# A Method for Optimal Kinematic Design of Five-bar Planar Parallel Manipulators

Tien Dung Le, Hee-Jun Kang and Quang Vinh Doan

*Abstract*— **In this paper, an optimal kinematic design method for symmetrical five-bar planar parallel manipulators is presented. The proposed design method is implemented in two steps. First, an optimal configuration is achieved, resulting in a set of closed-form parametric relationships. Second, a searching algorithm is proposed for finding the link lengths of the manipulator to maximize usable workspace. In this usable workspace there is no singularity configuration, and also a good dexterity is satisfied. A design example is included to illustrate the effectiveness of the proposed method.**

*Keywords*- Optimal kinematic design, Parallel robotic manipulator, Five-bar manipulator, Singularity, Workspace.

## I. Introduction

Parallel manipulators have advantages like high accuracy, high stiffness, high payload capability, low moving inertia, and so on. However, the main drawbacks of parallel manipulator are their relatively small workspace and many singular configurations in the workspace. In addition, the performance of parallel manipulators is strong dependent on their geometric parameters. For these reasons, the optimum kinematic design of the parallel manipulators is much complex and the adequacy and effectiveness of the design method become more critical.

Five-bar planar parallel manipulator is a typical parallel mechanism with 2 degrees of freedom (DOF) [1, 2]. It consists of two active joints and three passive joints. The five-bar planar parallel manipulator with symmetric structure which is also called 5R symmetrical mechanism [3-5] attracted the most researchers in the field. The optimal design of this kind of parallel manipulator has interested many researchers. In [6], a geometric model of the solution space was used to obtain analytical relationships between the link lengths of 2-DOF planar parallel manipulator and performance criteria based on the global conditioning and global velocity indices. The global behavior of the 2-DOF planar parallel manipulator was described and useful for analysis and design. However, since the workspace used in the analysis is theoretical, the result cannot be completely applied to a practical design. Tian Huang [7] proposed a hybrid method for the optimum kinematic design of 2 DOF parallel manipulators which was implemented in two steps to optimize the mechanism with respect to the local and global conditioning indices. In [8], an approach of switching working modes in the context of the development of a 2 DOF planar parallel robot with revolute actuators was proposed for optimization of workspace. The disadvantage of this method is that it is related to the control strategy of robot, so it is still complicated in practice. In [9], some applied performance indices such as the global condition index, the global velocity index, the global payload index, and the global stiffness index were defined and investigated for planar 5R symmetrical parallel mechanisms. The corresponding atlases were also graphically represented in the established design space. However, a design method was not reported in this paper. On the other hand, an optimum design method of the 5R symmetrical parallel manipulator with a surrounded workspace was proposed in [10]. In addition, an optimal design method to determine the geometric parameters of a PRRRP parallel mechanism was presented in [11]. And in [12], a parametric variations method and a simplex direct method were used to find the optimal solution of kinematic design of planar parallel manipulators. However, the above methods seem slow and use a lot of memory of computer. In [13], a unified formulation for optimal design of parallel manipulators was proposed. The optimal design problem was formulated to maximize the volume of regular-shape workspace, while subjected to dexterity constraints. However, in this method, the effective regular workspace which is optimized may contains the singularity configuration of the parallel manipulators. In addition, this method just controls the step-length and does not control the direction of searching, so it takes much time to handle the constraints and search the optimal solution. And the algorithm is still complicated and lacking efficiency. The convergence can be slow and does not guarantee to get the global optimal solution.

This paper presents an optimal kinematic design method for symmetrical five-bar parallel manipulators which enhance the method in [13]. There are two steps for implementation of the proposed design method. An optimal configuration is achieved in the first step, resulting in a set of closed-form parametric relationships. In the second step, a

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searching algorithm is proposed for finding the link lengths of robot to maximize usable workspace. There is no singularity configuration in this usable workspace, and also a good dexterity is satisfied.

The rest of paper is organized as follows. In section 2, the kinematic equations of a five-bar planar parallel manipulator with symmetrical structure are presented. The proposed optimal kinematic design method is described in section 3. In section 4, a design example is given. Finally, a conclusion is reached in section 5.

## II. KINEMATIC EQUATIONS

As depicted in Fig.1, a five-bar planar parallel manipulator with symmetrical structure and a reference frame Oxy is established in the workspace. The five-bar planar parallel manipulator is actuated by two active joints, $A_1$ and $A_2$, which are fixed on the base. Three passive revolute joints of the five-bar planar parallel manipulator are $P_1$, $P_2$ and E.We denote $\boldsymbol{\theta}_a=(\theta_{a1},\theta_{a2})^T$ and $\boldsymbol{\theta}_p=(\theta_{p1},\theta_{p2})^T$ corresponding to the active joint vector and passive joint vector, respectively; $\mathbf{X}=(x,y)^T$ is the Cartesian coordinates; and E($x$,$y$) is the end-effector of the parallel manipulators. The links length of the parallel manipulators are $A_1P_1 = A_2P_2 = l_1$, $P_1E = P_2E = l_2$, and $l_0=A_1A_2$.

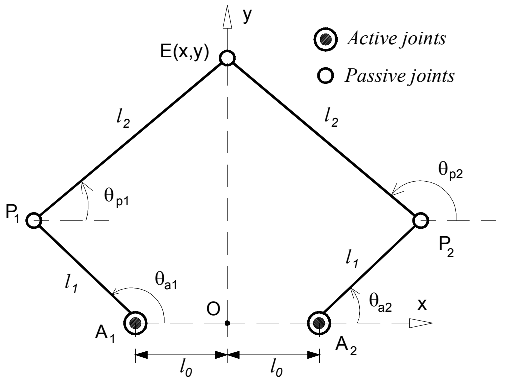


Fig. 1. The five-bar planar parallel manipulator.

### A. Forward kinematics

The forward kinematics problem is to obtain the output $\mathbf{E}(x,y)$ in the reference $A_1$xy with respect to a set of given input $\boldsymbol{\theta}_a$.

From Fig.1, we obtain:

$$x = -l_0 + l_1 \cos\theta_{a1} + l_2 \cos\theta_{p1} \quad (1)$$

$$x = l_0 + l_1 \cos\theta_{a2} + l_2 \cos\theta_{p2} \quad (2)$$

And:

$$y = l_1 \sin\theta_{a1} + l_2 \sin\theta_{p1} \quad (3)$$

$$y = l_1 \sin\theta_{a2} + l_2 \sin\theta_{p2} \quad (4)$$

Here we have four joint variables $\theta_{a1}$, $\theta_{a2}$, $\theta_{p1}$ and $\theta_{p2}$. Only $\theta_{a1}$ and $\theta_{a2}$ are independent and are the inputs of the controller for the parallel manipulator, while the rest joints $\theta_{p1}$ and $\theta_{p2}$ are functions of $\theta_{a1}$ and $\theta_{a2}$. We compute $\theta_{p1}$ and $\theta_{p2}$ depending on $\theta_{a1}$ and $\theta_{a2}$ as follows.

We assign:

$$m = 2l_0 + l_{21}(\cos\theta_{a2} - \cos\theta_{a1}) \quad (5)$$

$$n = l_1\left|\sin\theta_{a1} - \sin\theta_{a2}\right| \quad (6)$$

We compute:

$$\varphi_1 = \varphi_3 = \text{acos}\left(\frac{\sqrt{m^2+n^2}}{2l_2}\right) \quad (7)$$

$$\varphi_2 = \text{atan}\left(\frac{n}{m}\right) \quad (8)$$

There are two solutions to forward kinematics:

*Solution 1 (up-configuration):*

$$\theta_{p1} = \varphi_1 + \varphi_2 \quad (9)$$

$$\theta_{p2} = \pi - \varphi_3 + \varphi_2 \quad (10)$$

*Solution 2 (down-configuration):*

$$\theta_{p1} = 2\pi - \varphi_1 + \varphi_2 \quad (11)$$

$$\theta_{p2} = \pi + \varphi_2 + \varphi_3 \quad (12)$$

In this paper, we just consider the solution 1 (up-configuration) to the forward kinematics for analyzing the five-bar planar parallel manipulator.

### B. Inverse kinematics

In the inverse kinematics problem, we compute the input $\boldsymbol{\theta}_a$ from the given position of end-effector $\mathbf{E}(x,y)$. As depicted in Fig.2, there could be four different solutions to this problem with a desired position of the end-effector.

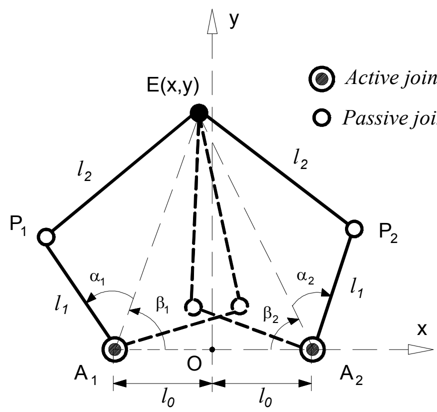


Fig. 2. Four solutions to the inverse kinematics of the five-bar planar parallel manipulator.

We compute:

$$\alpha_1 = \text{acos}\left(\frac{l_1^2 + \left((l_0+x)^2 + y^2\right) - l_2^2}{2l_1\sqrt{(l_0+x)^2 + y^2}}\right) \quad (13)$$

$$\alpha_2 = \text{acos}\left(\frac{l_1^2 + \left((l_0-x)^2 + y^2\right) - l_2^2}{2l_1\sqrt{(l_0-x)^2 + y^2}}\right) \quad (14)$$

$$\beta_1 = \text{atan}\left(\frac{y}{l_0+x}\right) \quad (15)$$

$$\beta_2 = \operatorname{atan}\left(\frac{y}{l_0 - x}\right) \tag{16}$$

The four solutions to inverse kinematics will be:

$$\theta_{a1} = \beta_1 \pm \alpha_1 \tag{17}$$

$$\theta_{a2} = \pi - \beta_2 \pm \alpha_2 \tag{18}$$

The four solutions to the inverse kinematic problem correspond to four working modes of the five-bar planar parallel manipulators as follows.

(1) Mode "+-" : $\theta_{a1} = \beta_1 + \alpha_1$ and $\theta_{a2} = \pi - \beta_2 - \alpha_2$

(2) Mode "-+": $\theta_{a1} = \beta_1 - \alpha_1$ and $\theta_{a2} = \pi - \beta_2 + \alpha_2$

(3) Mode "--": $\theta_{a1} = \beta_1 - \alpha_1$ and $\theta_{a2} = \pi - \beta_2 - \alpha_2$

(4) Mode "++": $\theta_{a1} = \beta_1 + \alpha_1$ and $\theta_{a2} = \pi - \beta_2 + \alpha_2$.

In this paper, we consider the solutions to the inverse kinematics of five-bar planar parallel manipulator with working mode "+-".

## III. OPTIMAL KINEMATIC DESIGN METHOD

In [13], a general approach to the optimal kinematic synthesis problem of parallel manipulators is presented based on a random search technique. When applied to the five-bar planar manipulator case, the algorithm was implemented in two steps. In the first step, a square effective workspace was taken for instance by choosing a point in workspace that satisfies all constraints as the center of the square workspace. The size length of this square was chosen adequately large and after that reduced gradually until all constraints were fulfilled. In the second step, the center of the square workspace was varied for finding the maximal size of the square effective workspace corresponding to the given link lengths of the five-bar manipulator. Repeat two steps above for different sets of parameters, they find the maximal effective workspace. A controlled random search technique was used for this searching process.

However, the disadvantages of the method presented in [13] are shown as follows: First, the arbitrary choosing of the center and adequately large side length of square workspace make the process become complex and take a lot of computations. Second, the algorithm has to handle many parameters such as the link lengths of parallel manipulators, the coordinate of center of the square regular workspace, the constraint parameters,... And finally, in the algorithm, the authors did not concern about the singular configurations of five-bar manipulator in the searching workspace.

In this paper, we present a hybrid method that avoids the disadvantages of the method in [13]. In our method, we find the optimal workspace inside the maximal inscribed circle (MIC). This circle is the usable workspace which is defined as the maximum continuous workspace that contains no singular loci inside but is bounded by singular loci outside [3]. By this way, we find the center of the square regular workspace as the center of MIC, and the initialization of the maximum square is the square inscribed of the MIC corresponding to a given set of parameters. In addition, by taking a local optimal approach in [7], the number of parameters that need to be determined in the design process is reduced. And good condition index in the workspace is guaranteed.

Based on the analysis in [3], the theoretical workspace or reachable workspace of symmetric five-bar planar parallel manipulator is the intersection region which is bounded by the four following circles:

$$\begin{cases} C_{1o} : (x + l_0)^2 + y^2 = (l_1 + l_2)^2 \\ C_{1i} : (x + l_0)^2 + y^2 = (l_1 - l_2)^2 \\ C_{2o} : (x - l_0)^2 + y^2 = (l_1 + l_2) \\ C_{2i} : (x - l_0)^2 + y^2 = (l_1 - l_2)^2 \end{cases} \tag{19}$$

The usable workspace is defined as the maximum continuous workspace that contains no singular loci inside but boundary by singular loci outside. And the *maximal inscribed circle* (MIC) is defined as the circle that is located at the y axis and tangent with the usable workspace boundary curves. The MIC is defined as [3]:

$$x^2 + (y - y_{MIC})^2 = r_{MIC}^2 \tag{20}$$

where $r_{MIC}$ is the radius and $(0, y_{MIC})$ is the center.

If $l_1+l_2 > l_0$, we will obtain:

$$y_{MIC} = \left[(l_1 + l_2 + y_{Col})^2 - l_0^2\right] / 2(l_1 + l_2 + y_{Col}) \tag{21}$$

$$y_{Col} = \sqrt{l_1^2 - (l_2 - l_0)^2} \tag{22}$$

$$r_{MIC} = |y_{MIC}| - y_{Col} \tag{23}$$

If $l_1+l_2 = l_0$ then the workspace will be equal to zero.

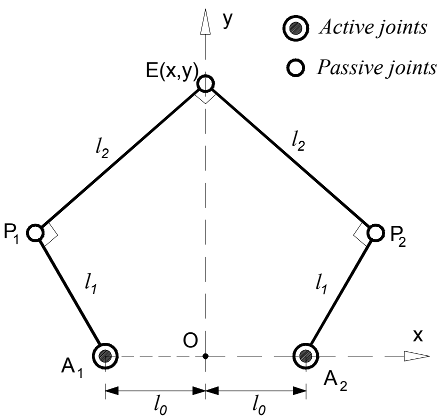


Fig.3. The optimal configuration of the five-bar planar parallel manipulator.

In the optimal kinematic design problem, we include considering the conditioning index which is defined by [14]:

$$\frac{1}{\kappa} = \frac{\sigma_{\min}}{\sigma_{\max}} \tag{24}$$

in which $0 \le 1/\kappa \le 1$ and $\sigma_{\min}$, $\sigma_{\max}$ are the minimum and maximum singular values of Jacobian, respectively. The bigger $1/\kappa$ is the better it is.

The result in [7] showed that, the condition for five-bar planar parallel manipulator achieves an optimal configuration when the following parametric relationship is satisfied:

$$l_0 = \frac{\sqrt{2}}{2}(l_2 - l_1) \quad (27)$$

At this optimal configuration, $\sigma_{min}$ takes the maximum and $\sigma_{max}$ takes the minimum values simultaneously in the overall workspace.

The position of the end-effector when the robot is at the optimal configuration is shown in Fig.3. And the coordinates of the end-effector E(x,y) at this position are as the following:

$$E(x,y) = \left(0, \frac{\sqrt{2}}{2}(l_1 + l_2)\right)^T \quad (28)$$

Let $\theta_{aimax}$ and $\theta_{aimin}$ be, respectively, the lower and upper bounds for $i$th actuator due to the actuator limits ($i$= 1,2). The optimal kinematic design problem for maximization of effective usable workspace of the five-bar planar parallel manipulator is formulated as follows.

***Optimization problem***:

*Find the link lengths $l_0$, $l_1$ and $l_2$ that maximize a workspace bounded by a square $W_o(\rho)$ inside the MIC and subjects to all constrains below:*

***(1)*** $1/\kappa \geq \gamma$ (29)

***(2)*** $l_0 = \frac{\sqrt{2}}{2}(l_2 - l_1)$ (30)

***(3)*** $\theta_{a1\min} \leq \theta_{a1} \leq \theta_{a1\max}$ (31)

***(4)*** $\theta_{a2\min} \leq \theta_{a2} \leq \theta_{a2\max}$ (32)

*where $\rho$ is the ratio of $l_2$ to $l_1$; $\gamma$ is a design constant.*

***Solution***:

In this paper, we consider the case of $0 \leq l_0 < l_1 + l_2$. So from equation (27) we have $\rho \geq 1$. The optimal design problem above becomes an one-dimensional optimization problem in which the objective function $W_o(\rho)$ cannot be expressed as a mathematic function. The algorithm for finding $W_o(\rho)$ at a given value of $\rho$ is described as follows:

- At a given value of $\rho \geq 1$ and $0 \leq \gamma \leq 1$, we could find the parameters $r_{MIC}$ and $y_{MIC}$ using the equations (21), (22) and (23).

- Choosing the inscribed square of the MIC as the maximal square workspace, the side length of this square is $a = r_{MIC}\sqrt{2}$. Then decrease $a$ gradually until all constrains are fulfilled. This obtained value of $a$ is therefore the largest side length of the square workspace $W_o(\rho)$ corresponding to the given value $\rho$. The flow chart of the algorithm to find $a_{MAX}$ is presented in Fig. 4.

And then, by changing the value of ρ, different largest side lengths of the square workspace $W_o(\rho)$ are obtained correspondingly. The maximal value of all those largest side lengths gives the maximal size for the square workspace $W_o(\rho)$ corresponding to the given value of $\rho$.

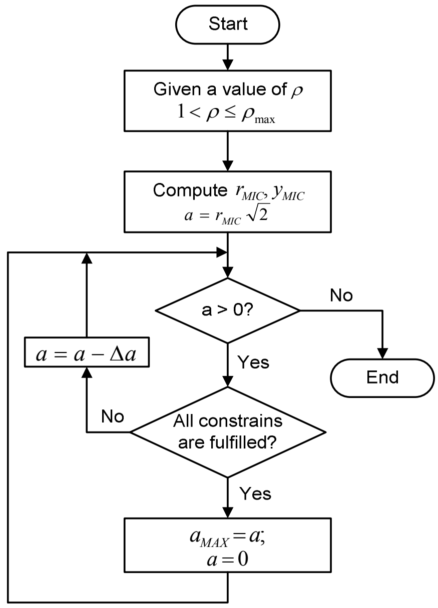


Fig.4. Flowchart of the algorithm to find $a_{MAX}$ .

## IV. DESIGN EXAMPLE

In the optimization problem described in section III, given γ = 0.5, $l_1$ = 1, $\theta_{a1max}$ = 4π/3, $\theta_{a1min}$ = -π/3, $\theta_{a2max}$ = 4π/3, $\theta_{a2min}$ = -π/3 as an example for the optimum kinematic design of the five-bar manipulator. We apply the proposed method for $\rho$ changes from 1.2 to 4, the plot of $a_{MAX}$ versus $\rho$ is given in Fig.5.

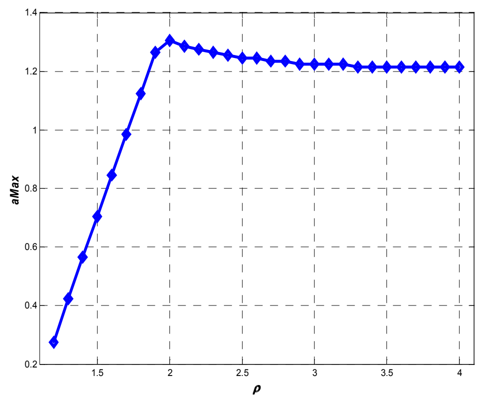


Fig.5. Plot of $a_{MAX}$ versus γ = 0.5 and ρ = 1.2 - 4.

Fig. 5 shows that the area of $W_o(\rho)$ which satisfies all constrains (23)-(26) of the optimization problem increases with the increase in $\rho$ from 1.2 to about 2, but this tendency decreases when $\rho$ > 2. We also can see that $a_{Max}$ has only one optimal value in the interval $\rho \in (1.8, 2.3)$. For finding exactly optimal solution, we can apply the Golden Section Search method or Fibonaci Search method.

Using Golden Section Search method, we find the optimal solution: $\rho \approx 1.9721$ and $a_{Max} \approx 1.3142$. The contour of conditional index is expressed in Fig. 6.

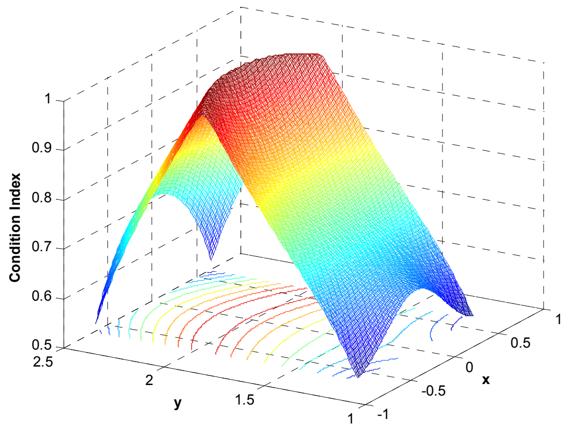


Fig.6. The condition index in the optimal workspace.

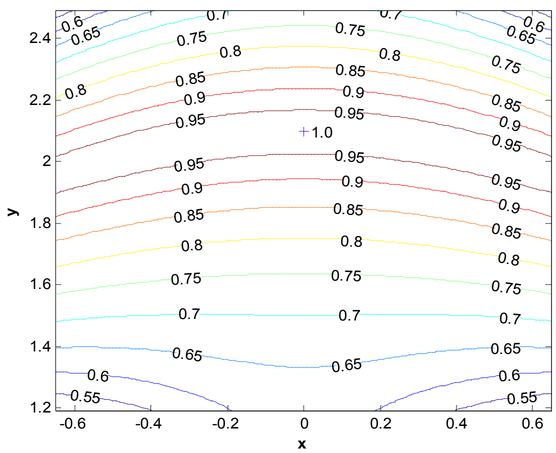


Fig.7. The contour of conditional index .

Fig. 6 presents the plot of the condition index. And Fig. 7 shows the distribution of the conditioning index inside the maximal usable workspace which was found by applying the proposed method. We can see that all the values of conditioning index of the five-bar manipulator are larger than the prescribed threshold, 0.5.

Compared with the results in [13], the values of the condition index in this paper are better. In Fig.6 The maximal of 1/κ is 1, corresponding to the optimal configuration of the five-bar manipulator. The method described in [13] cannot get the optimal configuration of the five-bar manipulator. So the results of the condition index showed in that paper are less than 1.

## V. CONCLUSION

In this paper, we presented an optimal kinematic design method for symmetrical five-bar parallel manipulators. The design method results in an optimal configuration in which the condition index has the maximal value. The number of parameters that needs to be determined in the design process is reduced. A searching algorithm is proposed for finding the link lengths of robot to maximize usable workspace. In this usable workspace not only there is no singularity configuration, but also a good dexterity is satisfied. An example was carried out and the results show the effectiveness of the proposed optimal kinematic design method for the manipulators.

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