Types of Limits

You can find different types of limits on the internet but basically, there are three types of limits.

- One-sided limit means that limit exists from one side, left or right.
- A two-sided limit means a limit exists from both sides.
- Infinite limit means function increases or decreases boundlessly.

Limit notation

The example above will be represented as:

Limits notation

This is called limit notation. It is read as “The limit of the function (insert function) as x (or the function’s variable) approaches to (insert undefined point).

How to apply limits?

Limits are easy to find. Just put the value in the function and you will have the limit. If you want to know “whether the limit exists or not” find both right and left-sided limits.

If the values are the same, a limit exists. Alternatively, you can use the Limit calculator to solve the limits.

Example:

Evaluate the limit from the right for function 1/(x-2) at 2?

Solution: Write the function:

$$f(x) = \frac{1}{x-2}$$

Apply the limit on the right side.

$$= \lim_{x \to 2^+} \left(\frac{1}{x-2} \right)$$

$$\lim_{x \to 2^+} \left(\frac{1}{x-2} \right) = \infty$$

The limit from the right is at infinity.

So, this was the basic introduction of limits. Let’s move to the second part of our topic.

One-sided limit means that limit exists from one side, left or right.

Left-Hand Limit

Let II be an open interval containing cc, and let ff be a function defined on II, except possibly at cc. The **limit of** f(x)f(x), **as** xx **approaches** cc **from the left**, is LL, or, **the left--hand limit of** ff **at** cc is LL, denoted by

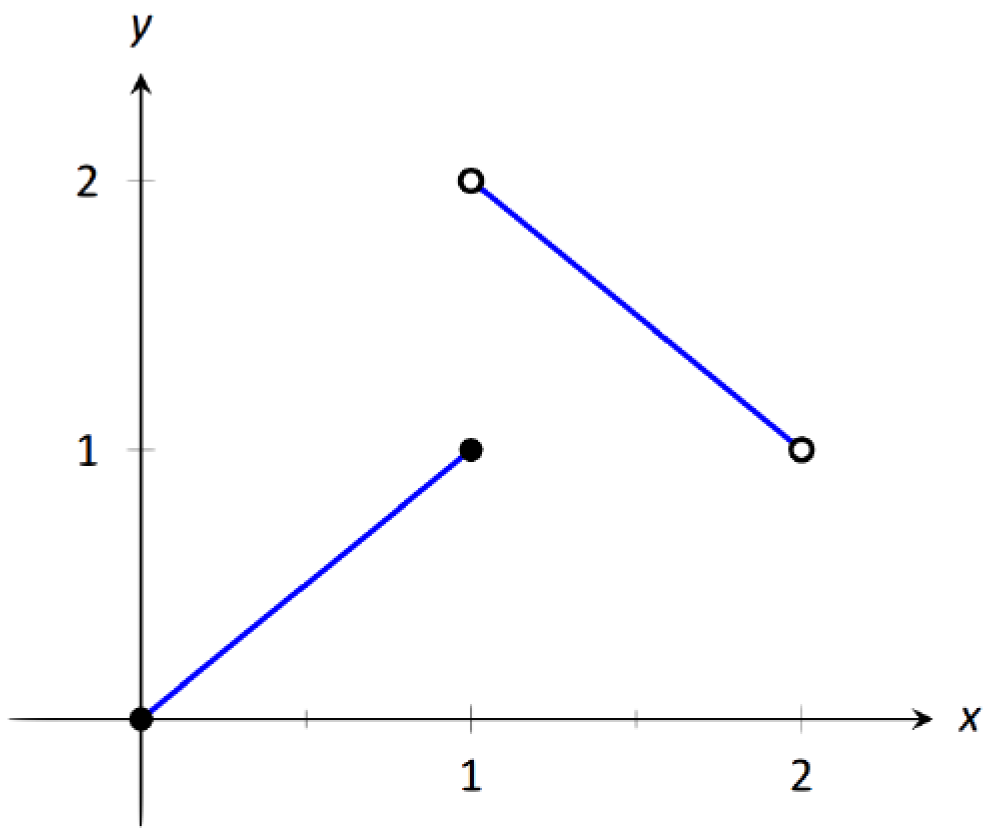
$$\lim_{x \to c^-} f(x) = L$$

Right-Hand Limit

Let II be an open interval containing cc, and let ff be a function defined on II, except possibly at cc. The **limit of** f(x)f(x), **as** xx **approaches** cc **from the right, is** LL, or, **the right--hand limit of** ff **at** cc **is** LL, denoted by

$$\lim_{x \to c^+} f(x) = L$$

➢ ***For example :***



$$f(x) = \begin{cases} x, & 0 \geq x \geq 1 \\ 3 - x, & 1 < x < 2 \end{cases}$$

1. $\lim\limits_{x \to 1^-} f(x)$ $\qquad\qquad$ $\lim\limits_{x \to 2^-} f(x)$

   $\lim\limits_{x \to 1^+} f(x)$

$\lim_{x \to 1^-} f(x) = 1$.

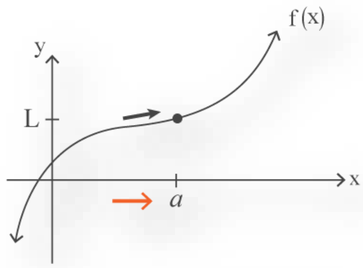
$\lim_{x \to 2^-} f(x) = 1$

$\lim f(x) = 2$

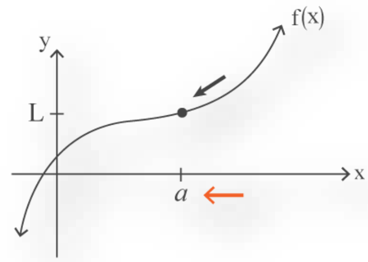
- A two-sided limit means a limit exists from both sides.

➢ Overview of Two-Sided Limits

Let f(x) be a function and limit of x from left-hand side is given by $L = \lim_{x \to a-} f(x)$ and the right-hand side is given R = $\lim_{x \to a+} f(x)$
Now if L = R then the value obtained by L = R is called double sided limit.



$$\lim_{x \to a^-} f(x) =$$ left-sided limit



$$\lim_{x \to a^+} f(x) =$$ right-sided limit

- Infinite limit means function increases or decreases boundlessly.

DEFINITION OF AN INFINITE LIMIT

Let f(x) be a function that can be defined on either side of a point a, and may or may not be defined at a:

$\lim_{x \to a} f(x) = +\infty$ ***or*** $\lim_{x \to a} f(x) = -\infty$

means as x approaches a, but not equal to a, the value of f(x) increase/decreases without bound.

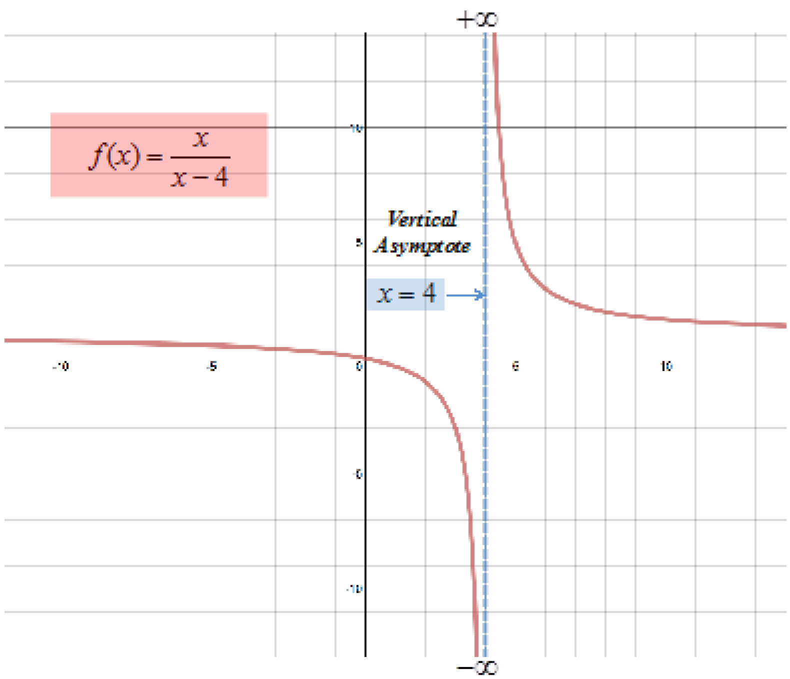
The line at which the limit of a function increases or decreases without bound is called a **vertical asymptote**.

DEFINITION OF A VERTICAL ASYMPTOTE :

The line x = a is a vertical asymptote of f(x) if one of the following is true:

$\lim_{x \to a} f(x) = \infty$ $\lim_{x \to a^-} f(x) = \infty$ $\lim_{x \to a^+} f(x) = \infty$

$\lim_{x \to a} f(x) = -\infty$ $\lim_{x \to a^-} f(x) = -\infty$ $\lim_{x \to a^+} f(x) = -\infty$



Because lim x→ 4 − f(x)=−∞ **;** lim x→ 4 + f(x)=∞ **, the line** x=4 **is a vertical asymptote.**

DEFINITION OF A VERTICAL ASYMPTOTE :

The line x = a is a vertical asymptote of f(x) if one of the following is true: