Mathematics problems

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1.

1.1 $\frac{x^{n+2}}{x^{n-2}} = x^{n+2-(n-2)} = x^{n+2-n+2} = \underline{x^4}$

1.2

$x^{-1} \cdot 8 = 2$

$\frac{8}{x} = 2$

$2x = 8$

$\underline{x = 4}$

1.3

$(a^5)^0 = \cancel{10^{5\cdot 0}} = (5^{10})^0 = 5^{10\cdot 0} = 5^0 = \underline{1}$

1.4

$\frac{\sqrt{4x}}{\sqrt{x}} = \frac{\sqrt{4}}{\sqrt{1}} = \frac{2}{1} = \underline{2}$

1.5

$x^2 + (x+1)^2 = (x+2)^2$

$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$

$x^2 - 2x - 3 = 0$

$\rightarrow (x-3)(x+1) = 0$

$\underline{x_1 = 3}$

$\underline{x_2 = -1}$

1.6 $2^x > 1024$

$2^x > 2^{10}$

$x > 10$

2.

2.1

If $C=0$ then $F=32$

If $C=100$ then $F=212$

$F = a + b \cdot C$

$212 = a + b \cdot 100$ $\quad 32 = a + b \cdot 0$

$212 = 32 + b \cdot 100$ $\quad \rightarrow a = 32$

$180 = b \cdot 100$

$1{,}8 = b$

$F = 32 + 1{,}8C$

$\boxed{F = C}$

$C = 32 + 1{,}8C$

$-32 = 0{,}8C \rightarrow C = -40$

$C = -40$

$F = -40$

2.2

$f(x) = 5x + 4$

$f(3) = y = ?$

$f(3) = 5 \cdot 3 + 4 = 19$

$y = 19$

2.3

$$x^2 - 4x + 3 = 0$$

$$\rightarrow \quad (x-3)(x-1) = 0$$

$$x_1 = 3$$
$$x_2 = 1$$

2.4

$i = 2\%$ $\quad R \cdot (1+i)^t$

$R = 10\text{€}$ ~~10 90~~

$t = 90y$ $\quad 10 \cdot 1.02^{90} = \underline{59{,}43\text{€}}$

2.5

$$e^{\ln 5} = 5$$

3.

3.1 $\sum_{i=1}^{\infty} \frac{12}{6^i}$ $\quad \sum_{i=1}^{\infty} a_i = \frac{ab}{1-b}$

$a_n = 12 \cdot \frac{1}{6^i}$ $\quad a = 12$, $b = \frac{1}{6}$

$$\sum_{i=1}^{\infty} \frac{12}{6^i} = \frac{12 \cdot \frac{1}{6}}{1 - \frac{1}{6}} = \frac{12/6}{5/6} = \frac{12}{5}$$

3.2

$$\lim_{x \to 1} \frac{6^{(1-x)}}{x} \Rightarrow \lim_{x \to 1} \frac{6^{(1-1)}}{1} = \frac{1}{1} = \underline{1}$$

3.3

$f(x) = x^5 - 8$ $\quad f'(x) = 5x^4$

$x = -3$ $\quad f'(-3) = 5 \cdot (-3)^4 = 5 \cdot 81 = \underline{405}$

3.4

$$\frac{d}{dx}\frac{x^3+2x-1}{x-2} = \frac{(3x^2+2)(x-2)-(x^3+2x-1)\cdot 1}{(x-2)^2} =$$

$$= \frac{3x^2-6x^2+2x-4-x^3-2x+1}{(x-2)^2} = \frac{2x^3-6x^2-3}{(x-2)^2}$$

3.5

$$\frac{d^2}{dx^2}4x^4+4x^2 \Rightarrow \frac{d}{dx}16x^3+8x \Rightarrow 48x^2+8$$

3.6

$$\frac{d}{dx}\frac{\ln x}{e^x} \Rightarrow \frac{\frac{1}{x}\cdot e^x - e^x\cdot \ln x}{(e^x)^2} = \frac{\frac{1}{x}-\ln x}{e^x}$$

3.7

$f(x) = 3x^2-5x+2$

$f'(x) = 6x-5$

$f''(x) = 6 > 0$ convex

$6x-5=0$

$6x=5$

$x=\frac{5}{6}$ local minimum

$f(x)=0$

$(3x-2)(x-1)=0$

$x_1=1$

$x_2=\frac{2}{3}$

x	$-\infty, \frac{2}{3} > x$	$\frac{2}{3}$	$\frac{2}{3} < x < \frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{6} < x < 1$	1	$1 < x, \infty$
f(x)	+	0	−	−	−	0	+
f'(x)	−	−	−	0	+	+	+
shape	↘	↘	↘	∅	↗	↗	↗
f''(x)	+	+	+	+	+	+	+
convexity	∪	∪	∪	inf	∪	∪	∪

3.8

$f(x,y) = x^2 + y^3$

$f(2,3) = 2^2 + 3^3 = 4 + 27 = \underline{31}$

3.9

$f(x,y) = \ln(x-y) \quad (x,y) \in \mathbb{R}$ if $x - y > 0$

$\underline{x > y}$

3.10

$\frac{\partial}{\partial x} x^5 + xy^3 \Rightarrow 5x^4 + y^3$

3.11

$f(x,y) = x^2y^2 + 10$

$f'_x(x,y) = 2xy^2 \qquad f''_x(x,y) = 2y^2$

$f'_y(x,y) = 2x^2y \qquad f''_y(x,y) = 2x^2$ $\quad \underline{x = 0 \quad y = 0}$

3.12 $\max x^2y^2$ s.t. $x + y = 10$

$\mathcal{L} = f(x,y) - \lambda g(x,y)$

$\mathcal{L} = x^2y^2 - \lambda(x + y - 10)$

$\frac{\partial \mathcal{L}}{\partial x} = 2x - \lambda = 0 \rightarrow 2x = \lambda$

$\frac{\partial \mathcal{L}}{\partial y} = 2y - \lambda = 0 \rightarrow 2y = \lambda$ $\Big\}$ $\boxed{x = y}$

$\frac{\partial \mathcal{L}}{\partial y \lambda} = x + y - 10 = 0$

$2y - 10 = 0$

$\underline{y = 5 = x}$

4.

4.1

		B: 12	B: 18	B: 27
A	26	14	50	26
A	51	7	13	37
A	19	19	73	25

$= A \cdot B$

4.2

		4	6
		1	3
B	191	39	59
B	212	10	16

$= B \cdot A$

4.3

$$A^T = \begin{bmatrix} 7,1 & 2 & 4 \\ 8,1 & 7,8 & 4,44 \\ 4,7 & 1,1 & 0 \end{bmatrix}$$

4.4

$$\det(A) = 8 - 18 = \underline{-10}$$

5.

5.1

$$\Omega = \{(1,1), (1,2), \ldots (6,6)\} \quad \#\Omega = 36$$

5.2

		T	F
use drugs: 1%	⊕	99%	0,05%
∅ drugs: 99%	⊖	99,5%	1%

$$P_1 = 0,01 \cdot 0,99 + 0,99 \cdot 0,05 = 0,01485 = \underline{1,485\%}$$

5.3

$$P_2(\text{us drugs} \mid P_1) = \frac{0,99 \cdot 0,01}{0,01485} = 0,66 = \underline{66\%}$$