say, where ε_0 is the vacuum permittivity. Now matching the solution of (1.2), $\phi \sim \exp(-r/\lambda_{\mathrm{D}})/r$, with the potential $\phi = Ze/4\pi\varepsilon_0 r$ as $r \to 0$ we see that

$$\phi(r) = \frac{Ze}{4\pi\varepsilon_0 r}\exp(-r/\lambda_{\mathrm{D}}) \tag{1.3}$$

where

$$\lambda_{\mathrm{D}} = \left(\frac{\varepsilon_0 k_{\mathrm{B}} T_{\mathrm{e}}}{n_{\mathrm{e}} e^2}\right)^{1/2} \simeq 7.43 \times 10^3 \left(\frac{T_{\mathrm{e}}(\mathrm{eV})}{n_{\mathrm{e}}}\right)^{1/2} \mathrm{m} \tag{1.4}$$

is called the *Debye shielding length*. Beyond a *Debye sphere*, a sphere of radius λ_{D}, centred at P, the plasma remains effectively neutral. By the same argument λ_{D} is also a measure of the penetration depth of external electrostatic fields, i.e. of the thickness of the boundary sheath over which charge neutrality may not be maintained.

The plausibility of the argument used to establish (1.3) requires that a large number of electrons be present within the Debye sphere, i.e. $n_{\mathrm{e}}\lambda_{\mathrm{D}}^3 \gg 1$. The inverse of this number is proportional to the ratio of potential energy to kinetic energy in the plasma and may be expressed as

$$g = \frac{e^2}{\varepsilon_0 k_{\mathrm{B}} T_{\mathrm{e}} \lambda_{\mathrm{D}}} = \frac{1}{n_{\mathrm{e}}\lambda_{\mathrm{D}}^3} \ll 1 \tag{1.5}$$

Since g plays a key role in the development of formal plasma theory it is known as the *plasma parameter*. Broadly speaking, the more particles there are in the Debye sphere the less likely it is that there will be a significant resultant force on any given particle due to 'collisions'. It is, therefore, a measure of the dominance of collective interactions over collisions.

The most fundamental of these collective interactions are the *plasma oscillations* set up in response to a charge imbalance. The strong electrostatic fields which drive the electrons to re-establish neutrality cause oscillations about the equilibrium position at a characteristic frequency, the *plasma frequency* ω_{p}. Since the imbalance occurs over a distance λ_{D} and the electron thermal speed V_{e} is typically $(k_{\mathrm{B}} T_{\mathrm{e}}/m_{\mathrm{e}})^{1/2}$ we may express the electron plasma frequency ω_{pe} by

$$\omega_{\mathrm{pe}} = \frac{(k_{\mathrm{B}} T_{\mathrm{e}}/m_{\mathrm{e}})^{1/2}}{\lambda_{\mathrm{D}}} = \left(\frac{n_{\mathrm{e}} e^2}{m_{\mathrm{e}}\varepsilon_0}\right)^{1/2} \tag{1.6}$$

which reduces to $\omega_{pe} \simeq 56.4 n_e^{1/2}\,\mathrm{s}^{-1}$. Note that any applied fields with frequencies less than the electron plasma frequency are prevented from penetrating the plasma by the more rapid electron response which neutralizes the field. Thus a plasma is not transparent to electromagnetic radiation of frequency $\omega < \omega_{pe}$. The corresponding frequency for ions, the *ion plasma frequency* ω_{pi}, is defined by

$$\omega_{pi} = \left(\frac{n_i (Ze)^2}{m_i \varepsilon_0}\right)^{1/2} \simeq 1.32 Z \left(\frac{n_i}{A}\right)^{1/2} \tag{1.7}$$

where Z denotes the charge state and A the atomic number.

1.4.1 Collisions and the plasma parameter

We have seen that the effective range of an electric field, and hence of a collision, is the Debye length λ_D. Thus any particle interacts at any instant with the large number of particles in its Debye sphere. Plasma collisions are therefore *many-body interactions* and since $g \ll 1$ collisions are predominantly weak, in sharp contrast with the strong, binary collisions that characterize a neutral gas. In gas kinetics a collision frequency ν_c is defined by $\nu_c = n V_{th} \sigma(\pi/2)$ where $\sigma(\pi/2)$ denotes the cross-section for scattering through $\pi/2$ and V_{th} is a thermal velocity. Such a deflection in a plasma would occur for particles 1 and 2 interacting over a distance b_0 for which $e_1 e_2 / 4\pi\varepsilon_0 b_0 \sim k_B T$ so that $\nu_c = (n V_{th} \pi b_0^2)$. However, the cumulative effect of the much more frequent weak interactions acts to increase this by a factor $\sim 8 \ln(\lambda_D / b_0) \approx 8 \ln(4\pi n \lambda_D^3)$. For electron collisions with ions of charge Ze it follows that the electron–ion collision time $\tau_{ei} \equiv \nu_{ei}^{-1}$ is given by

$$\tau_{ei} = \frac{2\pi \varepsilon_0^2 m_e^{1/2} (k_B T_e)^{3/2}}{Z^2 n_i e^4 \ln \Lambda} \tag{1.8}$$

where $\ln \Lambda = \ln 4\pi n \lambda_D^3$ is known as the *Coulomb logarithm*. For singly charged ions the electron–ion collision time is

$$\tau_{ei} = 3.44 \times 10^{11} \frac{T_e^{3/2}\,(\mathrm{eV})}{n_i \ln \Lambda}\,\mathrm{s}$$

in which we have replaced the factor 2π in (1.8) with the value found from a correct treatment of plasma transport in Chapter 12. The Coulomb logarithm is

$$\ln \Lambda = 6.6 - \frac{1}{2} \ln\left(\frac{n}{10^{20}}\right) + \frac{3}{2} \ln T_e\,(\mathrm{eV})$$

The *electron mean free path* $\lambda_e = V_e \tau_{ei}$ is

$$\lambda_e = 1.44 \times 10^{17} \frac{T_e^2 (\mathrm{eV})}{n_i \ln \Lambda}$$

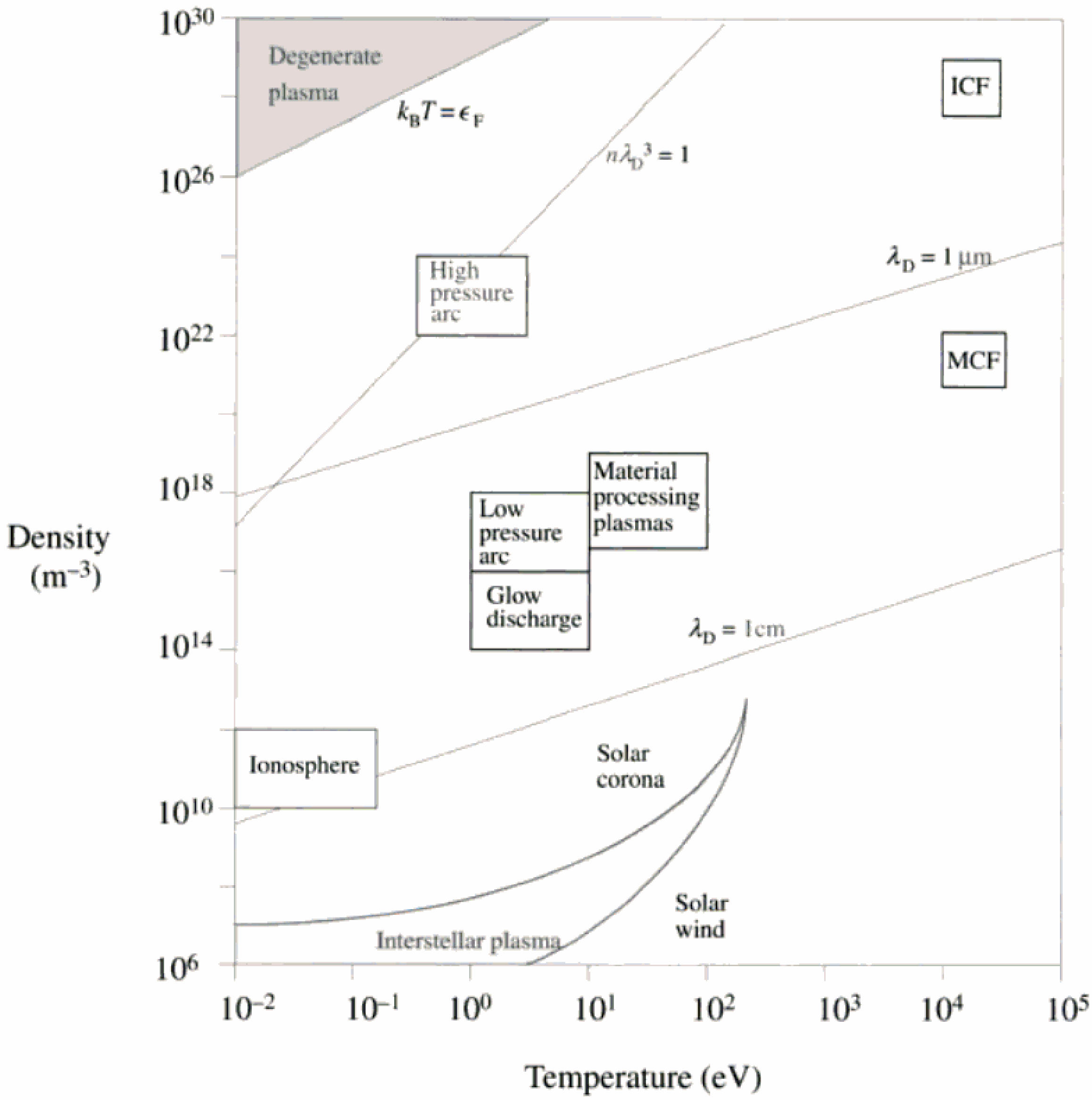

Fig. 1.4. Landmarks in the plasma universe.

Table 1.1 lists approximate values of various plasma parameters along with typical values of the magnetic field associated with each for a range of plasmas across the plasma universe. These and other representative plasmas are included in the diagram of parameter space in Fig. 1.4 which includes the parameter lines $\lambda_D = 1\ \mu m$, $1\ cm$ and $n\lambda_D^3 = 1$ together with the line marking the boundary at which plasmas become degenerate $k_B T = \epsilon_F$, where ϵ_F denotes the Fermi energy.

Table 1.1. *Approximate values of parameters across the plasma universe.*

Plasma	n (m^{-3})	T (keV)	B (T)	ω_{pe} (s^{-1})	λ_D (m)	$n\lambda_D^3$	ν_{ei} (Hz)
Interstellar	10^6	10^{-5}	10^{-9}	$6 \cdot 10^4$	0.7	$3 \cdot 10^5$	$4 \cdot 10^8$
Solar wind (1 AU)	10^7	10^{-2}	10^{-8}	$2 \cdot 10^5$	7	$4 \cdot 10^9$	10^{-4}
Ionosphere	10^{12}	10^{-4}	10^{-5}	$6 \cdot 10^7$	$2 \cdot 10^{-3}$	10^4	10^4
Solar corona	10^{12}	0.1	10^{-3}	$6 \cdot 10^7$	0.07	$4 \cdot 10^8$	0.5
Arc discharge	10^{20}	10^{-3}	0.1	$6 \cdot 10^{11}$	$7 \cdot 10^{-7}$	40	10^{10}
Tokamak	10^{20}	10	10	$6 \cdot 10^{11}$	$7 \cdot 10^{-5}$	$3 \cdot 10^7$	$4 \cdot 10^4$
ICF	10^{28}	10	—	$6 \cdot 10^{15}$	$7 \cdot 10^{-9}$	$4 \cdot 10^3$	$4 \cdot 10^{11}$