

Einführung in die Wahrscheinlichkeitstheorie

Übungsserie 3

Aufgabe 1:

$$\mathbb{P}(A^C) = 1 - \mathbb{P}(A) = \underline{\underline{\frac{3}{4}}}$$

$$\mathbb{P}(A^C \cup B) = \mathbb{P}((A \cap B^C)^C) = \mathbb{P}((A \setminus B)^C) = 1 - \left(\frac{1}{3} - \frac{1}{6}\right) = \underline{\underline{\frac{5}{6}}}$$

$$\mathbb{P}(A \cup B^C) = \mathbb{P}((B \setminus A)^C) = 1 - \left(\frac{1}{4} - \frac{1}{6}\right) = \underline{\underline{\frac{11}{12}}}$$

$$\mathbb{P}(A \cap B^C) = \mathbb{P}(A \setminus B) = \frac{1}{3} - \frac{1}{6} = \underline{\underline{\frac{1}{6}}}$$

$$\mathbb{P}(A \triangle B) = \mathbb{P}((A \setminus B) \cup (B \setminus A)) = \left(\frac{1}{3} - \frac{1}{6}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) = \underline{\underline{\frac{1}{4}}}$$

$$\mathbb{P}(A^C \cup B^C) = \mathbb{P}((A \cap B)^C) = 1 - \frac{1}{6} = \underline{\underline{\frac{5}{6}}}$$

Aufgabe 3:

$$(a) \quad 0 \leq \mathbb{P}(A \cap B) \leq \mathbb{P}(B) = 0 \implies \mathbb{P}(A \cap B) = 0$$

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \underbrace{\mathbb{P}(B)}_{=0} - \underbrace{\mathbb{P}(A \cap B)}_{=0} = \underline{\underline{\mathbb{P}(A)}}$$

$$\begin{aligned} \mathbb{P}(A \cup B) &= \mathbb{P}(A \setminus B) + \mathbb{P}(B \setminus A) + \mathbb{P}(A \cap B) \\ &= \underline{\mathbb{P}(A \setminus B)} + \underbrace{\mathbb{P}(B \cap A^C)}_{=0} + \underbrace{\mathbb{P}(A \cap B)}_{=0} = \underline{\underline{\mathbb{P}(A)}} \end{aligned} \quad (1)$$

$$(b) \quad \mathbb{P}(B^C) = 1 - \mathbb{P}(B) = 0$$

$$\mathbb{P}(A \cap B) = \mathbb{P}(A \setminus B^C) \stackrel{(1)}{=} \underline{\underline{\mathbb{P}(A)}} \quad (2)$$

$$\mathbb{P}(B \setminus A) = \mathbb{P}(B \cap A^C) \stackrel{(2)}{=} \underline{\underline{\mathbb{P}(A^C)}}$$

Aufgabe 5:

$$|A| = (n-1)!$$

$$|\Omega| = n!$$

$$\mathbb{P}(A) = \frac{|A|}{|\Omega|} = \underline{\underline{\frac{1}{n}}}$$