

# Automaten und Berechenbarkeit

## 9. Übungsserie

**Aufgabe 1:**

(a)  $G_a = (N_a, T, S_a, P_a)$

$$N_a = \{Q_0, Q_1, Q_2, Q_3, Q_4\}$$

$$T = \{a, b\}$$

$$S_a = Q_0$$

$$P_a = \{Q_0 \rightarrow \lambda, Q_0 \rightarrow aQ_1, Q_0 \rightarrow bQ_0, Q_1 \rightarrow aQ_2, Q_1 \rightarrow bQ_1, Q_2 \rightarrow aQ_3, Q_2 \rightarrow bQ_2, \\ Q_3 \rightarrow aQ_4, Q_3 \rightarrow bQ_3, Q_4 \rightarrow aQ_0, Q_4 \rightarrow bQ_4\}$$

$$G_{a2} = (N_{a2}, T, S_{a2}, P_{a2})$$

$$N_{a2} = \{Q_0, Q_1, Q_2, Q_3, Q_4\}$$

$$T = \{a, b\}$$

$$S_{a2} = Q_0$$

$$P_{a2} = \{Q_0 \rightarrow \lambda, Q_1 \rightarrow Q_0a, Q_0 \rightarrow Q_0b, Q_2 \rightarrow Q_1a, Q_1 \rightarrow Q_1b, Q_3 \rightarrow Q_2a, Q_2 \rightarrow Q_2b, \\ Q_4 \rightarrow Q_3a, Q_3 \rightarrow Q_3b, Q_0 \rightarrow Q_4a, Q_4 \rightarrow Q_4b\}$$

(b)  $G_b = (N_b, T, S_b, P_b)$

$$N_b = \{Q_0, Q_1, Q_2, Q_3\}$$

$$T = \{a, b\}$$

$$S_b = Q_0$$

$$P_b = \{Q_0 \rightarrow \lambda, Q_1 \rightarrow \lambda, Q_2 \rightarrow \lambda, Q_2 \rightarrow \lambda, Q_0 \rightarrow bQ_0, Q_0 \rightarrow aQ_1, Q_1 \rightarrow aQ_1, Q_1 \rightarrow bQ_2, \\ Q_2 \rightarrow aQ_3, Q_2 \rightarrow bQ_0, Q_3 \rightarrow aQ_3, Q_3 \rightarrow bQ_3\}$$

$$G_{b2} = (N_{b2}, T, S_{b2}, P_{b2})$$

$$N_{b2} = \{Q_0, Q_1, Q_2, Q_3, S\}$$

$$T = \{a, b\}$$

$$S_{b2} = S$$

$$P_{b2} = \{Q_0 \rightarrow \lambda, S \rightarrow Q_0, S \rightarrow Q_1, S \rightarrow Q_2, Q_0 \rightarrow Q_0b, Q_1 \rightarrow Q_0a, Q_1 \rightarrow Q_1a, Q_2 \rightarrow Q_1b, \\ Q_3 \rightarrow Q_2a, Q_0 \rightarrow Q_2b, Q_3 \rightarrow Q_3a, Q_3 \rightarrow Q_3b\}$$

(c)  $G_c = (N_c, T, S_c, P_c)$

$$N_c = \{Q_0, Q_1, Q_2, Q_3, Q_4\}$$

$$T = \{a, b\}$$

$$S_c = Q_0$$

$$P_c = \{Q_0 \rightarrow \lambda, Q_1 \rightarrow \lambda, Q_2 \rightarrow \lambda, Q_2 \rightarrow \lambda, Q_3 \rightarrow \lambda, Q_0 \rightarrow aQ_0, Q_0 \rightarrow bQ_1, Q_1 \rightarrow aQ_0, Q_1 \rightarrow bQ_2, \\ Q_2 \rightarrow aQ_3, Q_2 \rightarrow bQ_2, Q_3 \rightarrow aQ_4, Q_3 \rightarrow bQ_2, Q_4 \rightarrow aQ_4, Q_4 \rightarrow bQ_4\}$$

$$G_{c2} = (N_{c2}, T, S_{c2}, P_{c2})$$

$$N_{c2} = \{Q_0, Q_1, Q_2, Q_3, Q_4, S\}$$

$$T = \{a, b\}$$

$$S_{c2} = Q_0$$

$$P_{c2} = \{Q_0 \rightarrow \lambda, S \rightarrow Q_0, S \rightarrow Q_1, S \rightarrow Q_2, S \rightarrow Q_3, Q_0 \rightarrow Q_0a, Q_1 \rightarrow Q_0b, Q_0 \rightarrow Q_1b, Q_2 \rightarrow Q_1b, \\ Q_3 \rightarrow Q_2a, Q_2 \rightarrow Q_2b, Q_4 \rightarrow Q_3a, Q_2 \rightarrow Q_3b, Q_4 \rightarrow Q_4a, Q_4 \rightarrow Q_4b\}$$

**Aufgabe 2:**

Pumping-Lemma:  $z = 10^{n_L} 1^{n_L} \$ 10^{n_L-1} 10^{n_L}$ . Damit  $z_i = uv^i wx^i y \in L_{bin-bin+1}$ ,

muss  $|v| = |x|$  und  $w = 1^a \$ 10^b$  mit  $a, b \in \mathbb{N}$ . Da  $|vwx| \leq n_L$  und  $|vx| \geq 1$ , muss  $v = 1^c, x = 0^c$  mit  $c \geq 1$ .

$\implies z_0 = 10^{n_L} 1^{n_L-c} \$ 10^{n_L-(c+1)} 10^{n_L} \notin L_{bin-bin+1} \implies L_{bin-bin+1}$  ist nicht kontextfrei.

**Aufgabe 3:**

(a) jkdas

(b) Beispiel aus der Vorlesung:  $L = \{a^i b^j c^k \mid i = 0 \vee j = k\}$ , Nicht-Regularität kann mit Präfix-Version nicht nachgewiesen werden.

Suffix-Version:  $z = ab^{n_L} c^{n_L}, v = c^x, x \geq 1$ , da  $|vw| \leq n_L. \implies z_0 = ab^{n_L} c^{n_L-x} \notin L$