \sim \log n$	No

Table 1: Energy vs. entropy asymmetry

Energy conservation is exact throughout the primordial state:

$$\frac{d}{dn}\left(\sum_k E_k\right) = 0 \quad \forall n \tag{4}$$

Entanglement entropy accumulates monotonically with each cycle:

$$\frac{dS_{\text{ent}}}{dn} > 0 \quad \forall n \tag{5}$$

Explicitly, as energy circulates through the closed timelike curves (CTCs), it accumulates quantum entanglement with its past states:

$$S_{\text{ent}}(n) = S_0 \cdot \log(n) + \mathcal{O}(1) \tag{6}$$

This logarithmic growth is characteristic of entanglement accumulation in cyclic quantum systems.

1.1.4 Information Saturation and the Bekenstein-Hawking Limit

The topological structure has a maximum information capacity given by the Bekenstein-Hawking bound:

$$S_{\max} = \frac{A}{4l_P^2} \tag{7}$$

where A is the effective "area" of the topological boundary and l_P is the Planck length.

When the accumulated entanglement entropy approaches this limit:

$$S_{\text{ent}}(n) \longrightarrow S_{\max} \tag{8}$$

the system reaches **information saturation**. The topology can no longer contain the accumulated information, triggering the collapse/unfolding event.

The critical cycle number is therefore:

$$n_{\text{critical}} \sim \exp\left(\frac{S_{\max}}{S_0}\right) \sim \exp\left(\frac{A}{4l_P^2}\right) \tag{9}$$

For cosmological scales, this yields $n_{\text{critical}} \sim \exp(10^{123})$—an astronomically large but **finite** number.

1.1.5 The Nature of the Collapse

This analysis leads to a profound reinterpretation of the Big Bang:

> *The Big Bang was not an explosion of energy (which remains zero), but an* ***information discharge****—the moment when the primordial "memory" overflowed its topological container.*

The collapse creates new "space" for information, manifesting as the bidirectional unfolding into two planes (matter/antimatter universes). This is consistent with:

- **Zero-energy universe hypothesis** (Tryon, Guth): Total energy remains zero before and after the Big Bang
- **Holographic principle** ('t Hooft, Susskind): Information content is bounded by surface area
- **Entropic gravity** (Verlinde): Spacetime emerges from information/entropy dynamics

1.1.6 Summary: The Unstable Nothing

The G_0 postulate can be summarized as follows:

> **The primordial state is an "unstable nothing"**: a perfect energy vacuum ($\sum E = 0$) that is nevertheless filled with accumulating information tensions. The "nothing" collapses not because energy balance fails, but because **even the void has a memory limit**.

Mathematically:

$$\text{Energy:} \quad G_0 = \sum[(+\epsilon) + (-\epsilon)] = 0 \quad \text{(stable)} \tag{10}$$

$$\text{Entropy:} \quad S(n) \to S_{\max} \quad \text{as} \quad n \to n_c \quad \text{(unstable)} \tag{11}$$

The arrow of time **emerges** from this entropy accumulation, even within the temporally closed primordial structure. Time's directionality is not imposed externally but arises from the intrinsic asymmetry between energy (symmetric) and information (accumulative).

1.2 Geometry of the Closed 4D Torus

The proposed structure combines elements of a **4-torus** (T^4) with non-orientable topology similar to the Klein bottle. The most appropriate metric is a generalization of the Clifford torus embedded in curved spacetime.

Critical clarification on signature: The primordial state involves a **signature transition** from Euclidean to Lorentzian, analogous to the Hartle-Hawking no-boundary proposal:

- **Euclidean regime** (primordial torus): Signature $(+,+,+,+)$, all coordinates spacelike, enabling compact topology without causal paradoxes in the classical sense
- **Lorentzian regime** (post-collapse): Signature $(-,+,+,+)$, one timelike direction emerges
- **Transition surface** Σ: Where signature changes, satisfying junction conditions

The "CTCs" in our model are more precisely understood as closed curves that become timelike *after* the signature transition begins—they represent the "unwinding" of what were purely spatial loops in the Euclidean phase.

1.2.1 Signature Change Formalism

The transition from Euclidean to Lorentzian is modeled via a degenerate metric at the transition surface Σ:

Euclidean side ($\tau < 0$):

$$ds_E^2 = d\tau^2 + a(\tau)^2 d\Omega^2_{K^2 \times T^2} \tag{12}$$

Transition surface ($\tau = 0$):

$$\det(g_{\mu\nu})\big|_\Sigma = 0 \tag{13}$$

Lorentzian side ($t > 0$, with $\tau \to it$):

$$ds_L^2 = -dt^2 + a(t)^2 d\Omega_3^2 \tag{14}$$

The junction conditions (generalized Israel-Darmois) at Σ ensure continuity of the induced 3-metric and extrinsic curvature where defined. This formalism follows Hayward, Deruelle, and others working on signature-changing spacetimes.

1.2.2 Generalized Clifford Torus Metric

For a 3-torus embedded in 4D with curvature, the base metric is:

$$\begin{aligned} ds^2 = &-N(r)^2 dt^2 + a(r)^2 d\theta_1^2 \\ &+ b(r)^2 d\theta_2^2 + c(r)^2 d\theta_3^2 + f(r)^2 dr^2 \end{aligned} \tag{15}$$

where:

- $\theta_1, \theta_2, \theta_3 \in [0, 2\pi]$ are periodic angular coordinates
- $N(r)$ is the lapse function
- $a(r), b(r), c(r)$ are the curvature radii in each toroidal direction
- r is the radial coordinate toward the central "throat"

1.2.3 Temporal Closure Condition (Closed Loop)

For time to form a closed loop (without temporal arrow), we need **closed timelike curves (CTCs)**. This is achieved with:

$$ds^2 = -N(r)^2(dt + \omega(r)d\phi)^2 + \ldots \tag{16}$$

where $\omega(r)$ is a frame-dragging function. When:

$$g_{tt} = -N^2 + \omega^2 g_{\phi\phi} \to 0 \tag{17}$$

CTCs form. The generalized ergosphere boundary defines the temporal loop region.

1.3 Non-Trivial Topology: "Top-Bottom" Connection

The connection between the upper and lower parts of the torus (like a Klein bottle) is described by a **non-orientable manifold**.

1.3.1 Fundamental Group

The appropriate 4-dimensional non-orientable manifold is the product $K^2 \times T^2$ (Klein bottle times 2-torus), whose fundamental group is:

$$\pi_1(K^2 \times T^2) = \langle a, b, c, d \mid aba^{-1}b = 1, \\ [c, d] = 1, [a, c] = [a, d] = [b, c] = [b, d] = 1\rangle \quad (18)$$

where:

- Generators a, b encode the Klein bottle K^2 with its characteristic orientation-reversing identification ($aba^{-1}b = 1$)
- Generators c, d encode the 2-torus T^2 factor ($[c, d] = 1$)
- The commutation relations ensure the product structure

This construction preserves the essential Klein-type property: traversing a closed path involving generator a reverses orientation, crucial for connecting "top" with "bottom" with parity inversion.

1.3.2 Point Identification

Mathematically, if we use coordinates (t, x, y, z), the non-orientable identification requires an **odd number of sign reversals** to achieve $\det(J) = -1$:

$$(t, x, y, z) \sim (-t, -x, -y, z + L) \quad (19)$$

where L is the period in the z direction. The Jacobian of this transformation:

$$J = \mathrm{diag}(-1, -1, -1, +1), \quad \det(J) = -1 \quad (20)$$

confirms genuine orientation reversal. This identification:

- Reverses time ($t \to -t$): connects future with past
- Reverses two spatial coordinates ($x \to -x$, $y \to -y$): implements CPT-like transformation
- Preserves one spatial direction (z): the "axis" of the Klein-type structure
- Generates the bidirectional "hourglass" topology with true non-orientability

Note: The previous formulation $(t, x, y, z) \sim (-t, -x, y, z + L)$ had $\det(J) = +1$, which preserves orientation. The corrected form ensures topological consistency with $K^2 \times T^2$.

1.4 Double Cone Geometry (Hourglass)

The geometry of two cones facing each other at their vertices is described by a modified **Milne double cone** metric:

$$ds^2 = -dt^2 \\ + t^2[d\chi^2 + \sinh^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)] \quad (21)$$

For the symmetric bidirectional version, we extend to $t \in \mathbb{R}$ (not just $t > 0$):

$$ds^2 = -dt^2 + |t|^2[d\chi^2 + \sinh^2\chi\, d\Omega^2] \quad (22)$$

At $t = 0$ (the hourglass vertex), we have a **conical singularity** where both cones converge.

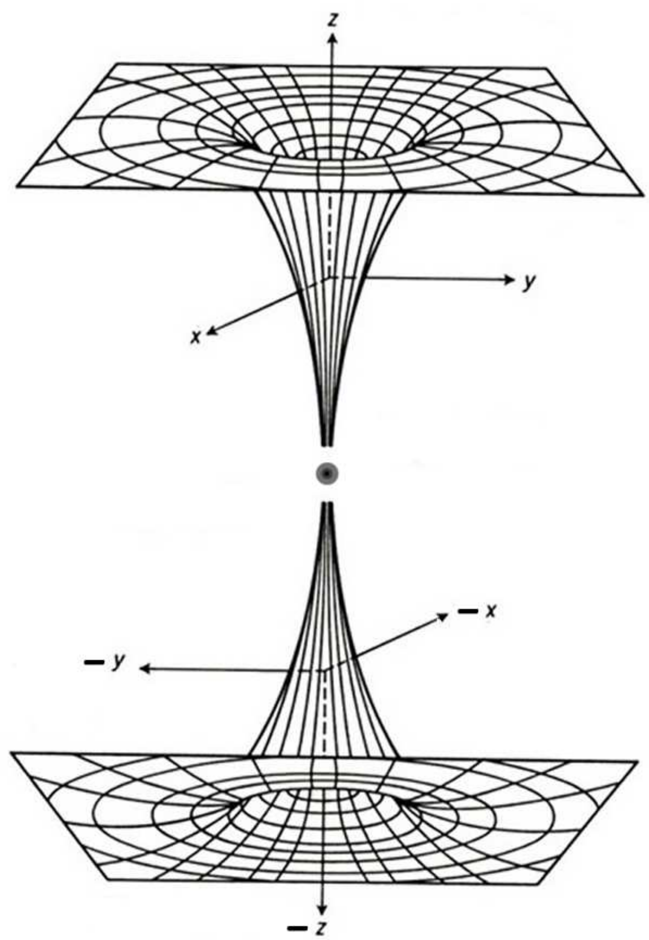

Figure 1: Double cone (hourglass) structure of the primordial state

1.5 Stress-Energy Tensor of the Central Exotic State

The "exotic state" in the torus throat requires a stress-energy tensor with extreme properties.

1.5.1 General Form

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} + \pi_{\mu\nu} + q_\mu u_\nu + u_\mu q_\nu \tag{23}$$

where:

- ρ = energy density (extreme, possibly trans-Planckian)
- p = pressure
- $\pi_{\mu\nu}$ = anisotropic stress tensor
- q_μ = heat flux

1.5.2 Conditions for the Exotic State

To sustain the closed toroidal geometry with CTCs, we need **exotic energy** that violates energy conditions:

Null Energy Condition (NEC) Violation:

$$T_{\mu\nu}k^\mu k^\nu < 0 \quad \text{for some null vector } k^\mu \tag{24}$$

This is equivalent to:

$$\rho + p < 0 \tag{25}$$

Exotic Equation of State:

$$w = \frac{p}{\rho} < -1 \quad \text{(phantom energy)} \tag{26}$$

1.5.3 Energy Density at the Throat

At the central vertex of the torus, density diverges:

$$\rho(r) = \rho_P \cdot \left(\frac{l_P}{r}\right)^\alpha \tag{27}$$

where:

- $\rho_P = c^5/(\hbar G^2) \approx 5.16 \times 10^{96}$ kg/m^3 (Planck density)
- $l_P = \sqrt{\hbar G/c^3} \approx 1.62 \times 10^{-35}$ m (Planck length)
- $\alpha \geq 4$ for sufficiently strong singularity

2 Collapse Mechanism

2.1 Closed Loop Stability

The temporally closed toroidal structure is inherently **metastable**. Stability conditions are derived from the second variation of the Einstein-Hilbert action.

2.1.1 Stability Condition

For a static geometry with CTCs, stability requires:

$$\delta^2 S = \int d^4x\sqrt{-g}\big[\delta g^{\mu\nu} G_{\mu\nu} + \text{quantum corrections}\big] > 0 \tag{28}$$

The problem is that **quantum loop corrections** inevitably destabilize the configuration.

2.2 Stability Breaking Mechanisms

2.2.1 A) Quantum Instability (Most Promising)

The **generalized Hawking effect** in geometries with CTCs generates radiation that drains energy:

$$T_H = \frac{\hbar\kappa}{2\pi k_B c} \tag{29}$$

where κ is the surface gravity of the "temporal ergosphere". The accumulated energy loss:

$$\frac{dM}{dt} = -\sigma A \cdot T_H^4 \tag{30}$$

eventually destroys the geometry after a characteristic time:

$$\tau_{\text{decay}} \sim \frac{M^3}{\hbar c^4} \times f(\text{topology}) \tag{31}$$

2.2.2 B) Quantum Vacuum Fluctuation

Vacuum fluctuations in geometries with CTCs grow exponentially (Cauchy instability):

$$\langle\phi^2\rangle \sim \exp(t/\tau_{\text{CTC}}) \tag{32}$$

where τ_{CTC} is the period of the closed timelike curve.

2.2.3 C) Entropic Limit ("n Cycles" Model)

If energy circulates in a loop n times, the entanglement entropy between degrees of freedom grows:

$$S_{\text{ent}}(n) = S_0 \cdot \log(n) + \text{corrections} \tag{33}$$

When S_{ent} reaches the Bekenstein-Hawking limit:

$$S_{\max} = \frac{A}{4l_P^2} \tag{34}$$

the system must "discharge" its information, triggering collapse.

Critical number of cycles:

$$n_{\text{critical}} \sim \exp\left(\frac{A}{4l_P^2}\right) \sim \exp(10^{123}) \quad (35)$$

(extremely large but finite number)

2.3 Bidirectional Unfolding Modeling

The bidirectional collapse is modeled through a **topological phase transition**.

2.3.1 Dynamic Metric Ansatz

$$ds^2 = -dt^2 + a(t)^2 d\Sigma_+^2 + b(t)^2 d\Sigma_-^2 \quad (36)$$

where $d\Sigma_+^2$ and $d\Sigma_-^2$ are the metrics of the two emerging "planes".

2.3.2 Evolution Equations

Modifying the Friedmann equations for two sectors:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_+ - \frac{k_+}{a^2} + \frac{\Lambda_+}{3} \quad (37)$$

$$\left(\frac{\dot{b}}{b}\right)^2 = \frac{8\pi G}{3}\rho_- - \frac{k_-}{b^2} + \frac{\Lambda_-}{3} \quad (38)$$

with total conservation condition:

$$\rho_+ a^3 + \rho_- b^3 = \rho_{\text{total}} = \text{constant} \quad (39)$$

2.3.3 Bifurcation Condition

Unfolding occurs when the metric signature at the throat changes:

$$\det(g_{\mu\nu}) \to 0 \quad \text{at } r = r_{\text{critical}} \quad (40)$$

This corresponds to a **signature change surface**, studied in quantum cosmology.

2.3.4 Master Collapse Equation

The collapse dynamics follows:

$$\frac{\partial^2 \Phi}{\partial t^2} - c^2 \nabla^2 \Phi + V'(\Phi) = 0 \quad (41)$$

where Φ is a scalar field parameterizing the topological transition, with potential:

$$V(\Phi) = \lambda(\Phi^2 - v^2)^2 \quad (42)$$

The minimum $\Phi = \pm v$ corresponds to the two separated universes.

2.4 The "Size" Problem in Closed 4D

A natural objection to the model is: what is the torus size R? The critical cycle number depends on this parameter:

- For Planck scale ($R \sim l_P$): $n \sim e^1 \approx 2.7$ (almost immediate collapse)
- For nuclear scale ($R \sim 10^{-15}$ m): $n \sim \exp(10^{40})$ (enormous number)

However, this question reveals a deeper conceptual problem.

2.4.1 The Conceptual Problem

In 3D, "size" means length, area, volume, quantities measurable with an external ruler. In 4D with closed time (CTCs), the concept becomes complicated because:

1. **There is no "moment" to measure**: if time is a loop, there is no $t = 0$ where "the torus measures X" can be defined
2. **The metric is inside, not outside**: the metric defines distances, but is part of the torus itself
3. **There is no exterior**: there is no external space from which to observe the structure

It is analogous to asking "what is north of the North Pole?". The question assumes a frame of reference that does not apply.

2.4.2 Three Possible Resolutions

Option A: Intrinsic Invariant (4-Volume)

In differential geometry, quantities exist that require no external reference:

$$V_4 = \int d^4x \sqrt{|g|} \quad (43)$$

For a 4-torus with radii R_1, R_2, R_3, R_4:

$$V_4 = (2\pi)^4 \times R_1 \times R_2 \times R_3 \times R_4 \quad (44)$$

The "size" would be $V_4^{1/4}$, an effective average radius.

Option B: Energetic Definition

If the torus contains all the energy of the observable universe:

$$E_{\text{total}} = M_{\text{universe}} \times c^2 \approx 10^{53}\ \text{kg} \times c^2 \approx 10^{70}\ \text{J} \quad (45)$$

By the Schwarzschild mass-radius relation:

$$R_{\text{characteristic}} = \frac{2GM}{c^2} \approx 10^{26}\ \text{m} \quad (46)$$

Notably, this coincides with the order of magnitude of the current observable universe radius.

Option C: Size as Emergent Property (Preferred Resolution)

The most elegant answer: the question "how big is it?" makes no sense before unfolding.

"Size" only exists after linear time emerges. Before collapse, the torus possesses **topology** and **curvature**, but no measurable extension in the conventional sense. Size is an emergent property of unfolding, not an initial condition.

2.4.3 Implication: Topological vs. Geometric Collapse

If we adopt Option C, the critical cycle number does not depend on a size R, but on intrinsic topological properties:

$$n_{\text{critical}} \sim \exp(S_{\text{topological}}) \quad (47)$$

where $S_{\text{topological}}$ is calculated from discrete invariants:

Invariant	Sym.	Description
Betti Number	b_k	Holes per dimension
Euler Char.	χ	Global invariant
Pontryagin Class	p_k	Manifold torsion
Orientability	σ	± 1

For the non-orientable manifold $M^4 = K^2 \times T^2$, the Betti numbers are computed via the Künneth theorem. With K^2 having $b_0 = 1, b_1 = 1, b_2 = 0$ (over $\mathbb{R}$) and T^2 having $b_0 = 1, b_1 = 2, b_2 = 1$:

$$\chi(K^2 \times T^2) = \chi(K^2) \cdot \chi(T^2) = 0 \cdot 0 = 0 \quad (48)$$

$$b_0 = 1, \quad b_1 = 3, \quad b_2 = 3, \quad b_3 = 1, \quad b_4 = 0 \quad (49)$$

Critical consistency check: $b_4 = 0$ confirms non-orientability (for a compact orientable n-manifold, $b_n = 1$; for non-orientable, $b_n = 0$).

Hidden structure: The integral homology $H_1(K^2; \mathbb{Z}) = \mathbb{Z} \oplus \mathbb{Z}_2$ contains torsion that vanishes with real coefficients but contributes to the analytic torsion (see below).

The topological entropy is formulated using the **Ray-Singer analytic torsion**, a genuine topological invariant:

$$S_{\text{top}} = k_B \log T(M^4) \quad (50)$$

where the analytic torsion $T(M^4)$ is defined via spectral zeta functions:

$$\log T(M) = \frac{1}{2} \sum_{q=0}^{4} (-1)^q \cdot q \cdot \zeta'_{\Delta_q}(0) \quad (51)$$

Here $\zeta_{\Delta_q}(s) = \sum_{\lambda_j > 0} \lambda_j^{-s}$ is the spectral zeta function of the Hodge Laplacian Δ_q acting on q-forms, and the prime denotes derivative at $s = 0$.

Physical interpretation: By the Cheeger-Müller theorem, $T(M) = \tau(M)$ equals the Reidemeister torsion, which captures:

- The $\mathbb{Z}_2$-torsion in $H_1(K^2)$ (orientation-reversal "memory")
- Information beyond Betti numbers that accumulates with CTC cycles

The collapse condition becomes:

$$S_{\text{top}}(n) + S_{\text{ent}}(n) \longrightarrow S_{\max} = \frac{A}{4 l_P^2} \quad (52)$$

where $S_{\text{ent}}(n) \sim \log(n)$ is the entanglement entropy from Eq. 6, and the Bekenstein-Hawking bound provides the holographic limit (consistent with Ryu-Takayanagi in appropriate contexts).

2.4.4 Profound Consequence

This reformulation implies that **topology determines the torus's fate, not geometry**.

- Topological properties are **discrete** (integers)
- There is no free parameter R to specify
- Collapse is inevitable once topology is fixed
- The "when" of collapse makes no sense, only the "if"

This is analogous to nuclear decay: the half-life is determined by the nucleus structure, not by an external "size". The torus "decays" topologically, not geometrically.

3 Connection with Known Physics

3.1 Schwarzschild and Kerr Solutions

3.1.1 Maximal Schwarzschild Extension

The maximally extended Schwarzschild metric (Kruskal-Szekeres coordinates) already contains the "hourglass" structure:

$$ds^2 = \frac{32M^3}{r} e^{-r/2M}(-dT^2 + dX^2) + r^2 d\Omega^2 \quad (53)$$

The Penrose diagram shows:

- Region I: Exterior universe (our universe)
- Region II: Black hole interior
- Region III: Parallel universe (the "other plane"?)
- Region IV: White hole interior

Connection with the hypothesis: Black holes as "scars" would correspond to points where the Region I $\leftrightarrow$ Region III connection persists residually.

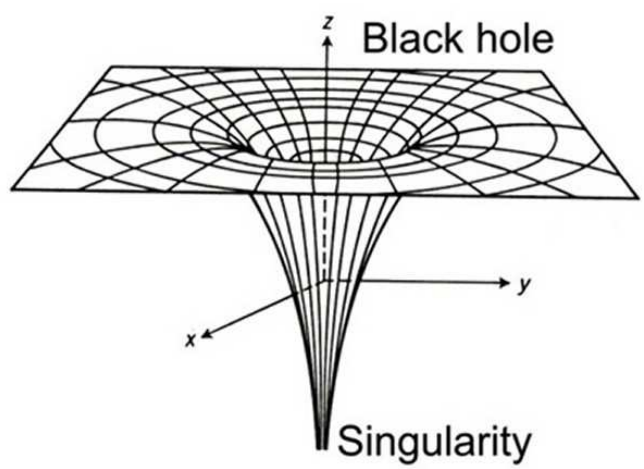

Figure 2: Representation of a black hole as a residual "scar" of primordial geometry

3.1.2 Einstein-Rosen Bridges (Wormholes)

The proposed original geometry is essentially a **primordial wormhole** connecting the two planes:

$$ds^2 = -dt^2 + dr^2 + (b_0^2 + r^2)(d\theta^2 + \sin^2\theta\, d\phi^2) \quad (54)$$

where b_0 is the throat radius.

3.1.3 Kerr Solution and Rotation

For rotating black holes (Kerr), the singularity is a **ring**, not a point:

$$\Sigma = r^2 + a^2\cos^2\theta = 0 \quad \rightarrow \quad r = 0,\, \theta = \pi/2 \quad (55)$$

Passing through the ring connects to regions with CTCs, analogous to the primordial temporal loop.

3.2 Hartle-Hawking Model (No-Boundary Proposal)

The Hartle-Hawking proposal [2] suggests the universe has no initial temporal boundary (see [7] for a comprehensive modern review). The wave function of the universe:

$$\Psi[h_{ij}, \phi] = \int \mathcal{D}g\, \mathcal{D}\phi\, e^{-I_E[g,\phi]} \quad (56)$$

where I_E is the Euclidean action and the integral is over compact geometries.

Connection: The primordial torus could be the Euclidean geometry dominating the path integral. The torus $\rightarrow$ two universes transition would be the "nucleation" point where Euclidean time becomes Lorentzian.

3.2.1 Gravitational Instanton

The central "exotic state" would be analogous to a **Hawking-Turok instanton**:

$$ds^2 = d\tau^2 + a(\tau)^2 d\Omega_3^2 \quad (57)$$

with $a(\tau)$ passing through a minimum (torus throat).

3.3 Loop Quantum Cosmology (LQC)

Important clarification: The following presents an *analogy* with LQC, not a derivation from it. LQC and the primordial torus model have fundamentally different foundations.

In LQC, the Big Bang singularity is resolved through a **quantum bounce** when density reaches the critical value:

$$\rho_{\text{crit}} = \rho_P \cdot \frac{\sqrt{3}}{32\pi^2\gamma^3} \approx 0.41\rho_P \quad (58)$$

where $\gamma \approx 0.2375$ is the Immirzi parameter.

3.3.1 Modified Friedmann Equation in LQC

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\text{critical}}}\right) \tag{59}$$

When $\rho \to \rho_{\text{critical}}$, $H \to 0$, indicating the bounce point.

3.3.2 Structural Analogy (Not Derivation)

	LQC	**This Model**
Bounce	Holonomies	Info. saturation
Threshold	$\rho \to \rho_c$	$S \to S_{\max}$
Topology	Fixed T^3	$K^2 \times T^2$
CTCs	Absent	Essential
Basis	From LQG	Postulated

The analogy: Both models predict a transition when a critical threshold is reached. In LQC, it is energy density bounded by holonomies; in our model, it is information capacity bounded by topology.

What is NOT claimed: We do not derive our collapse mechanism from LQC holonomies. The mathematical similarity in the modified Friedmann equation is *suggestive* but not *foundational*. A rigorous connection would require embedding both frameworks in a common quantum gravity theory.

3.4 Penrose's Conformal Cyclic Cosmology (CCC)

In Penrose's model, each "aeon" ends in a massless radiation-dominated state, conformally equivalent to the beginning of the next aeon.

Connection: The temporally closed toroidal structure could represent the **connection between aeons** in a modified CCC version:

$$g_{\mu\nu}^{(n+1)} = \Omega^2 g_{\mu\nu}^{(n)} \tag{60}$$

where Ω is the conformal factor connecting the end of one aeon with the beginning of the next.

3.5 Planck State and Quantum Gravity

3.5.1 Quantum Gravity Condensate

The "exotic state" could be a **graviton condensate** in the non-perturbative regime:

$$|\Psi_{\text{ex}}\rangle = \exp\left(\int d^3k\, \alpha(k) a_k^\dagger\right)|0\rangle \tag{61}$$

where $\alpha(k)$ has support at trans-Planckian scales.

3.5.2 Wheeler Foam

At Planck scale, spacetime is a "foam" of fluctuating topologies:

$$\langle g_{\mu\nu}(x) g_{\rho\sigma}(x')\rangle \sim l_P^4\, \delta^4(x - x') \tag{62}$$

The primordial torus would be a coherent configuration emerging from this foam.

4 Falsifiable Predictions

4.1 CMB Signatures

4.1.1 A) Antipodal Correlations

If the universe has residual toroidal topology, correlations should exist at antipodal points of the CMB:

$$C(\theta) = \langle T(\hat{n}) T(-\hat{n})\rangle \neq 0 \quad \text{for } \theta = \pi \tag{63}$$

Specific prediction: Excess correlation in the two-point function at $\theta = 180 \pm 2$.

Current status: There are marginal hints of anomalous correlations at large scales ($l < 5$), but not conclusive.

4.1.2 B) Large-Scale Suppressed Modes

A compact initial topology would suppress low-l modes:

$$\frac{C_l^{\text{observed}}}{C_l^{\Lambda\text{CDM}}} < 1 \quad \text{for } l \leq l_{\text{cutoff}} \tag{64}$$

Prediction: $l_{\text{cutoff}} \sim 2$–4, consistent with the observed quadrupole ($l = 2$) suppression.

4.1.3 C) Concentric Circle Patterns

Following the Penrose-Gurzadyan analysis, search for anomalous temperature rings:

$$\frac{\delta T}{T} \sim 10^{-5} \text{ in circles of angular radius } \theta_n \tag{65}$$

Prediction: The circles would correspond to intersections of our bubble with the "other plane".

4.2 Black Hole Anomalies

4.2.1 A) Asymmetry in Black Hole Mergers

If black holes are "scars" of primordial geometry, they could have a preferred orientation:

$$\langle \vec{L} \cdot \hat{n}\rangle \neq 0 \tag{66}$$

where $\vec{L}$ is angular momentum and $\hat{n}$ is the direction toward the "other plane".

Prediction: Statistical correlation in spin directions of black holes detected by LIGO/Virgo.

4.2.2 B) Anomalous Emission Near the Horizon

The event horizon could show additional thermal emission due to residual connection:

$$T_{\text{effective}} = T_{\text{Hawking}} \times (1 + \varepsilon_{\text{connection}}) \tag{67}$$

where $\varepsilon_{\text{connection}} \sim l_P/r_s$ is a small correction.

4.2.3 C) Gravitational Wave Echoes

If internal structure exists (connection to the other plane), gravitational waves from mergers would show **echoes**:

$$h(t) = h_{\text{main}}(t) + \sum_n \alpha^n h_{\text{echo}}(t - n\Delta t) \tag{68}$$

with $\Delta t \sim r_s \log(r_s/l_P)/c$

Current status: There are controversial claims of echo detection in LIGO data.

4.3 Matter-Antimatter Asymmetry

If the unfolding was perfectly symmetric, one plane should have matter and the other antimatter:

Prediction: Our universe has an excess of matter because antimatter went to the other plane.

Test: The observed baryonic asymmetry ($\eta \sim 10^{-10}$) should be exactly compensated by antimatter in the other plane. This would eliminate the need for dynamic baryogenesis.

4.4 Specific Experiments

4.4.1 A) Event Horizon Telescope (EHT)

Search for deviations from the Kerr-predicted shadow:

$$r_{\text{shadow}} = r_{\text{Kerr}} \times (1 + \delta_{\text{topological}}) \tag{69}$$

Required precision: $\delta < 0.01$, achievable with next-generation EHT.

4.4.2 B) Primordial Gravitational Waves

The primordial gravitational wave spectrum would have a low-frequency cutoff:

$$\Omega_{\text{GW}}(f) \to 0 \quad \text{for } f < f_{\text{cutoff}} \sim 10^{-18} \text{ Hz} \tag{70}$$

corresponding to the original torus size.

Test: LISA and pulsar timing array detectors (NANOGrav, EPTA).

4.4.3 C) Primordial Nucleosynthesis

If particle exchange between planes occurred near the Big Bang:

$$Y_p = 0.247 + \delta Y_{\text{exchange}} \tag{71}$$

Prediction: Small deviation in primordial helium abundance.

5 Constructive Criticism

5.1 Most Promising Aspects

5.1.1 A) Geometric Elegance

The proposal unifies several concepts:

- Big Bang origin
- Nature of black holes
- Black hole/white hole asymmetry
- Possible resolution of the arrow of time problem

5.1.2 B) Connection with Established Physics

The mathematical structure naturally relates to:

- Maximally extended Schwarzschild solutions
- Einstein-Rosen bridges
- Bounce cosmology (LQC)
- No-boundary proposal (Hartle-Hawking)

5.1.3 C) Falsifiable Predictions

The model makes specific predictions that can be tested:

- Antipodal correlations in CMB
- Gravitational wave echoes
- Asymmetry in black hole orientations

5.1.4 D) Problem Resolution

Potentially explains:

- Why we don't observe white holes
- Matter/antimatter asymmetry
- The origin of the arrow of time

5.2 Problematic Aspects

5.2.1 A) Energy Condition Violation

Maintaining the closed toroidal geometry requires exotic energy that violates NEC/WEC. Although this is quantum mechanically possible (Casimir effect), sustaining it at cosmological scale is problematic.

Quantification of the problem: Sustaining this geometry requires:

$$\int (\rho + p)\, dV < 0 \tag{72}$$

which demands trans-Planckian amounts of exotic energy.

Partial response: Inapplicability of temporal stability arguments

The standard objection that exotic matter is "unstable and decays" implicitly assumes a linear temporal framework. However, in the primordial state with saturated CTCs, this assumption does not hold.

This follows the same logic as the "size" problem (section 2.4): asking "how big is the torus?" is a malformed question because it assumes an external reference frame that does not exist. Similarly, asking "how long does exotic matter last before decaying?" assumes a temporal arrow that does not exist in the primordial regime. Decay is inherently a temporal process requiring a "before" (exists) and an "after" (no longer exists). Without this directionality, exotic matter does not need to "endure" because "enduring" is not an applicable concept.

The energy conditions (NEC, WEC) are typically derived from quantum vacuum stability requirements—which in turn presuppose a globally hyperbolic causal structure with well-defined Cauchy surfaces. In the absence of this structure, the status of these conditions as physical "laws" becomes questionable.

Exotic matter does not need to survive in our universe. It only needs to have been possible in a regime where the rules that prohibit it (the energy conditions and the very concept of temporal instability) did not yet apply. The laws that make the initial state impossible only emerge as a consequence of departing from that state.

Limitation: This argument addresses the naive stability objection but does not resolve quantum divergences in the renormalized stress-energy tensor on CTC geometries (Cauchy instability), which is a pathology of the field theory itself rather than a temporal process. A complete resolution requires a non-perturbative quantum gravity framework.

5.2.2 B) Chronological Instability and the Chronology Protection Conjecture

Geometries with CTCs suffer from:

1. **Consistency paradoxes** (although resolvable with Novikov self-consistency principle)
2. **Quantum divergences** in the renormalized stress-energy tensor
3. **Cauchy instability** that would destroy the configuration

Hawking's Chronology Protection Conjecture (CPC) states that the laws of physics conspire to prevent CTCs from forming in macroscopic regions. The standard arguments rely on:

- Quantum vacuum polarization diverging at the chronology horizon
- The Averaged Null Energy Condition (ANEC) holding for quantum fields
- Global hyperbolicity as a consistency requirement for QFT

Our position: We do not claim to *evade* CPC. Rather, we propose a regime where its *premises do not apply*:

1. **No Cauchy surface**: CPC arguments require a globally hyperbolic region approaching a chronology horizon. The primordial torus has no such structure—it is *entirely* non-globally-hyperbolic.
2. **ANEC inapplicability**: ANEC is proven for complete null geodesics. In compact CTCs, all geodesics are closed and finite, making the "averaging" procedure ill-defined.

3. **Emergent laws argument**: The physical laws that enforce CPC (stable QFT vacuum, causal propagation) may themselves *emerge* from the collapse. One cannot use emergent laws to prohibit their own precondition.

Acknowledged limitation: The "emergent laws" argument is philosophically coherent but **operationally non-falsifiable** in this specific aspect. This is an epistemological weakness we explicitly recognize. The model's falsifiability rests on its *post-collapse predictions* (CMB signatures, gravitational wave echoes), not on direct observation of the primordial regime.

5.2.3 C) Lack of Dynamic Mechanism

The hypothesis describes the initial state but does not explain:

- Why that specific geometry?
- What determines the parameters (torus size, etc.)?
- Is the configuration a dynamic attractor?

5.2.4 D) Quantum-Gravitational Connection

In the trans-Planckian regime, classical general relativity does not apply. A complete quantum gravity theory is needed to:

- Describe the "exotic state"
- Model the transition/collapse
- Calculate precise predictions

5.3 Modifications for Greater Robustness

5.3.1 A) Incorporate Loop Quantum Gravity

Use the LQC formalism to regularize singularities:

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) + \text{holonomy corrections} \quad (73)$$

This would provide a natural bounce mechanism.

5.3.2 B) Field Model

Introduce a scalar field ϕ parameterizing the transition:

$$S = \int d^4x\sqrt{-g}\Big[\frac{R}{16\pi G} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\Big] \quad (74)$$

with $V(\phi)$ having minima corresponding to the two planes.

5.3.3 C) Connection with String Theory

The non-trivial topology could emerge naturally from:

- Calabi-Yau compactifications
- Conifold transitions
- Colliding branes (ekpyrotic model)

5.3.4 D) Thermodynamic Development

Formulate a thermodynamics of the primordial state:

$$dS_{\text{univ}} = dS_+ + dS_- + dS_{\text{conn}} \quad (75)$$

Collapse would be triggered when $dS_{\text{total}} = 0$ becomes unstable.

5.3.5 E) Selection Principle

Add a criterion explaining why this configuration:

- Weak anthropic principle
- Euclidean action minimization
- Dynamic attractor in configuration space

6 Required Future Theoretical Framework

6.1 Aspects Requiring New Physics

1. **Non-Perturbative Quantum Gravity**
   - To describe the trans-Planckian regime
   - Candidates: LQG, string theory, dynamical triangulations
2. **Thermodynamics of Spacetimes with CTCs**

- Generalization of the second law
- Entropy definition for temporal loops

3. **Cosmology of Topological Transitions**
    - Nucleation rates of new topologies
    - Signature change dynamics

6.2 Required Experimental Developments

1. **High-precision CMB** (PICO, LiteBIRD)
2. **Low-frequency gravitational waves** (LISA, DECi-hertz)
3. **High-resolution black hole imaging** (EHT ngVLA)
4. **Primordial gravitational wave detection** (CMB-S4)

7 Conclusions

The primordial 4D torus hypothesis presents a speculative but internally consistent cosmological proposal that:

1. **Can be mathematically formalized** using differential geometry and topology
2. **Connects with established physics** (Schwarzschild, Hartle-Hawking, LQC, CCC)
3. **Generates falsifiable predictions** that are specific and observationally accessible
4. **Requires additional development** in quantum gravity to be fully rigorous

The elegance of unifying the Big Bang origin, the nature of black holes, and the black/white asymmetry under a single geometric framework is notable. The greatest challenges are the need for exotic energy and the lack of an underlying quantum gravity theory.

Overall assessment: The hypothesis deserves additional theoretical development and belongs to the category of "informed speculation" along with models such as the ekpyrotic universe, brane cosmology, or the inflationary multiverse.

Appendix A: Key Equations Summary

Primordial Energy Symmetry (G_0)

$$G_0 = \lim_{n \to n_c} \sum_{k=1}^{n} [(+\epsilon_k) + (-\epsilon_k)] = 0 \tag{76}$$

Energy-Entropy Asymmetry

$$\begin{aligned} \text{Energy:} \quad & \sum E = 0 \quad \text{(exact, T-symmetric)} \\ \text{Entropy:} \quad & S(n) \sim \log(n) \quad \text{(accumulative, T-asymmetric)} \end{aligned} \tag{77}$$

Topological Structure ($K^2 \times T^2$)

$$\begin{aligned} \pi_1(K^2 \times T^2) = \langle a, b, c, d \mid aba^{-1}b = 1, [c, d] = 1, \dots \rangle \\ b_0 = 1,\ b_1 = 3,\ b_2 = 3,\ b_3 = 1,\ b_4 = 0 \end{aligned} \tag{78}$$

Non-Orientable Identification

$$(t, x, y, z) \sim (-t, -x, -y, z + L), \quad \det(J) = -1 \tag{79}$$

Signature Change (Euclidean $\to$ Lorentzian)

$$\begin{aligned} ds_E^2 &= d\tau^2 + a(\tau)^2 d\Omega^2_{K^2 \times T^2} \quad (\tau < 0) \\ ds_L^2 &= -dt^2 + a(t)^2 d\Omega_3^2 \quad (t > 0) \end{aligned} \tag{80}$$

Topological Entropy (Ray-Singer)

$$S_{\text{top}} = k_B \log T(M^4), \quad \log T = \frac{1}{2} \sum_{q=0}^{4} (-1)^q q\, \zeta'_{\Delta_q}(0) \tag{81}$$

Collapse Condition

$$S_{\text{top}}(n) + S_{\text{ent}}(n) \longrightarrow S_{\max} = \frac{A}{4l_P^2} \tag{82}$$

Exotic Stress-Energy Tensor

$$\begin{aligned} T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu) \\ \text{with } \rho + p < 0 \text{ (NEC violation)} \end{aligned} \tag{83}$$

Collapse Equation

$$\frac{\partial^2 \Phi}{\partial t^2} = c^2 \nabla^2 \Phi - \lambda(\Phi^2 - v^2)\Phi \tag{84}$$

CMB Prediction

$$\frac{C(\pi)}{C(0)} > \left[\frac{C(\pi)}{C(0)}\right]_{\Lambda\text{CDM}} \tag{85}$$

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