Einführung in die Wahrscheinlichkeitstheorie
Übungsserie 3

Aufgabe 1:

$$\mathbb{P}(A^C) = 1 - \mathbb{P}(A) = \underline{\underline{\frac{2}{3}}}$$
$$\mathbb{P}(A^C \cup B) = \mathbb{P}((A \cap B^C)^C) = \mathbb{P}((A \setminus B)^C) = 1 - \left(\frac{1}{3} - \frac{1}{6}\right) = \underline{\underline{\frac{5}{6}}}$$
$$\mathbb{P}(A \cup B^C) = \mathbb{P}((B \setminus A)^C) = 1 - \left(\frac{1}{4} - \frac{1}{6}\right) = \underline{\underline{\frac{11}{12}}}$$
$$\mathbb{P}(A \cap B^C) = \mathbb{P}(A \setminus B) = \frac{1}{3} - \frac{1}{6} = \underline{\underline{\frac{1}{6}}}$$
$$\mathbb{P}(A \triangle B) = \mathbb{P}((A \setminus B) \cup (B \setminus A)) = \left(\frac{1}{3} - \frac{1}{6}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) = \underline{\underline{\frac{1}{4}}}$$
$$\mathbb{P}(A^C \cup B^C) = \mathbb{P}((A \cap B)^C) = 1 - \frac{1}{6} = \underline{\underline{\frac{5}{6}}}$$

Aufgabe 3:

(a)

$$0 \leq \mathbb{P}(A \cap B) \leq \mathbb{P}(B) = 0 \implies \mathbb{P}(A \cap B) = 0$$
$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \underbrace{\mathbb{P}(B)}_{=0} - \underbrace{\mathbb{P}(A \cap B)}_{=0} = \underline{\underline{\mathbb{P}(A)}}$$
$$\mathbb{P}(A \cup B) = \mathbb{P}(A \setminus B) + \mathbb{P}(B \setminus A) + \mathbb{P}(A \cap B)$$
$$= \underline{\mathbb{P}(A \setminus B)} + \underbrace{\mathbb{P}(B \cap A^C)}_{=0} + \underbrace{\mathbb{P}(A \cap B)}_{=0} = \underline{\underline{\underline{\mathbb{P}(A)}}} \tag{1}$$

(b)

$$\mathbb{P}(B^C) = 1 - \mathbb{P}(B) = 0$$
$$\mathbb{P}(A \cap B) = P(A \setminus B^C) \overset{(1)}{=} \underline{\underline{\mathbb{P}(A)}} \tag{2}$$
$$\mathbb{P}(B \setminus A) = P(B \cap A^C) \overset{(2)}{=} \underline{\underline{\mathbb{P}(A^C)}}$$

Aufgabe 5:

$$|A| = (n-1)!$$
$$|\Omega| = n!$$
$$\mathbb{P}(A) = \frac{|A|}{|\Omega|} = \underline{\underline{\frac{1}{n}}}$$