MMAE 500: Data Driven Modeling

Homework 3

Assigned: 25 February 2024
Due: 22 March 2024

1. Consider the data generated by the file hw3Q1.m (in Matlab) or hw3Q1.ipynb (Python notebook), which is assembled into the matrix X. This is the same data that has been used in the previous homeworks.
   (a) Using the QR pivoting method discussed in lecture 12, determine the best two spatial locations to measure this system in order to reconstruct the data. Can you relate these to the functions that are used to generate the data?
   (b) Compute the dynamic mode decomposition of X. (That is, find a matrix A that maps each column of X one step forward in time, and compute the eigendecomposition of this matrix). Plot the eigenvectors of A that correspond to nonzero eigenvalues. How do they compare with the singular vectors obtained from taking an SVD of the data?
   (c) How do the nonzero eigenvalues of A identified in part (b) relate to the parameters used to define the data? (HINT: you might want to convert the discrete-time eigenvalues λ_d to continuous-time eigenvalues λ_c via $\lambda_c = \log(\lambda_d)/dt$.)

2. Consider the data generated by the file hw3Q3.ipynb (Python notebook), which is assembled into the matrix X. This is similar to the data considered previously, but note that now the width of one of the component functions varies over time. For this question, the code from lecture 9 may be a helpful reference.
   (a) Design and train an autoencoder to find a reduced-dimension approximation of the data, using linear activation functions and an encoded dimension of 4. Visualize the autoencoder modes (latent variables), and compare them to the appropriate singular vectors from the SVD of the data.
   (b) Now consider an autoencoder with nonlinear activation functions. Are you able to find an autoencoder that does substantially better than the SVD at finding a four-dimensional respresentation/encoding of the data? You may need to explore the use of multiple hidden layers, and different types of activation functions.