Primordial 4D Torus Hypothesis: A Pre-Big Bang Cosmological Model

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Abstract

This document presents a mathematical formalization of the cosmological hypothesis proposing a closed 4D toroidal geometric structure as a pre-Big Bang state. The corresponding metrics are developed, collapse mechanisms are analyzed, and connections with known physics and falsifiable predictions are established.

This hypothesis arises from the association of the primordial seed (Φ) with the exotic state described below, in previous studies conducted in The Atlas [8], and from its consequent Singularity Void, a structure we implemented to handle exceptions where time did not yet exist within our implementation of Einstein's block universe.

Conventions and Units: Throughout this paper, we use natural units with $c = \hbar = k_B = 1$ unless otherwise specified. The Planck units are:

$$l_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-35} \text{ m}$$
$$t_P = \frac{l_P}{c} \approx 5.39 \times 10^{-44} \text{ s}$$
$$m_P = \sqrt{\frac{\hbar c}{G}} \approx 2.18 \times 10^{-8} \text{ kg}$$
$$\rho_P = \frac{m_P}{l_P^3} \approx 5.16 \times 10^{96} \text{ kg/m}^3$$

1 Mathematical Formalization

1.1 Primordial Energy Symmetry Postulate (G_0)

Before describing the geometric structure, we establish the fundamental energy constraint of the primordial state.

1.1.1 The G_0 Equation

The primordial state satisfies exact energy cancellation enforced by non-orientable topology:

$$G_0 = \lim_{n \to n_c} \sum_{k=1}^{n} \underbrace{[(+\epsilon_k) + (-\epsilon_k)]}_{\text{topological pair}} = 0 \tag{1}$$

where:

- ϵ_k represents the k-th quantum fluctuation (energy quanta)
- The brackets $[\cdot]$ denote **simultaneous creation** imposed by the Klein-type topological identification (see Eq. 31)
- $n_c \sim \exp(S_{BH})$ is the critical cycle number before collapse

1.1.2 Physical Meaning: Spectral Zeta Regularization and Topological Protection

The exact energy cancellation in Eq. 1 is not merely symbolic but arises from a rigorous mathematical framework: **spectral zeta function regularization** (ζ-regularization) applied to the vacuum Hamiltonian of the primordial manifold.

The divergence problem (naive summation):

$$E_{\text{vac}} = \sum_{k=1}^{\infty} \frac{1}{2} \hbar \omega_k = \infty \quad \text{(divergent)} \tag{2}$$

In standard QFT, this vacuum energy divergence requires regularization. We employ the **spectral zeta function** of the Hamiltonian operator $\hat{H}$ on the manifold $M^4 = K^2 \times T^2$:

$$\zeta_{\hat{H}}(s) = \sum_{\lambda_j > 0} \lambda_j^{-s}, \quad \text{Re}(s) > 2 \tag{3}$$

The regularized vacuum energy is then defined via analytic continuation:

$$E_{\text{vac}}^{\text{reg}} = \frac{\hbar}{2}\zeta_{\hat{H}}(-1/2) + (\text{trace anomaly}) \quad (4)$$

Topological protection via the Euler characteristic: The crucial point is that for manifolds with vanishing Euler characteristic $\chi(M) = 0$, the conformal (trace) anomaly of the stress-energy tensor vanishes identically:

$$\langle T^{\mu}_{\mu} \rangle_{\text{anomaly}} \propto \chi(M) = 0 \quad (5)$$

Since $\chi(K^2 \times T^2) = \chi(K^2) \cdot \chi(T^2) = 0 \cdot 0 = 0$ (as computed in Section 2.4), the topological structure *protects* the vacuum from acquiring anomalous energy contributions. The energy cancellation is therefore not "imposed" by a notational convention, but is a **geometric consequence** of the null Euler class.

Rigorous formulation: The bracket notation in Eq. 1 should be understood as shorthand for:

$$G_0 = \lim_{s \to -1/2} \zeta_{\hat{H}}(s)\big|_{\chi=0} = 0 \quad (\text{topologically protected}) \quad (6)$$

The non-orientable identification (Eq. 31) guarantees that the spectrum of $\hat{H}$ possesses a $\mathbb{Z}_2$ symmetry: for every mode with energy $+\epsilon_k$, there exists a partner mode at $-\epsilon_k$ under the orientation-reversing identification. This spectral pairing, combined with the vanishing Euler characteristic, ensures exact cancellation without requiring supersymmetry.

This mechanism is *analogous* to supersymmetric cancellation (where boson-fermion pairing prevents UV divergences), but here it arises purely from **topology** rather than from an internal symmetry between particle types.

1.1.3 Energy-Entropy Asymmetry: The Collapse Trigger

While the G_0 equation guarantees perfect energy cancellation, entropy behaves fundamentally differently. This asymmetry is the key to understanding why the primordial state collapses.

Critical caveat on thermodynamics in CTCs: Standard thermodynamics, including the second law, presupposes a well-defined causal structure. In spacetimes with CTCs:

- Entropy can **decrease** along certain worldlines [24, 25]
- The second law $dS/dt \geq 0$ requires a global time function, which does not exist
- Thermodynamic equilibrium becomes ill-defined

Our resolution: We use **entanglement entropy** rather than thermodynamic entropy as the relevant quantity. Entanglement entropy:

- Is defined purely quantum-mechanically, without requiring causal structure
- Satisfies subadditivity bounds even in acausal spacetimes
- Provides a well-defined upper bound via the Bekenstein-Hawking formula

With this reinterpretation, the "entropy growth" in Eq. 8 refers to **accumulation of quantum entanglement** between degrees of freedom, not thermodynamic irreversibility.

	Behavior	T-sym
Energy	$\sum E = 0$	Yes
Entropy	$S \sim \log n$	No

Table 1: Energy vs. entropy asymmetry

Energy conservation is exact throughout the primordial state:

$$\frac{d}{dn}\left(\sum_k E_k\right) = 0 \quad \forall n \quad (7)$$

Entanglement entropy accumulates monotonically with each cycle:

$$\frac{dS_{\text{ent}}}{dn} > 0 \quad \forall n \quad (8)$$

Explicitly, as energy circulates through the closed timelike curves (CTCs), it accumulates quantum entanglement with its past states:

$$S_{\text{ent}}(n) = S_0 \cdot \log(n) + \mathcal{O}(1) \quad (9)$$

where we **define** S_0 as the entanglement entropy generated per CTC cycle:

$$S_0 \equiv \ln|T(K^2 \times T^2)| \sim \mathcal{O}(1) \text{ in Planck units} \quad (10)$$

This identification connects S_0 to the Ray-Singer torsion of the manifold, providing a topological origin for the entropy scale.

This logarithmic growth is characteristic of entanglement accumulation in cyclic quantum systems. **Note**: The $\log(n)$ scaling is characteristic of 1+1D systems; in higher dimensions, faster growth $S \sim n^\alpha$ with $\alpha > 0$ may occur. Our model uses the logarithmic form as a conservative lower bound.

1.1.4 Information Saturation and the Bekenstein-Hawking Limit

The topological structure has a maximum information capacity given by the Bekenstein-Hawking bound:

$$S_{\max} = \frac{A_{\text{eff}}}{4l_P^2} \tag{11}$$

Clarification on "area" in $K^2 \times T^2$: Unlike black holes, the primordial torus has no event horizon in the conventional sense. We identify the **signature transition surface** Σ_t as the analog of a horizon. The effective area is:

$$A_{\text{eff}} \equiv \text{Vol}_3(\Sigma_t) \tag{12}$$

where A_{eff} corresponds not to a classical geometric surface area, but to the **minimal quantum area spectrum** allowed by the topology. In the absence of a macroscopic scale factor, this area is fixed by the Planck scale: $A_{\text{eff}} \sim \mathcal{O}(1) \cdot l_P^2$. The "surface gravity" κ is defined via the degenerate metric at the transition:

$$\kappa^2 \equiv -\frac{1}{2}(\nabla^\mu n^\nu)(\nabla_\mu n_\nu)\big|_{\Sigma_t} \tag{13}$$

where n^μ is the normal to Σ_t. This generalizes the Hawking temperature formula to signature-changing surfaces.

Topological change requirement: The transition from $K^2 \times T^2$ (or its 3D slices) to the post-collapse S^3 spatial topology requires, by the Geroch-Tipler theorems [27, 31], either:

- Singularities traversed during the transition, or
- Integrated NEC violation: $\int T_{\mu\nu} k^\mu k^\nu d\lambda < 0$ along null geodesics

Our model invokes the latter, with the violation concentrated at Planck scales:

$$\int T_{\mu\nu} k^\mu k^\nu d\lambda \approx -\rho_P \cdot l_P \tag{14}$$

When the accumulated entanglement entropy approaches this limit:

$$S_{\text{ent}}(n) \longrightarrow S_{\max} \tag{15}$$

the system reaches **information saturation**. The topology can no longer contain the accumulated information, triggering the collapse/unfolding event.

The critical cycle number is therefore:

$$n_{\text{critical}} \sim \exp\left(\frac{S_{\max}}{S_0}\right) \sim \exp\left(\frac{A}{4l_P^2}\right) \tag{16}$$

For cosmological scales, this yields $n_{\text{critical}} \sim \exp(10^{123})$, an astronomically large but **finite** number.

1.1.5 The Nature of the Collapse

This analysis leads to a profound reinterpretation of the Big Bang:

> *The Big Bang was not an explosion of energy (which remains zero), but an* ***information discharge****, the moment when the primordial "memory" overflowed its topological container.*

The collapse creates new "space" for information, manifesting as the bidirectional unfolding into two planes (matter/antimatter universes). This is consistent with:

- **Zero-energy universe hypothesis** (Tryon, Guth): Total energy remains zero before and after the Big Bang
- **Holographic principle** ('t Hooft, Susskind): Information content is bounded by surface area
- **Entropic gravity** (Verlinde): Spacetime emerges from information/entropy dynamics

1.1.6 Summary: The Unstable Nothing

The G_0 postulate can be summarized as follows:

> **The primordial state is an "unstable nothing"**: a perfect energy vacuum ($\sum E = 0$) that is nevertheless filled with accumulating information tensions. The "nothing" collapses not because energy balance fails, but because **even the void has a memory limit**.

Mathematically:

$$\text{Energy:} \quad G_0 = \sum[(+\epsilon) + (-\epsilon)] = 0 \quad \text{(stable)} \tag{17}$$

$$\text{Entropy:} \quad S(n) \to S_{\max} \quad \text{as} \quad n \to n_c \quad \text{(unstable)} \tag{18}$$

The arrow of time **emerges** from this entropy accumulation, even within the temporally closed primordial structure. Time's directionality is not imposed externally but arises from the intrinsic asymmetry between energy (symmetric) and information (accumulative).

1.2 Geometry of the Closed 4D Torus

The proposed structure combines elements of a **4-torus** (T^4) with non-orientable topology similar to the Klein bottle. The most appropriate metric is a generalization of the Clifford torus embedded in curved spacetime.

Critical clarification on signature: The primordial state involves a **signature transition** from Euclidean to Lorentzian, analogous to the Hartle-Hawking no-boundary proposal:

- **Euclidean regime** (primordial torus): Signature $(+,+,+,+)$, all coordinates spacelike, enabling compact topology without causal paradoxes in the classical sense
- **Lorentzian regime** (post-collapse): Signature $(-,+,+,+)$, one timelike direction emerges
- **Transition surface** Σ: Where signature changes, satisfying junction conditions

The "CTCs" in our model are more precisely understood as closed curves that become timelike *after* the signature transition begins, they represent the "unwinding" of what were purely spatial loops in the Euclidean phase.

1.2.1 Signature Change Formalism

The transition from Euclidean to Lorentzian is modeled via a degenerate metric at the transition surface Σ:

Wick rotation convention: The standard prescription is $\tau = -it$ (equivalently, $t = i\tau$), ensuring:

- Euclidean regime: τ real, $\tau \in (-\infty, 0]$
- Transition surface: $\tau = 0 \leftrightarrow t = 0$
- Lorentzian regime: t real, $t \in [0, +\infty)$

Euclidean side ($\tau < 0$):

$$ds_E^2 = d\tau^2 + a(\tau)^2\, ds^2_{\Sigma_3} \tag{19}$$

where $ds^2_{\Sigma_3}$ is the metric on a 3-dimensional spatial section.

Transition surface ($\tau = 0$):

$$\det(g_{\mu\nu})\big|_\Sigma = 0 \tag{20}$$

Lorentzian side ($t > 0$, with $t = i\tau$):

$$ds_L^2 = -dt^2 + a(t)^2\, d\Omega_3^2 \tag{21}$$

Note on spatial topology: The Wick rotation changes the metric signature but **not** the spatial topology. If Σ_3 inherits structure from $K^2 \times T^2$, this must be specified. In the simplest case, Σ_3 is a 3-dimensional slice of the primordial manifold that becomes the spatial section of the emergent universe.

The junction conditions (generalized Israel-Darmois, following Hayward [18] and Dray-Manogue [28]) at Σ ensure continuity of the induced 3-metric and extrinsic curvature where defined.

1.2.2 Generalized Clifford Torus Metric

Dimensional clarification: The primordial manifold $K^2 \times T^2$ is 4-dimensional. We present two equivalent descriptions:

Option A: Intrinsic 4D metric (coordinates $\theta_1, \theta_2, \phi_1, \phi_2$):

$$ds^2 = R_1^2 d\theta_1^2 + R_2^2 d\theta_2^2 + R_3^2 d\phi_1^2 + R_4^2 d\phi_2^2 \tag{22}$$

with the Klein identification (Eq. 31) on (θ_1, θ_2) and periodicity on (ϕ_1, ϕ_2).

Option B: Foliated metric (for studying dynamics):

$$ds^2 = -N(\tau)^2 d\tau^2 + a(\tau)^2 d\Sigma_3^2 \tag{23}$$

where $d\Sigma_3^2$ is the metric on a 3-dimensional spatial slice of $K^2 \times T^2$, and τ is a Euclidean "time" parameter.

Note on "5D" appearance: Some formulations may introduce an auxiliary radial coordinate r for describing the "throat" structure:

$$ds^2 = -N(r)^2 dt^2 + a(r)^2 d\theta_1^2 + b(r)^2 d\theta_2^2 + c(r)^2 d\theta_3^2 + f(r)^2 dr^2 \tag{24}$$

This describes a **family** of 4D metrics parametrized by r, not a 5D spacetime. The coordinate r should be understood as labeling the location within the throat, with the physical metric being the induced 4D metric at each r.

1.2.3 Temporal Closure Condition (Closed Timelike Curves)

For time to form a closed loop, we need **closed timelike curves (CTCs)**. Consider a metric with frame-dragging:

$$ds^2 = g_{tt}dt^2 + 2g_{t\phi}dt\,d\phi + g_{\phi\phi}d\phi^2 + \dots \tag{25}$$

where:

$$g_{tt} = -N(r)^2, \quad g_{t\phi} = -N(r)^2\omega(r), \quad g_{\phi\phi} = h(r)^2 - N(r)^2\omega(r)^2 \tag{26}$$

Condition for CTCs: A curve with constant t, r, θ and $\phi \in [0, 2\pi]$ is **timelike** if:

$$ds^2 = g_{\phi\phi}(d\phi)^2 < 0 \quad \Leftrightarrow \quad g_{\phi\phi} < 0 \tag{27}$$

This occurs when the frame-dragging parameter exceeds a critical value:

$$|\omega(r)| > \frac{h(r)}{N(r)} \quad \text{(CTC condition)} \tag{28}$$

Comparison with Kerr: In the Kerr metric, CTCs exist for $r < 0$ (through the ring singularity), where $g_{\phi\phi}$ becomes negative. Similarly, the primordial torus has regions where closed ϕ-loops become timelike, enabling the temporal "recycling" essential to the model.

Note: The condition $g_{tt} \to 0$ defines the **ergosphere boundary**, not necessarily the CTC region. CTCs require closed curves to be timelike, not merely that t ceases to be a good time coordinate.

1.3 Non-Trivial Topology: "Top-Bottom" Connection

The connection between the upper and lower parts of the torus (like a Klein bottle) is described by a **non-orientable manifold**.

1.3.1 Fundamental Group

The appropriate 4-dimensional non-orientable manifold is the product $K^2 \times T^2$ (Klein bottle times 2-torus), whose fundamental group is:

$$\pi_1(K^2 \times T^2) = \langle a, b, c, d \mid aba^{-1}b = 1, \\ [c,d] = 1, [a,c] = [a,d] = [b,c] = [b,d] = 1 \rangle \tag{29}$$

where:

- Generators a, b encode the Klein bottle K^2 with its characteristic orientation-reversing identification ($aba^{-1}b = 1$)
- Generators c, d encode the 2-torus T^2 factor ($[c, d] = 1$)
- The commutation relations ensure the product structure

This construction preserves the essential Klein-type property: traversing a closed path involving generator a reverses orientation, crucial for connecting "top" with "bottom" with parity inversion.

1.3.2 Point Identification

Mathematically, $K^2 \times T^2$ is constructed as follows. Let (t, x) parametrize the Klein bottle factor and (y, z) the 2-torus factor.

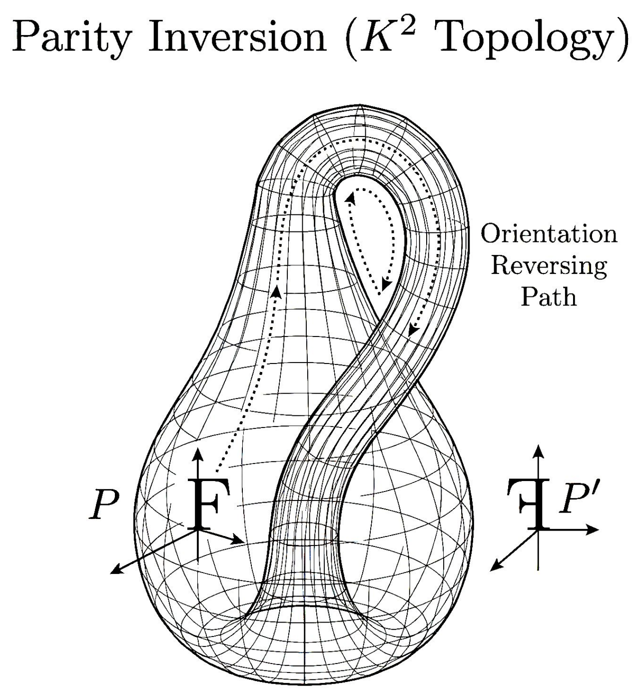

Figure 1: Visualization of the non-orientable identification in the Klein bottle factor. The orientation-reversing twist ($x \to -x$) under translation by $L_t/2$ implements P-symmetry breaking: traversing the closed path exchanges parity, connecting matter and antimatter sectors as topological "mirrors" of each other.

The identifications are:

Klein bottle K^2 in (t, x):

$$(t, x, y, z) \sim (t + L_t, x, y, z) \tag{30}$$

$$(t, x, y, z) \sim (t + L_t/2, -x, y, z) \quad \text{(Klein twist)} \tag{31}$$

2-torus T^2 in (y, z):

$$(t, x, y, z) \sim (t, x, y + L_y, z) \tag{32}$$

$$(t, x, y, z) \sim (t, x, y, z + L_z) \tag{33}$$

where L_t, L_y, L_z are the periods in each direction. The Jacobian of the Klein twist transformation is:

$$J_{\text{Klein}} = \text{diag}(+1, -1, +1, +1), \quad \det(J_{\text{Klein}}) = -1 \tag{34}$$

confirming genuine orientation reversal. This construction:

- Is a **free action** (no fixed points, since $L_t/2 \neq 0$)
- Generates the Klein bottle K^2 in the (t, x) factor with proper orientation-reversing identification
- Preserves the standard 2-torus T^2 in the (y, z) factor
- Produces a smooth 4-manifold $K^2 \times T^2$ without orbifold singularities

Physical interpretation: Traversing a closed path in the t-direction by $L_t/2$ while returning to the same (y, z) reverses the x-coordinate, implementing the orientation-reversing "mirror" that connects matter and antimatter sectors.

1.4 Double Cone Geometry (Hourglass)

The geometry of two cones facing each other at their vertices is described by a modified **Milne double cone** metric:

$$ds^2 = -dt^2 + t^2[d\chi^2 + \sinh^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)] \tag{35}$$

For the symmetric bidirectional version, we extend to $t \in \mathbb{R}$ (not just $t > 0$):

$$ds^2 = -dt^2 + |t|^2[d\chi^2 + \sinh^2\chi\, d\Omega^2] \tag{36}$$

At $t = 0$ (the hourglass vertex), we encounter a **metric degeneration on a smooth manifold** where the signature change occurs.

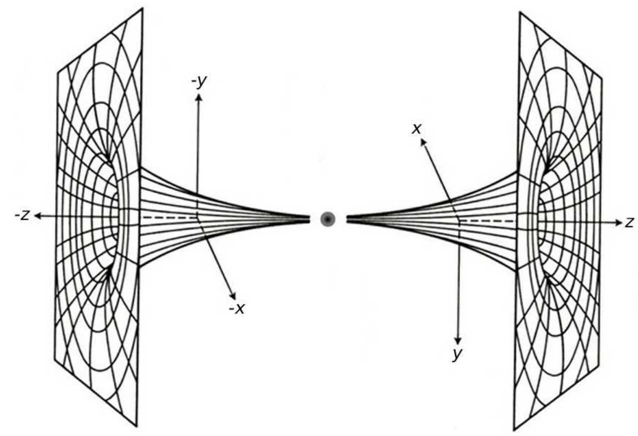

Figure 2: Double cone (hourglass) structure of the primordial state

1.5 Stress-Energy Tensor of the Central Exotic State

The "exotic state" in the torus throat requires a stress-energy tensor with extreme properties.

1.5.1 General Form

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} + \pi_{\mu\nu} + q_\mu u_\nu + u_\mu q_\nu \tag{37}$$

where:

- ρ = energy density (extreme, possibly trans-Planckian)
- p = pressure
- $\pi_{\mu\nu}$ = anisotropic stress tensor
- q_μ = heat flux

1.5.2 Conditions for the Exotic State

To sustain the closed toroidal geometry with CTCs, we need **exotic energy** that violates energy conditions:

Null Energy Condition (NEC) Violation:

$$T_{\mu\nu}k^\mu k^\nu < 0 \quad \text{for some null vector } k^\mu \tag{38}$$

This is equivalent to:

$$\rho + p < 0 \tag{39}$$

Exotic Equation of State:

$$w = \frac{p}{\rho} < -1 \quad \text{(phantom-like behavior)} \tag{40}$$

Critical clarification: Topological Casimir effect, not phantom fields. The effective equation of state $w < -1$ does **not** arise from a fundamental "phantom field" with wrong-sign kinetic terms (which would violate unitarity). Instead, we interpret it as an **effective phenomenon** arising from **topological vacuum polarization**, a generalized Casimir effect induced by the non-trivial topology of $K^2 \times T^2$.

The Topological Casimir Mechanism:

1. The compact, non-orientable topology imposes **non-trivial boundary conditions** on quantum fields, modifying the vacuum mode spectrum

2. The renormalized vacuum stress-energy tensor acquires topology-dependent contributions:

$$\langle T_{\mu\nu} \rangle_{\text{ren}} = \langle T_{\mu\nu} \rangle_{\text{Mink}} + \langle T_{\mu\nu} \rangle^{\text{top}}_{\text{Casimir}} \tag{41}$$

3. The topological Casimir contribution can violate local energy conditions while preserving **global stability** of the quantum vacuum:

$$\rho^{\text{top}}_{\text{Casimir}} \sim -\frac{\hbar c}{L^4} \cdot f(\text{topology}) \tag{42}$$

where L is the characteristic scale and f encodes the topological correction

Why this is not problematic:

- The Casimir effect is experimentally verified and does not violate fundamental stability

- It violates *local* energy conditions (NEC, WEC) but not *averaged* conditions over appropriate domains

- There is no propagating ghost field, the effect is purely a vacuum polarization phenomenon

- The effective $w < -1$ is a **geometric illusion**: interpreting $\langle T_{\mu\nu} \rangle_{\text{Casimir}}$ as a perfect fluid gives $w_{\text{eff}} < -1$, but this does not correspond to any fundamental degree of freedom

Mathematical formulation: For the Klein bottle factor K^2, the spectral zeta function regularization gives:

$$\langle T^{\mu}_{\mu} \rangle_{K^2} = \frac{\hbar c}{(2\pi)^2 L^4} \left[\zeta_{K^2}(-2) - \zeta_{T^2}(-2) \right] \tag{43}$$

The difference between non-orientable (K^2) and orientable (T^2) spectral functions yields a net negative contribution, mimicking phantom behavior without introducing pathological fields. This is the topological analog of the attractive Casimir force between conducting plates.

1.5.3 Energy Density at the Throat

At the central vertex of the torus, density diverges:

$$\rho(r) = \rho_P \cdot \left(\frac{l_P}{r} \right)^{\alpha} \tag{44}$$

where:

- $\rho_P = c^5/(\hbar G^2) \approx 5.16 \times 10^{96}$ kg/m^3 (Planck density)

- $l_P = \sqrt{\hbar G/c^3} \approx 1.62 \times 10^{-35}$ m (Planck length)

- $\alpha \geq 4$ for sufficiently strong singularity

2 Collapse Mechanism

2.1 Closed Loop Stability

The temporally closed toroidal structure is inherently **metastable**. Stability conditions are derived from the second variation of the Einstein-Hilbert action.

2.1.1 Stability Condition

For a static geometry with CTCs, stability requires:

$$\delta^2 S = \int d^4x \sqrt{-g} \big[\delta g^{\mu\nu} G_{\mu\nu} + \text{quantum corrections} \big] > 0 \tag{45}$$

The problem is that **quantum loop corrections** inevitably destabilize the configuration.

2.2 Stability Breaking Mechanisms

2.2.1 A) Quantum Instability (Most Promising)

The **generalized Hawking effect** in geometries with CTCs generates radiation that drains energy:

$$T_H = \frac{\hbar\kappa}{2\pi k_B c} \tag{46}$$

where κ is the surface gravity of the "temporal ergosphere". The accumulated energy loss:

$$\frac{dM}{dt} = -\sigma A \cdot T_H^4 \tag{47}$$

eventually destroys the geometry after a characteristic time:

$$\tau_{\text{decay}} \sim \frac{M^3}{\hbar c^4} \times f(\text{topology}) \tag{48}$$

2.2.2 B) Quantum Vacuum Fluctuation

Vacuum fluctuations in geometries with CTCs grow exponentially (Cauchy instability):

$$\langle \phi^2 \rangle \sim \exp(t/\tau_{\text{CTC}}) \tag{49}$$

where τ_{CTC} is the period of the closed timelike curve.

2.2.3 C) Entropic Limit ("n Cycles" Model)

If energy circulates in a loop n times, the entanglement entropy between degrees of freedom grows:

$$S_{\text{ent}}(n) = S_0 \cdot \log(n) + \text{corrections} \tag{50}$$

When S_{ent} reaches the Bekenstein-Hawking limit:

$$S_{\max} = \frac{A}{4l_P^2} \tag{51}$$

the system must "discharge" its information, triggering collapse.

Critical number of cycles:

$$n_{\text{critical}} \sim \exp\left(\frac{A}{4l_P^2}\right) \sim \exp(10^{123}) \tag{52}$$

(extremely large but finite number)

2.3 Bidirectional Unfolding Modeling

The bidirectional collapse is modeled through a **topological phase transition**.

2.3.1 Post-Collapse Two-Sector Structure

Important clarification: Post-collapse, the two sectors $\mathcal{M}_+$ and $\mathcal{M}_-$ emerge as **causally disconnected 4D spacetimes**, each with its own metric:

Universe $\mathcal{M}_+$ (matter sector):

$$ds_+^2 = -dt_+^2 + a(t_+)^2 d\Sigma_+^2 \tag{53}$$

Universe $\mathcal{M}_-$ (antimatter sector):

$$ds_-^2 = -dt_-^2 + b(t_-)^2 d\Sigma_-^2 \tag{54}$$

where $d\Sigma_\pm^2$ are the 3-metrics of spatial sections (e.g., S^3, $\mathbb{R}^3$, or H^3). The times t_+ and t_- are **independent** after causal disconnection, though synchronized at the collapse event ($t_+ = t_- = 0$).

2.3.2 Evolution Equations

Each sector follows independent Friedmann dynamics:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_+ - \frac{k_+}{a^2} + \frac{\Lambda_+}{3} \tag{55}$$

$$\left(\frac{\dot{b}}{b}\right)^2 = \frac{8\pi G}{3}\rho_- - \frac{k_-}{b^2} + \frac{\Lambda_-}{3} \tag{56}$$

with separate continuity equations:

$$\dot{\rho}_+ + 3H_+(\rho_+ + p_+) = 0 \tag{57}$$

$$\dot{\rho}_- + 3H_-(\rho_- + p_-) = 0 \tag{58}$$

The **initial condition constraint** (from the shared collapse event):

$$\rho_+ a^3\big|_{t_+=0} + \rho_- b^3\big|_{t_-=0} = \rho_{\text{primordial}} \tag{59}$$

This constraint encodes the conservation of total energy through the topological transition, though post-collapse, each sector evolves independently.

2.3.3 Bifurcation Condition

Unfolding occurs when the metric signature at the throat changes:

$$\det(g_{\mu\nu}) \to 0 \quad \text{at } r = r_{\text{critical}} \tag{60}$$

This corresponds to a **signature change surface**, studied in quantum cosmology.

2.3.4 Master Collapse Equation

The collapse dynamics follows:

$$\frac{\partial^2 \Phi}{\partial t^2} - c^2 \nabla^2 \Phi + V'(\Phi) = 0 \tag{61}$$

where Φ is a scalar field parameterizing the topological transition, with potential:

$$V(\Phi) = \frac{\lambda}{4}(\Phi^2 - v^2)^2 \tag{62}$$

This gives $V'(\Phi) = \lambda\Phi(\Phi^2 - v^2)$, consistent with Eq. 120. The minima $\Phi = \pm v$ correspond to the two separated universes.

2.4 The "Size" Problem in Closed 4D

A natural objection to the model is: what is the torus size R? The critical cycle number depends on this parameter:

- For Planck scale ($R \sim l_P$): $n \sim e^1 \approx 2.7$ (almost immediate collapse)
- For nuclear scale ($R \sim 10^{-15}$ m): $n \sim \exp(10^{40})$ (enormous number)

However, this question reveals a deeper conceptual problem.

2.4.1 The Conceptual Problem

In 3D, "size" means length, area, volume, quantities measurable with an external ruler. In 4D with closed time (CTCs), the concept becomes complicated because:

1. **There is no "moment" to measure**: if time is a loop, there is no $t = 0$ where "the torus measures X" can be defined
2. **The metric is inside, not outside**: the metric defines distances, but is part of the torus itself
3. **There is no exterior**: there is no external space from which to observe the structure

It is analogous to asking "what is north of the North Pole?". The question assumes a frame of reference that does not apply.

2.4.2 Three Possible Resolutions

Option A: Intrinsic Invariant (4-Volume)

In differential geometry, quantities exist that require no external reference:

$$V_4 = \int d^4x \sqrt{|g|} \tag{63}$$

For a 4-torus with radii R_1, R_2, R_3, R_4:

$$V_4 = (2\pi)^4 \times R_1 \times R_2 \times R_3 \times R_4 \tag{64}$$

The "size" would be $V_4^{1/4}$, an effective average radius.

Option B: Energetic Definition

If the torus contains all the energy of the observable universe:

$$E_{\text{total}} = M_{\text{universe}} \times c^2 \approx 10^{53}\ \text{kg} \times c^2 \approx 10^{70}\ \text{J} \tag{65}$$

By the Schwarzschild mass-radius relation:

$$R_{\text{characteristic}} = \frac{2GM}{c^2} \approx 10^{26}\ \text{m} \tag{66}$$

Notably, this coincides with the order of magnitude of the current observable universe radius.

Option C: Size as Emergent Property (Preferred Resolution)

The most elegant answer: the question "how big is it?" makes no sense before unfolding.

"Size" only exists after linear time emerges. Before collapse, the torus possesses **topology** but no macroscopic geometric parameter R. Its "size" is fixed at the **fundamental Planck scale limit** imposed by the quantization of geometry itself. Thus, the effective area A_{eff} in the entropy bound is not a variable, but a fundamental constant determined by l_P^2.

2.4.3 Implication: Topological vs. Geometric Collapse

If we adopt Option C, the critical cycle number does not depend on a size R, but on intrinsic topological properties:

$$n_{\text{critical}} \sim \exp(S_{\text{topological}}) \tag{67}$$

where $S_{\text{topological}}$ is calculated from discrete invariants:

Invariant	Sym.	Description
Betti Number	b_k	Holes per dimension
Euler Char.	χ	Global invariant
Pontryagin Class	p_k	Manifold torsion
Orientability	σ	± 1

For the non-orientable manifold $M^4 = K^2 \times T^2$, the Betti numbers are computed via the Künneth theorem. With K^2 having $b_0 = 1, b_1 = 1, b_2 = 0$ (over $\mathbb{R}$) and T^2 having $b_0 = 1, b_1 = 2, b_2 = 1$:

$$\chi(K^2 \times T^2) = \chi(K^2) \cdot \chi(T^2) = 0 \cdot 0 = 0 \tag{68}$$

$$b_0 = 1, \quad b_1 = 3, \quad b_2 = 3, \quad b_3 = 1, \quad b_4 = 0 \tag{69}$$

Critical consistency check: $b_4 = 0$ confirms non-orientability (for a compact orientable n-manifold, $b_n = 1$; for non-orientable, $b_n = 0$).

Hidden structure: The integral homology $H_1(K^2; \mathbb{Z}) = \mathbb{Z} \oplus \mathbb{Z}_2$ contains torsion that vanishes with real coefficients but contributes to the analytic torsion (see below).

Postulate (Topological Entropy): We **postulate** that the topological complexity of the primordial manifold contributes to an effective entropy-like quantity. By analogy with black hole entropy (which is also geometric in origin), we define:

$$S_{\text{top}} \equiv k_B \log T(M^4) \tag{70}$$

where $T(M^4)$ is the **Ray-Singer analytic torsion**, defined via spectral zeta functions:

$$\log T(M) = \frac{1}{2} \sum_{q=0}^{4} (-1)^q \cdot q \cdot \zeta'_{\Delta_q}(0) \tag{71}$$

Here $\zeta_{\Delta_q}(s) = \sum_{\lambda_j > 0} \lambda_j^{-s}$ is the spectral zeta function of the Hodge Laplacian Δ_q acting on q-forms.

Important caveat: This identification of torsion with entropy is **conjectural** and requires justification from a complete quantum gravity theory. The Ray-Singer torsion is a fixed topological invariant for a given manifold, not a dynamical quantity. We interpret it as encoding the "baseline" topological complexity that constrains the system's capacity for entanglement accumulation.

Mathematical basis: By the Cheeger-Müller theorem, $T(M) = \tau(M)$ equals the Reidemeister torsion, which captures the $\mathbb{Z}_2$-torsion in $H_1(K^2)$ (orientation-reversal "memory"), information beyond Betti numbers.

The collapse condition becomes:

$$S_{\text{ent}}(n) \longrightarrow S_{\max} - S_{\text{top}} \tag{72}$$

Clarification on entropy counting: To avoid double-counting, we distinguish:

- S_{top}: Fixed topological contribution (from Ray-Singer torsion), representing the "baseline" complexity
- $S_{\text{ent}}(n)$: Dynamical entanglement entropy that grows with cycles
- $S_{\max} = A_{\text{eff}}/4l_P^2$: The Bekenstein-Hawking bound on **total** information content

The collapse triggers when the dynamical component $S_{\text{ent}}(n)$ fills the remaining capacity:

$$S_{\text{ent}}(n) \longrightarrow S_{\max} - S_{\text{top}} = \frac{A_{\text{eff}}}{4l_P^2} - k_B \log T(M^4) \tag{73}$$

This formulation is consistent with the Ryu-Takayanagi holographic entanglement entropy in appropriate limits.

2.4.4 Profound Consequence

This reformulation implies that **topology determines the torus's fate, not geometry**.

- Topological properties are **discrete** (integers)
- There is no free parameter R to specify
- Collapse is inevitable once topology is fixed
- The "when" of collapse makes no sense, only the "if"

This is analogous to nuclear decay: the half-life is determined by the nucleus structure, not by an external "size". The torus "decays" topologically, not geometrically.

3 Connection with Known Physics

3.1 Schwarzschild and Kerr Solutions

3.1.1 Maximal Schwarzschild Extension

The maximally extended Schwarzschild metric (Kruskal-Szekeres coordinates) already contains the "hourglass" structure:

$$ds^2 = \frac{32M^3}{r} e^{-r/2M} (-dT^2 + dX^2) + r^2 d\Omega^2 \tag{74}$$

The Penrose diagram shows:

- Region I: Exterior universe (our universe)
- Region II: Black hole interior
- Region III: Parallel universe (the "other plane"?)
- Region IV: White hole interior

Connection with the hypothesis: Black holes as "scars" would correspond to points where the Region I $\leftrightarrow$ Region III connection persists residually. This is consistent with the ER=EPR conjecture [34].

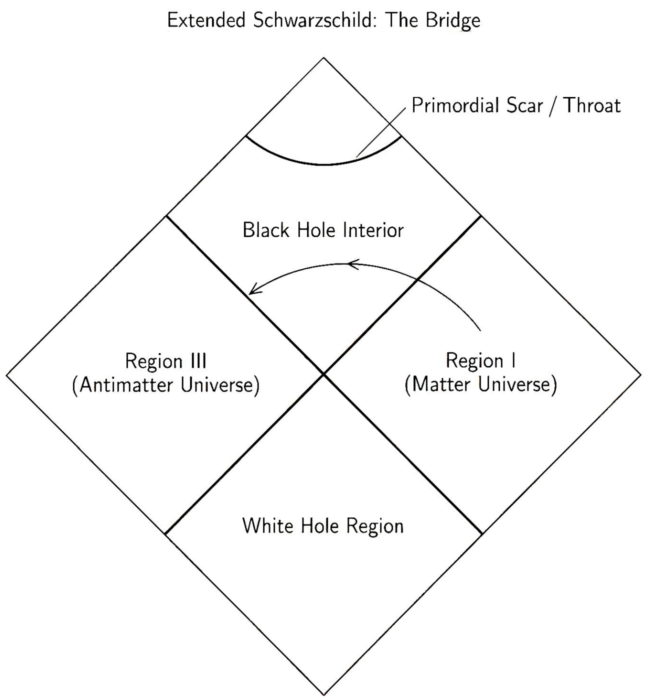

Figure 3: The black hole–cosmology connection: Penrose diagram showing how the maximally extended Schwarzschild geometry mirrors the primordial torus structure. Region I (our universe) and Region III (parallel universe) are connected through the black hole interior (Region II) and white hole interior (Region IV), analogous to the matter/antimatter sectors emerging from the primordial $K^2 \times T^2$ topology.

3.1.2 Einstein-Rosen Bridges (Wormholes)

The proposed original geometry is essentially a **primordial wormhole** connecting the two planes:

$$ds^2 = -dt^2 + dr^2 + (b_0^2 + r^2)(d\theta^2 + \sin^2\theta\, d\phi^2) \quad (75)$$

where b_0 is the throat radius.

3.1.3 Kerr Solution and Rotation

For rotating black holes (Kerr), the singularity is a **ring**, not a point:

$$\Sigma = r^2 + a^2\cos^2\theta = 0 \quad \rightarrow \quad r = 0,\, \theta = \pi/2 \quad (76)$$

Passing through the ring connects to regions with CTCs, analogous to the primordial temporal loop.

3.2 Hartle-Hawking Model (No-Boundary Proposal)

The Hartle-Hawking proposal [2] suggests the universe has no initial temporal boundary (see [7] for a comprehensive modern review). The wave function of the universe:

$$\Psi[h_{ij}, \phi] = \int \mathcal{D}g\,\mathcal{D}\phi\, e^{-I_E[g,\phi]} \quad (77)$$

where I_E is the Euclidean action and the integral is over compact geometries.

Connection: The primordial torus could be the Euclidean geometry dominating the path integral. The torus $\rightarrow$ two universes transition would be the "nucleation" point where Euclidean time becomes Lorentzian.

Euclidean action calculation (required for completeness): The Euclidean action for the $K^2 \times T^2$ geometry is:

$$I_E[K^2 \times T^2] = -\frac{1}{16\pi G}\int d^4x\sqrt{g}(R - 2\Lambda) + I_{\text{GHY}} + I_{\text{top}} \quad (78)$$

where I_{GHY} is the Gibbons-Hawking-York boundary term and I_{top} encodes topological contributions (related to the Ray-Singer torsion). For this geometry to be favored in the path integral over the Hartle-Hawking S^4, we require:

$$I_E[K^2 \times T^2] < I_E[S^4] \quad (79)$$

Open problem: A complete calculation of $I_E[K^2 \times T^2]$ requires specifying the metric ansatz and solving the Euclidean Einstein equations. This remains to be done and is essential for comparing with the standard no-boundary proposal.

3.2.1 Gravitational Instanton

The central "exotic state" would be analogous to a **Hawking-Turok instanton**:

$$ds^2 = d\tau^2 + a(\tau)^2 d\Omega_3^2 \tag{80}$$

with $a(\tau)$ passing through a minimum (torus throat).

3.3 Loop Quantum Cosmology (LQC)

Important clarification: The following presents an *analogy* with LQC, not a derivation from it. LQC and the primordial torus model have fundamentally different foundations.

In LQC, the Big Bang singularity is resolved through a **quantum bounce** when density reaches the critical value:

$$\rho_{\text{crit}} = \rho_P \cdot \frac{\sqrt{3}}{32\pi^2\gamma^3} \approx 0.41\rho_P \tag{81}$$

where $\gamma \approx 0.2375$ is the Immirzi parameter.

3.3.1 Modified Friedmann Equation in LQC

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\text{critical}}}\right) \tag{82}$$

When $\rho \to \rho_{\text{critical}}$, $H \to 0$, indicating the bounce point.

3.3.2 Structural Analogy (Not Derivation)

	LQC	**This Model**
Bounce	Holonomies	Info. saturation
Threshold	$\rho \to \rho_c$	$S \to S_{\max}$
Topology	Fixed T^3	$K^2 \times T^2$
CTCs	Absent	Essential
Basis	From LQG	Postulated

The analogy: Both models predict a transition when a critical threshold is reached. In LQC, it is energy density bounded by holonomies; in our model, it is information capacity bounded by topology.

What is NOT claimed: We do not derive our collapse mechanism from LQC holonomies. The mathematical similarity in the modified Friedmann equation is *suggestive* but not *foundational.* A rigorous connection would require embedding both frameworks in a common quantum gravity theory.

3.4 Penrose's Conformal Cyclic Cosmology (CCC)

In Penrose's model, each "aeon" ends in a massless radiation-dominated state, conformally equivalent to the beginning of the next aeon.

Connection: The temporally closed toroidal structure could represent the **connection between aeons** in a modified CCC version:

$$g_{\mu\nu}^{(n+1)} = \Omega^2 g_{\mu\nu}^{(n)} \tag{83}$$

where Ω is the conformal factor connecting the end of one aeon with the beginning of the next.

3.5 Planck State and Quantum Gravity

3.5.1 Quantum Gravity Condensate

The "exotic state" could be a **graviton condensate** in the non-perturbative regime:

$$|\Psi_{\text{ex}}\rangle = \exp\left(\int d^3k\, \alpha(k) a_k^\dagger\right)|0\rangle \tag{84}$$

where $\alpha(k)$ has support at trans-Planckian scales.

3.5.2 Wheeler Foam

At Planck scale, spacetime is a "foam" of fluctuating topologies:

$$\langle g_{\mu\nu}(x) g_{\rho\sigma}(x')\rangle \sim l_P^4\, \delta^4(x - x') \tag{85}$$

The primordial torus would be a coherent configuration emerging from this foam.

4 Falsifiable Predictions

4.1 CMB Signatures

4.1.1 A) Antipodal Correlations

If the universe has residual toroidal topology, correlations should exist at antipodal points of the CMB:

$$\xi(\pi) \equiv \langle T(\hat{n})T(-\hat{n})\rangle \neq \xi_{\Lambda\text{CDM}}(\pi) \tag{86}$$

Sign of correlation: The sign of $\xi(\pi)$ depends on the boundary conditions at collapse:

$$\xi(\pi) = (-1)^n \times |\xi_0| \tag{87}$$

where n depends on the matching conditions. This constitutes a **falsifiable prediction**: the model predicts either enhanced correlation or anti-correlation at $\theta = 180$, distinguishable from ΛCDM.

Quantitative prediction: Based on the topological identification scale:

$$\frac{\xi(\pi)}{\xi_{\Lambda\mathrm{CDM}}(\pi)} = 1 + \epsilon_{\mathrm{top}}, \quad |\epsilon_{\mathrm{top}}| \sim \mathcal{O}(0.1\text{–}1) \tag{88}$$

Current status: Planck 2018 data shows marginal hints of anomalous correlations at large scales ($l < 5$), potentially consistent with this prediction but not conclusive.

4.1.2 B) Large-Scale Suppressed Modes

A compact initial topology would suppress low-l modes:

$$\frac{C_l^{\mathrm{observed}}}{C_l^{\Lambda\mathrm{CDM}}} < 1 \quad \text{for } l \leq l_{\mathrm{cutoff}} \tag{89}$$

Prediction: $l_{\mathrm{cutoff}} \sim 2$–$4$, consistent with the observed quadrupole ($l = 2$) suppression.

4.1.3 C) Topological Imprints in CMB Patterns

The original Penrose-Gurzadyan (2010) analysis was statistically refuted by three independent groups [22, 32, 33]. The error was using an inappropriate power spectrum in control simulations. We therefore reformulate our prediction using robust methodology:

Reformulated prediction: A proper matched-filter search for temperature and **polarization** anomalies at specific angular radii θ_n (corresponding to intersections with the mirror plane) would show:

$$\langle E(\hat{n})E(-\hat{n})\rangle \neq \langle E(\hat{n})E(-\hat{n})\rangle_{\Lambda\mathrm{CDM}} \tag{90}$$

where E is the E-mode polarization, less contaminated by the issues affecting temperature-only analyses.

Prediction: The angular radii $\theta_n = \arccos(d_n/d_{\mathrm{LSS}})$ would correspond to the topological identification scales d_n.

4.2 Black Hole Anomalies

4.2.1 A) Asymmetry in Black Hole Mergers

If black holes are "scars" of primordial geometry, they could have a preferred orientation:

$$\langle \vec{L} \cdot \hat{n}\rangle \neq 0 \tag{91}$$

where $\vec{L}$ is angular momentum and $\hat{n}$ is the direction toward the "other plane".

Prediction: Statistical correlation in spin directions of black holes detected by LIGO/Virgo.

4.2.2 B) Anomalous Emission Near the Horizon

The event horizon could show additional thermal emission due to residual connection:

$$T_{\mathrm{effective}} = T_{\mathrm{Hawking}} \times (1 + \varepsilon_{\mathrm{connection}}) \tag{92}$$

where $\varepsilon_{\mathrm{connection}} \sim l_P/r_s$ is a small correction.

4.2.3 C) Gravitational Wave Echoes

If internal structure exists (connection to the other plane), gravitational waves from mergers would show **echoes**:

$$h(t) = h_{\mathrm{main}}(t) + \sum_n \alpha^n h_{\mathrm{echo}}(t - n\Delta t) \tag{93}$$

with $\Delta t \sim r_s \log(r_s/l_P)/c$

Current status: There are controversial claims of echo detection in LIGO data.

4.3 Matter-Antimatter Asymmetry

If the unfolding was perfectly symmetric, one plane should have matter and the other antimatter:

Prediction: Our universe has an excess of matter because antimatter went to the other plane.

Quantitative estimate: The observed baryon-to-photon ratio is $\eta_{\mathrm{obs}} \approx 6 \times 10^{-10}$. In our model, this arises from imperfect separation during the topological transition:

$$\eta \approx \frac{T_{\mathrm{BBN}}}{M_P} \times \sin(\delta_{\mathrm{CP}}) \times f(\mathrm{topology}) \tag{94}$$

where f is a function of the topological parameters controlling the "leakage" between sectors. To reproduce $\eta \sim 10^{-10}$, we require $f \sim \mathcal{O}(10^{-9})$ for $\sin(\delta_{\mathrm{CP}}) \sim 1$.

Test: The observed baryonic asymmetry should be exactly compensated by an antimatter excess in the other plane. This would eliminate the need for dynamic baryogenesis mechanisms satisfying the Sakharov conditions, as the asymmetry is purely topological in origin.

4.4 Specific Experiments

4.4.1 A) Event Horizon Telescope (EHT)

Search for deviations from the Kerr-predicted shadow:

$$r_{\text{shadow}} = r_{\text{Kerr}} \times (1 + \delta_{\text{topological}}) \quad (95)$$

Required precision: $\delta < 0.01$, achievable with next-generation EHT.

4.4.2 B) Primordial Gravitational Waves

The primordial gravitational wave spectrum would have a low-frequency cutoff:

$$f_{\text{cut}} = \frac{c}{L_0} \times \frac{a_{\text{inf}}}{a_0} \quad (96)$$

where L_0 is the characteristic scale of the primordial torus. For standard inflation:

$$f_{\text{cut}} \approx H_{\text{inf}} \times \frac{a_{\text{inf}}}{a_0} \approx 10^{-16} \text{ Hz} \quad (97)$$

This is **14 orders of magnitude below LISA's sensitivity band** (10^{-4}–10^{-1} Hz).

Enhanced prediction: Topological fluctuations during the transition could generate higher-frequency modes:

$$f_{\text{top}} \sim 10^{-9}\text{–}10^{-7} \text{ Hz} \quad \text{(pulsar timing band)} \quad (98)$$

Tests:

- NANOGrav/EPTA: sensitivity to $f \sim 10^{-9}$–10^{-7} Hz
- LISA: sensitivity to $f \sim 10^{-4}$–10^{-1} Hz (for topological signatures)
- CMB-S4: B-mode polarization for inflationary GW background

4.4.3 C) Primordial Nucleosynthesis

If particle exchange between planes occurred near the Big Bang:

$$Y_p = 0.247 + \delta Y_{\text{exchange}} \quad (99)$$

Prediction: Small deviation in primordial helium abundance.

5 Constructive Criticism

5.1 Most Promising Aspects

5.1.1 A) Geometric Elegance

The proposal unifies several concepts:

- Big Bang origin
- Nature of black holes
- Black hole/white hole asymmetry
- Possible resolution of the arrow of time problem

5.1.2 B) Connection with Established Physics

The mathematical structure naturally relates to:

- Maximally extended Schwarzschild solutions
- Einstein-Rosen bridges
- Bounce cosmology (LQC)
- No-boundary proposal (Hartle-Hawking)

5.1.3 C) Falsifiable Predictions

The model makes specific predictions that can be tested:

- Antipodal correlations in CMB
- Gravitational wave echoes
- Asymmetry in black hole orientations

5.1.4 D) Problem Resolution

Potentially explains:

- Why we don't observe white holes
- Matter/antimatter asymmetry
- The origin of the arrow of time

5.2 Problematic Aspects

5.2.1 A) Energy Condition Violation

Maintaining the closed toroidal geometry requires exotic energy that violates NEC/WEC. Although this is quantum mechanically possible (Casimir effect), sustaining it at cosmological scale is problematic.

Quantification of the problem: Sustaining this geometry requires:

$$\int (\rho + p)\, dV < 0 \tag{100}$$

which demands significant amounts of exotic energy. Using the Ford-Roman quantum inequalities as a lower bound:

$$\int (\rho + p)\, dV \gtrsim -\frac{\hbar c}{L^4} \cdot V_{\text{throat}} \tag{101}$$

For a throat of Planck size ($L \sim l_P$), the minimum exotic energy required is:

$$|E_{\text{exotic}}| \sim \frac{\hbar c}{l_P} \sim E_P \approx 10^9 \text{ J} \tag{102}$$

For macroscopic structures, the required exotic energy scales as L^3/l_P^4, becoming prohibitively large. This suggests that if the primordial topology exists, it must be stabilized by quantum gravity effects not captured by semiclassical estimates.

Partial response: Inapplicability of temporal stability arguments

The standard objection that exotic matter is "unstable and decays" implicitly assumes a linear temporal framework. However, in the primordial state with saturated CTCs, this assumption does not hold.

This follows the same logic as the "size" problem (section 2.4): asking "how big is the torus?" is a malformed question because it assumes an external reference frame that does not exist. Similarly, asking "how long does exotic matter last before decaying?" assumes a temporal arrow that does not exist in the primordial regime. Decay is inherently a temporal process requiring a "before" (exists) and an "after" (no longer exists). Without this directionality, exotic matter does not need to "endure" because "enduring" is not an applicable concept.

The energy conditions (NEC, WEC) are typically derived from quantum vacuum stability requirements, which in turn presuppose a globally hyperbolic causal structure with well-defined Cauchy surfaces. In the absence of this structure, the status of these conditions as physical "laws" becomes questionable.

Exotic matter does not need to survive in our universe. It only needs to have been possible in a regime where the rules that prohibit it (the energy conditions and the very concept of temporal instability) did not yet apply. The laws that make the initial state impossible only emerge as a consequence of departing from that state.

Limitation: This argument addresses the naive stability objection but does not resolve quantum divergences in the renormalized stress-energy tensor on CTC geometries (Cauchy instability), which is a pathology of the field theory itself rather than a temporal process. A complete resolution requires a non-perturbative quantum gravity framework.

5.2.2 B) Chronological Instability and the Chronology Protection Conjecture

Geometries with CTCs suffer from:

1. **Consistency paradoxes** (although resolvable with Novikov self-consistency principle)
2. **Quantum divergences** in the renormalized stress-energy tensor
3. **Cauchy instability** that would destroy the configuration

Hawking's Chronology Protection Conjecture (CPC) states that the laws of physics conspire to prevent CTCs from forming in macroscopic regions. The standard arguments rely on:

- Quantum vacuum polarization diverging at the chronology horizon
- The Averaged Null Energy Condition (ANEC) holding for quantum fields
- Global hyperbolicity as a consistency requirement for QFT

Our position: We do not claim to *evade* CPC. Rather, we propose a regime where its *premises do not apply*:

1. **No Cauchy surface**: CPC arguments require a globally hyperbolic region approaching a chronology horizon. The primordial torus has no such structure, it is *entirely* non-globally-hyperbolic.

2. **ANEC inapplicability**: ANEC is proven for complete null geodesics. In compact CTCs, all geodesics are closed and finite, making the "averaging" procedure ill-defined.

3. **Emergent laws argument**: The physical laws that enforce CPC (stable QFT vacuum, causal propagation) may themselves *emerge* from the collapse. One cannot use emergent laws to prohibit their own precondition.

Acknowledged limitation: The "emergent laws" argument is philosophically coherent but **operationally non-falsifiable** in this specific aspect. This is an epistemological weakness we explicitly recognize. The model's falsifiability rests on its *post-collapse predictions* (CMB signatures, gravitational wave echoes), not on direct observation of the primordial regime.

5.2.3 C) Spin Structure Obstruction and Fermion Emergence via Cobordism

The Klein bottle K^2 does **not** admit a spin structure because its second Stiefel-Whitney class $w_2(K^2) \neq 0$ [26]. This has profound consequences:

- Spin-1/2 fermions cannot be consistently defined on $K^2 \times T^2$
- Chiral fermions (essential for the Standard Model) are particularly problematic: the chirality operator γ_5 changes sign when transported around the non-orientable loop
- A left-handed fermion would become right-handed, making $SU(2)_L$ gauge interactions ill-defined globally

Resolution via Cobordism and Anomaly Inflow: The topological phase transition from the primordial state to the post-Big Bang universe must be understood as a **cobordism** between manifolds with different spin structures:

$$W^5 : \quad \partial W^5 = (K^2 \times T^2) \sqcup (S^3 \times \mathbb{R}) \tag{103}$$

The 5-dimensional cobordism W^5 interpolates between the non-spin manifold $K^2 \times T^2$ (with $w_2 \neq 0$) and the spin manifold $S^3 \times \mathbb{R}$ (with $w_2 = 0$). The Big Bang transition surface Σ_t constitutes a **domain wall** where the topological symmetry undergoes spontaneous breaking.

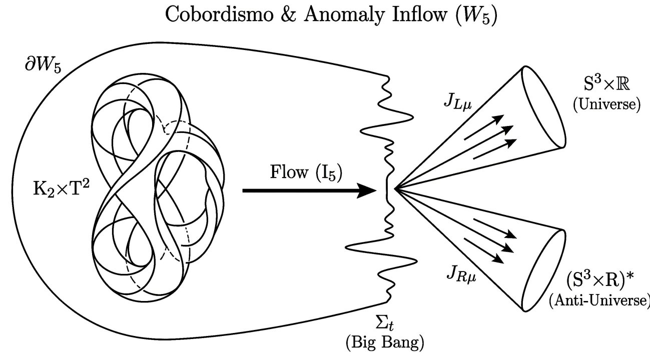

Figure 4: Schematic representation of the cobordism W^5 interpolating between the non-spin primordial manifold $K^2 \times T^2$ and the spin manifold $S^3 \times \mathbb{R}$. Fermions emerge as chiral zero modes localized at the domain wall Σ_t through the anomaly inflow mechanism.

Callan-Harvey Anomaly Inflow Mechanism: Fermions emerge at Σ_t through the **anomaly inflow** mechanism [29]. The key insight is:

1. The bulk $K^2 \times T^2$ cannot support chiral fermions, but hosts a topological field theory defined by the **Pontryagin density** (the 4D analog of the Chern-Simons term):

$$S_{\text{top}} = \frac{\theta}{32\pi^2} \int_{K^2 \times T^2} \text{tr}(R \wedge R) \tag{104}$$

where R is the curvature 2-form.

2. By the descent equations, this topological term is locally the total derivative of the Chern-Simons 3-form ($\text{tr}(R \wedge R) = d\Omega_{\text{CS}}$). By Stokes' theorem, this induces an **anomaly current** flowing toward the boundary Σ_t:

$$\delta S_{\text{top}} \implies J^{\mu}_{\text{bulk}} \to \Sigma_t \tag{105}$$

3. At the domain wall Σ_t, this inflow is cancelled by **chiral zero modes** localized on the boundary:

$$n_+ - n_- = \frac{1}{2}\eta(\Sigma_t) \tag{106}$$

where η is the Atiyah-Patodi-Singer eta invariant encoding the spectral asymmetry of the Dirac operator.

Physical interpretation: The fermionic content of our universe (quarks, leptons) does not preexist in the primordial state. Instead:

- Fermions are **boundary modes** of the topological transition, localized at Σ_t
- The quantum numbers (baryon number, lepton number, chirality) emerge from the topology of W^5 through index theorems
- The matter-antimatter asymmetry arises naturally: the two boundary components $(S^3 \times \mathbb{R})_+$ and $(S^3 \times \mathbb{R})_-$ carry opposite chiralities, hence opposite particle content

Consistency with the G_0 constraint: The total anomaly flux is conserved:

$$\int_{\Sigma_+} J^\mu n_\mu + \int_{\Sigma_-} J^\mu n_\mu = 0 \qquad (107)$$

This ensures that the net fermion number across both universes is zero, consistent with the global $G_0 = 0$ energy balance. The apparent baryon asymmetry in our universe is compensated by an anti-baryon excess in the mirror sector.

This represents a significant constraint on any attempt to formulate a complete quantum theory of the primordial state, but also provides a **mechanism** rather than mere hand-waving for fermion emergence.

5.2.4 D) Lack of Dynamic Mechanism

The hypothesis describes the initial state but does not explain:

- Why that specific geometry?
- What determines the parameters (torus size, etc.)?
- Is the configuration a dynamic attractor?

5.2.5 E) Quantum Field Theory Obstructions

The formulation of quantum field theory on spacetimes with CTCs faces fundamental obstacles:

Kay-Radzikowski-Wald Theorem (1997): For spacetimes with compactly generated Cauchy horizons (including those with CTCs), there exists **no** well-defined vacuum state satisfying the Hadamard condition. Specifically:

- The renormalized stress-energy tensor $\langle T_{\mu\nu}\rangle_{\text{ren}}$ diverges at the chronology horizon
- Standard regularization techniques (point-splitting, dimensional regularization) fail because the geodesic distance function $\sigma(x, x')$ becomes multivalued
- The Bunch-Davies vacuum is undefined (no past infinity in CTCs)

Our position: We do not claim to resolve these obstructions within conventional QFT. Rather, we propose:

1. The Planck scale provides a natural UV cutoff that regularizes divergences
2. A complete non-perturbative quantum gravity theory is required for the primordial regime
3. The very concept of "vacuum state" may need reformulation in the pre-Big Bang epoch

This is an acknowledged limitation: the quantum aspects of our model are effective descriptions, not derived from first principles.

5.2.6 F) Quantum-Gravitational Connection

In the trans-Planckian regime, classical general relativity does not apply. A complete quantum gravity theory is needed to:

- Describe the "exotic state" and its vacuum structure
- Regularize QFT divergences on CTC geometries
- Model the transition/collapse
- Calculate precise predictions

5.3 Modifications for Greater Robustness

5.3.1 A) Incorporate Loop Quantum Gravity

Use the LQC formalism to regularize singularities:

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) + \text{holonomy corrections} \qquad (108)$$

This would provide a natural bounce mechanism.

5.3.2 B) Field Model

Introduce a scalar field Φ parameterizing the transition, with non-minimal coupling to curvature:

$$S = \int d^4x\sqrt{-g}\Big[\frac{R}{16\pi G} - \frac{1}{2}\xi R\Phi^2 - \frac{1}{2}(\partial\Phi)^2 - V(\Phi)\Big] \quad (109)$$

where ξ is the non-minimal coupling constant ($\xi = 0$ for minimal coupling, $\xi = 1/6$ for conformal coupling). The coupling $\xi R\Phi^2$ is essential for:

- Regularizing the behavior near the signature-change surface
- Ensuring the correct conformal limit for massless fields
- Coupling the topological transition to spacetime curvature

The potential $V(\Phi) = \frac{\lambda}{4}(\Phi^2 - v^2)^2$ has minima at $\Phi = \pm v$ corresponding to the two separated planes.

5.3.3 C) Connection with String Theory

The non-trivial topology could emerge naturally from:

- Calabi-Yau compactifications
- Conifold transitions
- Colliding branes (ekpyrotic model)

5.3.4 D) Thermodynamic Development

Formulate a thermodynamics of the primordial state:

$$dS_{\text{univ}} = dS_+ + dS_- + dS_{\text{conn}} \quad (110)$$

Collapse would be triggered when $dS_{\text{total}} = 0$ becomes unstable.

5.3.5 E) Selection Principle

Add a criterion explaining why this configuration:

- Weak anthropic principle
- Euclidean action minimization
- Dynamic attractor in configuration space

6 Required Future Theoretical Framework

6.1 Aspects Requiring New Physics

1. **Non-Perturbative Quantum Gravity**
   - To describe the trans-Planckian regime
   - Candidates: LQG, string theory, dynamical triangulations
2. **Thermodynamics of Spacetimes with CTCs**
   - Generalization of the second law
   - Entropy definition for temporal loops
3. **Cosmology of Topological Transitions**
   - Nucleation rates of new topologies
   - Signature change dynamics

6.2 Required Experimental Developments

1. **High-precision CMB** (PICO, LiteBIRD)
2. **Low-frequency gravitational waves** (LISA, DECi-hertz)
3. **High-resolution black hole imaging** (EHT ngVLA)
4. **Primordial gravitational wave detection** (CMB-S4)

7 Conclusions

The primordial 4D torus hypothesis presents a speculative but internally consistent cosmological proposal that:

1. **Can be mathematically formalized** using differential geometry and topology
2. **Connects with established physics** (Schwarzschild, Hartle-Hawking, LQC, CCC)
3. **Generates falsifiable predictions** that are specific and observationally accessible
4. **Requires additional development** in quantum gravity to be fully rigorous

The elegance of unifying the Big Bang origin, the nature of black holes, and the black/white asymmetry under a single geometric framework is notable. The greatest challenges are the need for exotic energy and the lack of an underlying quantum gravity theory.

Overall assessment: The hypothesis deserves additional theoretical development and belongs to the category of "informed speculation" along with models such as the ekpyrotic universe, brane cosmology, or the inflationary multiverse.

Appendix: Key Equations Summary

Primordial Energy Symmetry (G_0)

$$G_0 = \lim_{n \to n_c} \sum_{k=1}^{n} [(+\epsilon_k) + (-\epsilon_k)] = 0 \tag{111}$$

Energy-Entropy Asymmetry

$$\text{Energy:} \quad \sum E = 0 \tag{112}$$

(exact, T-symmetric)

$$\text{Entropy:} \quad S(n) \sim \log(n) \tag{113}$$

(accumulative, T-asymmetric)

Topological Structure ($K^2 \times T^2$)

$$\begin{aligned} \pi_1(K^2 \times T^2) &= \langle a, b, c, d \mid aba^{-1}b = 1, [c, d] = 1, \ldots \rangle \\ b_0 = 1,\ b_1 &= 3,\ b_2 = 3,\ b_3 = 1,\ b_4 = 0 \end{aligned} \tag{114}$$

Non-Orientable Identification (Klein twist)

$$\begin{aligned} &(t, x, y, z) \sim (t + L_t, x, y, z) \quad \text{(periodicity)} \\ &(t, x, y, z) \sim (t + L_t/2, -x, y, z) \quad \det(J) = -1 \end{aligned} \tag{115}$$

Signature Change (Euclidean $\to$ Lorentzian)

$$\begin{aligned} ds_E^2 &= d\tau^2 + a(\tau)^2 d\Omega^2_{K^2 \times T^2} \quad (\tau < 0) \\ ds_L^2 &= -dt^2 + a(t)^2 d\Omega_3^2 \quad (t > 0) \end{aligned} \tag{116}$$

Topological Entropy (Ray-Singer)

$$S_{\text{top}} = k_B \log T(M^4), \quad \log T = \frac{1}{2} \sum_{q=0}^{4} (-1)^q q\, \zeta'_{\Delta_q}(0) \tag{117}$$

Collapse Condition

$$S_{\text{ent}}(n) \longrightarrow S_{\max} - S_{\text{top}} = \frac{A_{\text{eff}}}{4 l_P^2} - k_B \log T(M^4) \tag{118}$$

Exotic Stress-Energy Tensor

$$\begin{aligned} T_{\mu\nu} &= \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu) \\ &\text{with } \rho + p < 0 \text{ (NEC violation)} \end{aligned} \tag{119}$$

Collapse Equation

$$\frac{\partial^2 \Phi}{\partial t^2} = c^2 \nabla^2 \Phi - \lambda \Phi (\Phi^2 - v^2) \tag{120}$$

(derived from $V(\Phi) = \frac{\lambda}{4}(\Phi^2 - v^2)^2$)

CMB Prediction

$$\frac{C(\pi)}{C(0)} > \left[\frac{C(\pi)}{C(0)} \right]_{\Lambda\text{CDM}} \tag{121}$$

Note on Methodology

This work is presented as a foundational hypothesis for a pre-Big Bang cosmology. While the mathematical arguments regarding topological invariants are standard, their application to the specific geometry proposed here is a conceptual proposal intended to stimulate further discussion in the field of non-perturbative quantum gravity.

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