MAXIMUM LIKELIHOOD ESTIMATION OF MULTINOWIAL DISTRIBUTION Setting: - X = {x₁, x₂,..., x_N} N observations. x: :- a category out of K categories. :- [0,0,0,...,1,0,...,0] kth index :- One hot representation - Assume an underlying Multinomial distribution. — X was generated by sampling N' times from the multihornial. Question: What is the multinomial distribution that generated X? Some notation: Let $m = [m_1, m_2, ..., m_K]$ be a count vector on K categories. $m_1 + m_2 + \cdots + m_K = N$

then a multinomial distribution is a 2 parameter distribution

$$P(m|\Pi,N) = Multinomial(m|\Pi,N)$$

$$= (N|m_1 m_2 ... m_k) |\Pi_k^{m_k}|$$

$$m = n_{N} = [0, 0, 0, \dots, 1, 0, \dots, 0]$$

$$N = 1$$

$$\text{and} \qquad \left(\frac{1}{0 \cdot 0 \cdot \dots \cdot 1 \cdot \cdot \cdot 0}\right) = 1$$

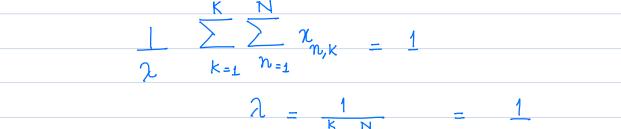
So
$$P(\chi_{m} \mid \overline{\Pi}, 1) = \frac{K}{\prod_{k=1}^{n} \chi_{n,k}}$$

Now writing
$$\lambda(\Pi)$$
 for the dataset χ

$$\lambda(\Pi) = \log \prod P(\chi_m | \Pi, 1)$$

$$\chi(\Pi) = \log \prod P(\chi_m | \Pi, 1)$$

$$= \log \prod_{N=1}^{K} \prod_{k=1}^{N_{n,k}}$$



$$\frac{\lambda - 1}{\sum_{k=1}^{K} \sum_{m=1}^{N} n_{m,k}} = \frac{1}{N}$$

Hence
$$T_k = \sum_{m=1}^{N} x_{m,k} / N$$