Einführung in die Wahrscheinlichkeitstheorie
Übungsserie 4

Aufgabe 3:

$$\begin{aligned}
A &= \{\text{alle 3 Farben gezogen}\} \\
A^C &= B_{weiss} \cup B_{rot} \cup B_{schwarz} \\
B_i &= \{\text{Farbe } i \text{ nicht gezogen}\} \\
\mathbb{P}(A^C) &= \mathbb{P}(B_{weiss}) + \mathbb{P}(B_{rot}) + \mathbb{P}(B_{schwarz}) \\
&\quad - (\mathbb{P}(B_{weiss} \cap B_{rot}) + \mathbb{P}(B_{weiss} \cap B_{schwarz}) + \mathbb{P}(B_{schwarz} \cap B_{rot})) \\
&\quad + \mathbb{P}(B_{weiss} \cap B_{rot} \cap B_{schwarz}) \\
\mathbb{P}(A^C) &= \frac{\binom{13}{5}}{\binom{20}{5}} + \frac{\binom{12}{5}}{\binom{20}{5}} + \frac{\binom{15}{5}}{\binom{20}{5}} \\
&\quad - \left(\frac{\binom{5}{5}}{\binom{20}{5}} + \frac{\binom{8}{5}}{\binom{20}{5}} + \frac{\binom{7}{5}}{\binom{20}{5}} \right) \\
&\quad + 0 \\
\mathbb{P}(A) &= \underline{\underline{1 - \mathbb{P}(A^C)}}
\end{aligned}$$

Aufgabe 5:

(a) $\Omega = \{(w, w, u_i), (w, s, u_i), (s, w, u_i), (s, s, u_i) \mid i \in [1, 3]\} \setminus \{(s, s, u_1)\}$

(b)

$$\begin{aligned}
U_i &= \{\text{Urne } i \text{ wurde ausgewählt}\} \\
A_{xy} &= \{\text{Farbe } x, \text{ dann } y \text{ wurde gezogen}\}
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}(A_{ww}) &= \mathbb{P}(U_1)\mathbb{P}(A_{ww}|U_1) + \mathbb{P}(U_2)\mathbb{P}(A_{ww}|U_2) + \mathbb{P}(U_3)\mathbb{P}(A_{ww}|U_3) \\
&= \frac{1}{3}(\mathbb{P}(A_{ww}|U_1) + \mathbb{P}(A_{ww}|U_2) + \mathbb{P}(A_{ww}|U_3)) \\
&= \underline{\frac{1}{3}\left(\frac{\binom{2}{5}}{\binom{2}{6}} + \frac{\binom{2}{4}}{\binom{2}{6}} + \frac{\binom{2}{5}}{\binom{2}{6}} \right)}
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}(A_{ss}) &= \frac{1}{3}(\mathbb{P}(A_{ss}|U_1) + \mathbb{P}(A_{ss}|U_2) + \mathbb{P}(A_{ss}|U_3)) \\
&= \underline{\frac{1}{3}\left(0 + \frac{\binom{2}{2}}{\binom{2}{6}} + \frac{\binom{2}{3}}{\binom{2}{6}} \right)}
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}(A_{ws} \cup A_{sw}) &= \mathbb{P}(A_{ws}) + \mathbb{P}(A_{sw}) \\
&= \frac{1}{3}\left(\frac{\binom{1}{5}}{\binom{1}{6}} \cdot \frac{\binom{1}{1}}{\binom{1}{5}} + \frac{\binom{1}{4}}{\binom{1}{6}} \cdot \frac{\binom{1}{2}}{\binom{1}{5}} + \frac{\binom{1}{3}}{\binom{1}{6}} \cdot \frac{\binom{1}{3}}{\binom{1}{5}} \right) \\
&\quad + \frac{1}{3}\left(\frac{\binom{1}{1}}{\binom{1}{6}} \cdot \frac{\binom{1}{5}}{\binom{1}{5}} + \frac{\binom{1}{2}}{\binom{1}{6}} \cdot \frac{\binom{1}{4}}{\binom{1}{5}} + \frac{\binom{1}{3}}{\binom{1}{6}} \cdot \frac{\binom{1}{3}}{\binom{1}{5}} \right) \\
&= \underline{\frac{2}{3}\left(\frac{\binom{1}{5}}{\binom{1}{6}} \cdot \frac{\binom{1}{1}}{\binom{1}{5}} + \frac{\binom{1}{4}}{\binom{1}{6}} \cdot \frac{\binom{1}{2}}{\binom{1}{5}} + \frac{\binom{1}{3}}{\binom{1}{6}} \cdot \frac{\binom{1}{3}}{\binom{1}{5}} \right)}
\end{aligned}$$