Data Mining II

Sequential Patterns

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Summary

Sequential Patterns
Basic Concepts

Motivation

- For Association Rules Mining the order of transactions is not important.
- But, for many applications the order might be significant.
  - Commerce: customer buys a computer, a printer and then photographic paper.
  - Web: sequences of pages visited are useful to find navigational patterns.
  - Text: order of words in a sentence is useful to find linguistic patterns.
  - ...

Sequences

- Given a set of items $I = \{i_1, i_2, \cdots, i_m\}$, a **sequence** is a ordered list of itemsets, i.e. $s =< a_1 a_2 \cdots a_r >$ where a_i is an itemset.
- The items in each element (event) a_i are in lexicographic order.
- A given item can occur only once in a element, but can occur multiple times in a sequence.
- Example:
  - $I = \{a, b, c, d, \cdots\}$
  - $s =< \{a, b\}, \{a, c\}, \{d\}, \{b, c, d\} >$

Sequences (cont.)

- The size of a sequence is the number of elements (itemsets) in the sequence.
- The length of a sequence os the number of items in the sequence.
- A sequence of length k is called a k-sequence
- Example:
  - $s =< \{a, b\}, \{a, c\}, \{d\}, \{b, c, d\} >$
  - $size(s) = 4$
  - $length(s) = 8$

Sequences (cont.)

- A sequence $s_1 = < a_1 a_2 \cdots a_r >$ is a subsequence of $s_2 = < b_1 b_2 \cdots b_v >$ if there exist integers $1 \leq j_1 < j_2 < \cdots < j_{r-1} < j_r \leq v$ such that $a_1 \subseteq b_{j_1}, a_2 \subseteq b_{j_2}, \cdots, a_r \subseteq b_{j_r}$
- In that case s_2 is also said to be a supersequence of s_1.
- Example
    - $s = < \{a\}, \{a, b, c\}, \{a, c\}, \{d\}, \{c, f\} >$

subsequences of s

- $< \{a\}, \{a\}, \{a, c\}, \{d\}, \{c\} >$
- $< \{a, c\}, \{a, c\}, \{d\}, \{c, f\} >$
- $< \{a\}, \{c\} >$

not subsequences of s

- $< \{d\}, \{f\}, \{c, f\} >$
- $< \{c, f\}, \{d\} >$
- $< \{a, b, c\}, \{d\}, \{c\}, \{f\} >$

Sequences (cont.)

- A sequence database consists of ordered elements or events
- Transaction database vs sequence database

Transaction database

TID	Itemsets
1	{a, b, c}
2	{a, c, d}
3	{a, d, e}
4	{b, e, f}

Sequence database

SID	Sequences
1	<{a},{a, b, c},{a,c},{d},{c,f}>
2	<{a,d},{c},{b,c},{a,e}>
3	<{e,f},{a,b},{d,f},{c},{b}>
4	<{e},{g},{a,f},{c},{b},{c}>

Sequences (cont.)

- The support of a sequence is the fraction of total data sequences that contain that sequence.

- Example

Sequence database

SID	Sequences
1	<{a},{a, b, c},{a,c},{d},{c,f}>
2	<{a,d},{c},{b,c},{a,e}>
3	<{e,f},{a,b},{d,f},{c},{b}>
4	<{e},{g},{a,f},{c},{b},{c}>

- sup(<{a}>) = 4
- sup(<{a},{c}>) = 4
- sup(<{a,b},{c}>) = 2

Sequential Pattern Mining Task

- Given
  - a sequence database *S*
  - minimum support *minsup*
- Find
  - **all** frequent sequences, i.e. sequences with support above *minsup*

Sequential Pattern Mining Task (cont.)

- Example

Transactions Database

Customer ID	Transactions
1	30
1	90
2	10,20
2	30
2	10,40,60,70
3	30,50,70,80
4	30
4	30,40,70,80
4	90
5	90

Sequences Database

Customer ID	Data Sequences
1	<{30},{90}>
2	<{10,20},{30},{10,40,60,70}>
3	<{30,50,70,80}>
4	<{30},{30,40,70,80},{90}>
5	<{90}>

	Sequential Patterns with minsup=25% (2)
1-sequences	<{30}>,<{40}>,<{70}>,<{80}>,<{90}>
2-sequences	<{30},{40}>,<{30},{70}>,<{30},{90}>, <{30,70}>,<{30,80}>,<{40,70}>,<{70,80}>
3-sequences	<{30},{40,70}>, <{30,70,80}>

Sequential Pattern Mining Task (cont.)

- Challenges
  - A huge number of possible sequential patterns are hidden in databases
  - A mining algorithm should
    - find the complete set of patterns, when possible, satisfying the minimum support (frequency) threshold
    - be highly efficient, scalable, involving only a small number of database scans
    - be able to incorporate various kinds of user-specific constraints

Mining Sequential Patterns

Methods for Mining Sequential Patterns

- Apriori-based Approaches
  - **GSP** [Srikant and Agrawal, 1996]
  - SPADE [Zaki, 2001]: vertical format-based mining
  - ...
- Pattern-Growth-based Approaches
  - **PrefixSpan** [Pei et al., 2001]
  - CloSpan [Yan et al., 2003]: mining closed sequential patterns
  - ...

GSP Algorithm

GSP: Generalized Sequential Pattern Mining Algorithm

[Srikant and Agrawal, 1996]

- very similar to Apriori
- C_k: set of all candidate k-sequences
- F_k: set of all frequent k-sequences
- uses Apriori Property to prune candidates:
    - if a sequence s is not frequent, then none of the supersequences of s is frequent.
    - Example ($minsup = 25\%(2)$)

Seq. ID	Sequence
1	<(bd)cb(ac)>
2	<(bf)(ce)b(fg)>
3	<(ah)(bf)abf>
4	<(be)(ce)d>
5	<a(bd)bcb(ade)>

<hb> is infrequent, so <hab> and <(ah)b>

(simplified notation: <a(bc)> is <{a},{b,c}>)

- Example:

SID	Data Sequences
1	<3,9>
2	<(12),3,(1467)>
3	<(3578)>
4	<3,(3478),9>
5	<9>

- $minsup = 25\%(2)$
- $C_1 = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- $F_1 = \{\langle 3\rangle, \langle 4\rangle, \langle 7\rangle, \langle 8\rangle, \langle 9\rangle\}$

GSP Algorithm: identifying frequent sequences (cont.)

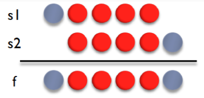
- Example:

SID	Data Sequences
1	<3,9>
2	<(12),3,(1467)>
3	<(3578)>
4	<3,(3478),9>
5	<9>

- $F_1 = \{\langle 3\rangle, \langle 4\rangle, \langle 7\rangle, \langle 8\rangle, \langle 9\rangle\}$
- $C_2 = \{\langle 3, 3\rangle, \langle 34\rangle, \langle 3, 4\rangle, \langle 4, 3\rangle, \langle 37\rangle, \langle 3, 7\rangle, \cdots\}$
- how to generate candidate sequences?

GSP Algorithm: identifying frequent sequences (cont.)

Candidate k-sequence generation



Join step

- $s_1, s_2 \in F_{k-1}$
- drop first item A in s_1 and last item Z in s_2
- if (common = s_1-A) == s_2-Z
  - candidate $c = A + common + Z$
  - two possibilities
    - Z is separate if it was separate in s_2
    - Z is added to the last set if it was part of last set in s_2

Prune step

- Candidate k-sequence is pruned if any of its (k-1)-subsequences is infrequent.

GSP Algorithm: identifying frequent sequences (cont.)

- $F_1 = \{\langle 3\rangle, \langle 4\rangle, \langle 7\rangle, \langle 8\rangle, \langle 9\rangle\}$
- Join step (no prune)
  $C_2 = \{\langle 3,3\rangle, \langle 3,4\rangle, \langle 34\rangle, \langle 3,7\rangle, \langle 37\rangle, \cdots \langle 4,3\rangle, \cancel{\langle 43\rangle}, \cdots, \langle 89\rangle\}$
- Join $\langle a\rangle$ and $\langle b\rangle$
- common part is $\langle\rangle$
- combinations $a + \langle\rangle + b$
  - b was separate in $\langle b\rangle$, so we have $\langle a, b\rangle$
  - b is also part of the last set in $\langle b\rangle$, so we have $\langle ab\rangle$
- if $a = b$ then $\langle ab\rangle$ is excluded because ab is a set.
- if we have $\langle ab\rangle$, $\langle ba\rangle$ is excluded because ab is a set.

GSP Algorithm: identifying frequent sequences (cont.)

- $F_3 = \{\langle 12, 4\rangle, \langle 12, 5\rangle, \langle 1, 45\rangle, \langle 14, 6\rangle, \langle 2, 45\rangle, \langle 2, 4, 6\rangle\}$
- Join step
  $C_4 = \{\langle 12, 4, 6\rangle, \langle 12, 45\rangle\}$
  - Join $\langle 12, 4\rangle$ and $\langle 2, 4, 6\rangle$, common part is $\langle 2, 4\rangle$
  - combination $1 + \langle 2, 4\rangle + 6$
  - 6 is separate, so we have $\langle 12, 4, 6\rangle$
  - Join $\langle 12, 4\rangle$ and $\langle 2, 45\rangle$, common part is $\langle 2, 4\rangle$
  - combination $1 + \langle 2, 4\rangle + 5$
  - 6 part of 45, so we have $\langle 12, 45\rangle$
- Prune step
  - delete each item at a tim and check if subseq is frequent
  - $\xcancel{\langle 12, 4, 6\rangle}$ because $\langle 1, 4, 6\rangle \notin F_3$
- $C_4 = \{\langle 12, 45\rangle\}$

GSP Algorithm: Exercises

1. Given $I = \{1, 2, 3, 4\}$ and $S = \{\langle 1, 2\rangle, \langle 12, 3\rangle, \langle 2, 13, 4\rangle, \langle 2, 12, 3\rangle\}$
   Determine the following sets with $minsup = 50\%$
   - C_1, F_1
   - C_2, F_2
   - C_3, F_3
2. Given the following transactions data base, mine the frequent sequences with $minsup = 70\%$

Customer ID	Itemsets
1	milk,bread
1	coffee, meat
2	water, bread
2	coffee, meat
3	milk, bread
3	coffee, soup

GSP Algorithm Bottlenecks

- GSP benefits from Apriori pruning
  - reduces search space
- GSP Bottlenecks
  - generates a huge set of candidate sequences (especially 2-item candidate sequences)
  - example: for 6 frequent 1-item sequences, it generates $6 * 6 + 6 * 5/2 = 51$ 2-item candidates sequences.

	<a>	<b>	<c>	<d>	<e>	<f>
<a>	<aa>	<ab>	<ac>	<ad>	<ae>	<af>
<b>	<ba>	<bb>	<bc>	<bd>	<be>	<bf>
<c>	<ca>	<cb>	<cc>	<cd>	<ce>	<cf>
<d>	<da>	<db>	<dc>	<dd>	<de>	<df>
<e>	<ea>	<eb>	<ec>	<ed>	<ee>	<ef>
<f>	<fa>	<fb>	<fc>	<fd>	<fe>	<ff>

	<a>	<b>	<c>	<d>	<e>	<f>
<a>		<(ab)>	<(ac)>	<(ad)>	<(ae)>	<(af)>
<b>			<(bc)>	<(bd)>	<(be)>	<(bf)>
<c>				<(cd)>	<(ce)>	<(cf)>
<d>					<(de)>	<(df)>
<e>						<(ef)>
<f>						

GSP Algorithm Bottlenecks (cont.)

- GSP bottlenecks (cont.)
  - scans the database multiple times
  - the length of each candidate grows by one at each database scan
  - a long pattern grows up from short patterns
  - an exponential number of short candidates
  - inefficient for mining long sequential patterns

PrefixSpan: Prefix-Projected Sequential Pattern Growth

[Pei et al., 2001]

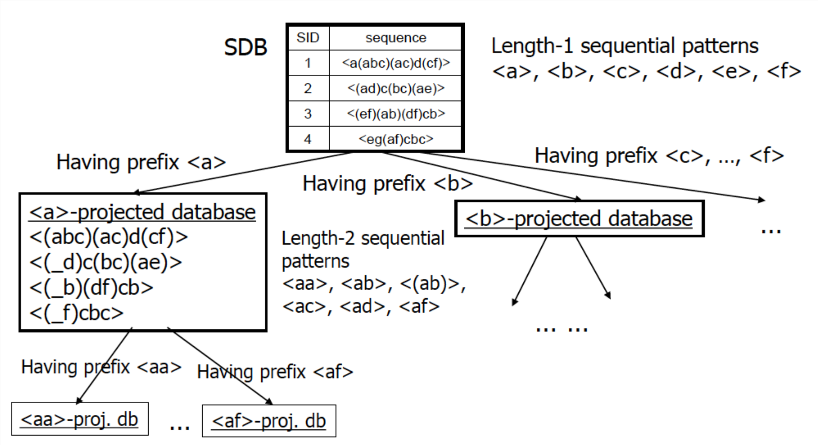
- More efficient and less memory hungry than GSP
- It does not generate candidates
- GSP performs breath-first search
- PrefixSpan performs depth-first search
- It uses prefix-based projection: less projections and quickly shrinking sequences

PrefixSpan Algorithm: Prefix and Suffix (Projection)

- Given the sequence $\langle a(abc)(ac)d(cf)\rangle$
  - $\langle a\rangle$, $\langle aa\rangle$, $\langle a(ab)\rangle$, $\langle a(abc)\rangle$ are **prefixes** of the sequence
  - $\langle ab\rangle$, $\langle a(bc)\rangle$ are **not prefixes** of the sequence
  - For a prefix, we can obtain prefix-projected suffix

Prefix	*Suffix* (Prefix-Based *Projection)*
<a>	<(abc)(ac)d(cf)>
<aa>	<(_bc)(ac)d(cf)>
<ab>	<(_c)(ac)d(cf)>

PrefixSpan Algorithm: Divide-and-Conquer Approach



PrefixSpan Algorithm: Example

- Find length-1 frequent sequential patterns

SID	sequence
1	<a(abc)(ac)d(cf)>
2	<(ad)c(bc)(ae)>
3	<(ef)(ab)(df)cb>
4	<eg(af)cbc>

- $minsup = 50\%$ in support descending order
- $\langle a \rangle$:4, $\langle b \rangle$:4, $\langle c \rangle$:4, $\langle d \rangle$:3, $\langle e \rangle$:3, $\langle f \rangle$:3 $\cancel{\langle g \rangle}$:1

- Divide the search space in 6 (one for each frequent item) to find subsets of sequential patterns with:
  - prefix $\langle a \rangle$ using $\langle a \rangle$-projected database
  - prefix $\langle b \rangle$ using $\langle b \rangle$-projected database
  - ...
  - prefix $\langle f \rangle$ using $\langle f \rangle$-projected database

PrefixSpan Algorithm: Example (cont.)

- Find subsets of sequential patterns with prefix $\langle a \rangle$
  - consider only the subsequences prefixed with the first occurrence of a
  - build $\langle a \rangle$-projected database

    (a: a was a separate element; _: a was a part of the same element)

SID	sequence
1	<a(abc)(ac)d(cf)>
2	<(ad)c(bc)(ae)>
3	<(ef)(ab)(df)cb>
4	<eg(af)cbc>

<a>-projected database
<(abc)(ac)d(cf)>
<(_d)c(bc)(ae)>
<(_b)(df)cb>
<(_f)cbc>

Support counts on the projected database:

- a: 2, b: 4, c: 4, d: 2, $\cancel{e}$: 1, f: 2
- $(\_b)$: 2, $\cancel{(\_c)}$: 1 $\cancel{(\_d)}$: 1, $\cancel{(\_f)}$: 1

- Find all length-2 frequent sequences with prefix $\langle a \rangle$:
  - $\langle aa \rangle$: 2, $\langle ab \rangle$: 4, $\langle (ab) \rangle$: 2, $\langle ac \rangle$: 4, $\langle ad \rangle$: 2, $\langle af \rangle$: 2
- We already have $\langle a \rangle$ prefixed sequences of length 1 and 2.
- Now we partition again by length-2 sequences

PrefixSpan Algorithm: Example (cont.)

- Find frequent sequences with prefix $\langle aa \rangle$
  - $\langle aa \rangle$-projected database

SID	sequence
1	<a(abc)(ac)d(cf)>
2	<(ad)c(bc)(ae)>
3	<(ef)(ab)(df)cb>
4	<eg(af)cbc>

<a>-projected database

<(abc)(ac)d(cf)>
<(_d)c(bc)(ae)>
<(_b)(df)cb>
<(_f)cbc>

<aa>-projected database

<(_bc)(ac)d(cf)>
<(_e)>

  - $\cancel{a}$: 1, $\cancel{b}$: 0, $\cancel{c}$: 1, $\cancel{d}$: 1, $\cancel{e}$: 0, $\cancel{f}$: 1
  - $\cancel{(\_b)}$: 1, $\cancel{(\_c)}$: 1, $\cancel{(\_d)}$: 0, $\cancel{(\_e)}$: 1, $\cancel{(\_f)}$: 0
  - no hope to generate frequent $\langle aa \rangle$ prefix sequences

PrefixSpan Algorithm: Example (cont.)

- Find frequent sequences with prefix $\langle ab \rangle$
  - $\langle ab \rangle$-projected database

SID	sequence
1	<a(abc)(ac)d(cf)>
2	<(ad)c(bc)(ae)>
3	<(ef)(ab)(df)cb>
4	<eg(af)cbc>

<a>-projected database

<(abc)(ac)d(cf)>
<(_d)c(bc)(ae)>
<(_b)(df)cb>
<(_f)cbc>

<ab>-projected database

<(_c)(ac)d(cf)>
<(_c)(ae)>
<(c)>

  - frequent "items": (_c): 2, a:2, c: 2
  - frequent $\langle ab \rangle$ prefix sequences: $\langle a(bc) \rangle$:2, $\langle aba \rangle$:2, $\langle abc \rangle$:2

- Find frequent sequences with prefix $\langle a(bc) \rangle$
  - $\langle a(bc) \rangle$-projected database

SID	sequence
1	<a(abc)(ac)d(cf)>
2	<(ad)c(bc)(ae)>
3	<(ef)(ab)(df)cb>
4	<eg(af)cbc>

<a>-projected database

<(abc)(ac)d(cf)>
<(_d)c(bc)(ae)>
<(_b)(df)cb>
<(_f)cbc>

<ab>-projected database

<(_c)(ac)d(cf)>
<(_c)(ae)>
<(c)>

<a(bc)>-projected database

<(ac)d(cf)>
<(ae)>

  - frequent “items”: a:2
  - frequent $\langle a(bc) \rangle$ prefix sequences: $\langle a(bc)a \rangle$:2

PrefixSpan Algorithm: Example (cont.)

- Find frequent sequences with prefix $\langle aba \rangle$
  - $\langle aba \rangle$-projected database

SID	sequence
1	<a(abc)(ac)d(cf)>
2	<(ad)c(bc)(ae)>
3	<(ef)(ab)(df)cb>
4	<eg(af)cbc>

<a>-projected database

<(abc)(ac)d(cf)>
<(_d)c(bc)(ae)>
<(_b)(df)cb>
<(_f)cbc>

<ab>-projected database

<(_c)(ac)d(cf)>
<(_c)(ae)>
<(c)>

<aba>-projected database

<(_c)d(cf)>
<(_e)>

  - no frequent “items”

PrefixSpan Algorithm: Example (cont.)

- Find frequent sequences with prefix $\langle (ab) \rangle$
  - $\langle (ab) \rangle$-projected database

SID	sequence
1	<a(abc)(ac)d(cf)>
2	<(ad)c(bc)(ae)>
3	<(ef)(ab)(df)cb>
4	<eg(af)cbc>

<a>-projected database

<(abc)(ac)d(cf)>
<(_d)c(bc)(ae)>
<(_b)(df)cb>
<(_f)cbc>

<(ab)>-projected database

<(_c)(ac)d(cf)>
<(df)(cb)>

  - frequent "items": d:2 , c:2 , f:2
  - frequent $\langle (ab) \rangle$ prefix sequences: $\langle (ab)d \rangle$:2, $\langle (ab)c \rangle$:2, $\langle (ab)f \rangle$:2

PrefixSpan Algorithm: Example (cont.)

- Find frequent sequences with prefix $\langle (ab)d \rangle$
  - $\langle (ab)d \rangle$-projected database

SID	sequence
1	<a(abc)(ac)d(cf)>
2	<(ad)c(bc)(ae)>
3	<(ef)(ab)(df)cb>
4	<eg(af)cbc>

<a>-projected database
<(abc)(ac)d(cf)>
<(_d)c(bc)(ae)>
<(_b)(df)cb>
<(_f)cbc>

<(ab)>-projected database
<(_c)(ac)d(cf)>
<(df)(cb)>

<(ab)d>-projected database
<(cf)>
<(_f)(cb)>

  - frequent “items”: c:2
  - frequent $\langle (ab)d \rangle$ prefix sequences: $\langle (ab)dc \rangle$:2

PrefixSpan Algorithm: Example (cont.)

- At the end, for the sequences database

Sequence_id	Sequence
10	$\langle a(abc)(ac)d(cf)\rangle$
20	$\langle (ad)c(bc)(ae)\rangle$
30	$\langle (ef)(ab)(df)cb\rangle$
40	$\langle eg(af)cbc\rangle$

- with *minsup* $=$ 50%, we get the following frequent sequences:

Prefix	Projected (postfix) database	Sequential patterns
$\langle a\rangle$	$\langle(abc)(ac)d(cf)\rangle$, $\langle(\_d)c(bc)(ae)\rangle$, $\langle(\_b)(df)cb\rangle$, $\langle(\_f)cbc\rangle$	$\langle a\rangle$, $\langle aa\rangle$, $\langle ab\rangle$, $\langle a(bc)\rangle$, $\langle a(bc)a\rangle$, $\langle aba\rangle$, $\langle abc\rangle$, $\langle(ab)\rangle$, $\langle(ab)c\rangle$, $\langle(ab)d\rangle$, $\langle(ab)f\rangle$, $\langle(ab)dc\rangle$, $\langle ac\rangle$, $\langle aca\rangle$, $\langle acb\rangle$, $\langle acc\rangle$, $\langle ad\rangle$, $\langle adc\rangle$, $\langle af\rangle$
$\langle b\rangle$	$\langle(\_c)(ac)d(cf)\rangle$, $\langle(\_c)(ae)\rangle$, $\langle(df)cb\rangle$, $\langle c\rangle$	$\langle b\rangle$, $\langle ba\rangle$, $\langle bc\rangle$, $\langle(bc)\rangle$, $\langle(bc)a\rangle$, $\langle bd\rangle$, $\langle bdc\rangle$, $\langle bf\rangle$
$\langle c\rangle$	$\langle(ac)d(cf)\rangle$, $\langle(bc)(ae)\rangle$, $\langle b\rangle$, $\langle bc\rangle$	$\langle c\rangle$, $\langle ca\rangle$, $\langle cb\rangle$, $\langle cc\rangle$
$\langle d\rangle$	$\langle(cf)\rangle$, $\langle c(bc)(ae)\rangle$, $\langle(\_f)cb\rangle$	$\langle d\rangle$, $\langle db\rangle$, $\langle dc\rangle$, $\langle dcb\rangle$
$\langle e\rangle$	$\langle(\_f)(ab)(df)cb\rangle$, $\langle(af)cbc\rangle$	$\langle e\rangle$, $\langle ea\rangle$, $\langle eab\rangle$, $\langle eac\rangle$, $\langle eacb\rangle$, $\langle eb\rangle$, $\langle ebc\rangle$, $\langle ec\rangle$, $\langle ecb\rangle$, $\langle ef\rangle$, $\langle efb\rangle$, $\langle efc\rangle$, $\langle efcb\rangle$.
$\langle f\rangle$	$\langle(ab)(df)cb\rangle$, $\langle cbc\rangle$	$\langle f\rangle$, $\langle fb\rangle$, $\langle fbc\rangle$, $\langle fc\rangle$, $\langle fcb\rangle$

PrefixSpan Algorithm: Exercise

1. Given the following sequences database

Sequences Database

Customer ID	Data Sequences
1	<{30},{90}>
2	<{10,20},{30},{10,40,60,70}>
3	<{30,50,70,80}>
4	<{30},{30,40,70,80},{90}>
5	<{90}>

find all the frequent $\langle 30 \rangle$ prefix sequences with *minsup* $= 25\%$.

PrefixSpan Efficiency

- No candidate sequence needs to be generated
- Projected databases keep shrinking
- Major cost of PrefixSpan: constructing projected databases with recursively similar suffixes.
  - if it fits on memory, keeping pointers to the suffix offset of the sequence, avoids physical copy.
  - can also be improved by bi-level projections

Summary

- Sequential Pattern Mining is useful in many application, e.g. weblog analysis, financial market prediction, BioInformatics, etc.
- It is similar to the frequent itemsets mining, but with consideration of ordering.
- We have looked at different approaches that are descendants from two popular algorithms in mining frequent itemsets
  - Candidates Generation: AprioriAll and GSP
  - Pattern Growth: FreeSpan and PrefixSpan

Advanced Topics

Advanced Topics: Sequence Mining Constraints

- Item constraint
  - Find web log patterns only about online-bookstores
- Length constraint
  - Find patterns having at least 20 items
- Super pattern constraint
  - Find super patterns of “PC,digital camera”
- Aggregate constraint
  - Find patterns that the average price of items is over $100

Advanced Topics: Sequence Mining Constraints (cont.)

- Regular expression constraint
  - Find patterns “starting from Yahoo homepage, search for hotels in Washington DC area”
  - Yahootravel(WashingtonDC|DC)(hotel|motel|lodging)
- Duration constraint
  - Find patterns about ±24 hours of a shooting
- Gap constraint
  - Find purchasing patterns such that “the gap between each consecutive purchases is less than 1 month”

Advanced Topics: Generating Rules

- In classic sequential pattern mining, no rules are generated.
- It is, however possible to define and generate many type of rules
  - Sequential rules $X \rightarrow Y$, where Y is a sequence and X is a proper subsequence of Y
  - Class sequential rules $X \rightarrow y$, where X is a sequence and y is a class

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