Einführung in die Wahrscheinlichkeitstheorie

Übungsserie 6

Aufgabe 2:

(a) $\mathbb{P}((A \cap B) \cup (A \cap C)) = \mathbb{P}(A \cap B) + \mathbb{P}(A \cap C) - \mathbb{P}(A \cap B \cap C) = \frac{1}{9} + \frac{1}{9} - \frac{1}{27} = \underline{\underline{\frac{5}{27}}}$

(b) $\mathbb{P}(A \setminus (B \cup C)) = \mathbb{P}((A \cap B^C) \cup (A \cap C^C)) = \mathbb{P}(A \cap B^C) + \mathbb{P}(A \cap C^C) - \mathbb{P}(A \cap B^C \cap C^C) = \frac{2}{9} + \frac{2}{9} - \frac{4}{27} = \underline{\underline{\frac{8}{27}}}$

Aufgabe 3:

$$\begin{aligned}
\mathbb{P}(A) &= \frac{\binom{6}{4}}{2^6} = \frac{15}{2^6} \\
\mathbb{P}(B) &= \frac{1}{2} \\
\mathbb{P}(A \cap B) &= \frac{1}{2^2} \cdot \frac{1}{2^4} + \frac{1}{2^2} \cdot \frac{\binom{4}{2}}{2^4} = \frac{7}{2^6} \\
&\neq \underline{\underline{\mathbb{P}(A)\mathbb{P}(B)}}
\end{aligned}$$

$\implies$ Die beiden Ereignisse sind nicht unabhängig voneinander.

Aufgabe 5:

(a) $\mathbb{P}(A \cap (B \cup C)) = \mathbb{P}((A \cap B) \cup (A \cap C)) = \mathbb{P}(A) \cdot \mathbb{P}(B) + \mathbb{P}(A) \cdot \mathbb{P}(C) - \underbrace{\mathbb{P}(A \cap B \cap C)}_{=0 \ (B \cap C = \emptyset)}$

$= \mathbb{P}(A)(\mathbb{P}(B) + \mathbb{P}(C)) = \underline{\underline{\mathbb{P}(A)\mathbb{P}(B \cup C)}}$

(b) $\mathbb{P}(A \cap (B \cup C)) = \mathbb{P}(A)\mathbb{P}(B) + \mathbb{P}(A)\mathbb{P}(C) - \mathbb{P}(A)\mathbb{P}(B)\mathbb{P}(C) = \mathbb{P}(A)(\mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(B)\mathbb{P}(C)) = \underline{\underline{\mathbb{P}(A)\mathbb{P}(B \cup C)}}$

Aufgabe 6:

(c) $\mathbb{P}(A) = 1 : \mathbb{P}(A \cap B) \overset{\text{ÜS 3}}{=} \mathbb{P}(B) = \underline{\underline{\mathbb{P}(A)\mathbb{P}(B)}}$

$\mathbb{P}(A) = 0 : \mathbb{P}(A \cap B) \overset{\text{ÜS 3}}{=} \mathbb{P}(A) = \underline{\underline{\mathbb{P}(A)\mathbb{P}(B)}}$