**Calculation of Lie transformation .**

This transformation is equal (formulae (12) from [1])



with “perturbed” Hamiltonian



where



Map  is defined as Lie transformation:



where matrix  is defined by unperturbed Hamiltonian :



Two vectors  and  of the canonic dynamic variables for ion and electron are correspondingly



Hamiltonian  gives the following equations of the motion:



and not necessary present the changing of the phase , because it does not affect the dynamics of the ion-electron scattering event.

As is known, Lie transformation of an arbitrary function  of dynamic variables is characterized by the composition property:



So, Lie transformation  is described by two independent matrices  and  with the following nonzero entries:



It means that



In this expression velocities  and coordinate  are used.

Let’s input the following values:



and then with  Hamiltonians  and  are equal to



Further



and for these reasons



i.e. the changing of the ion parameters due to a collision with an electron are equal to





And, quite similarly, one find that



The changing of the electron parameters due to a collision with the ion can be found as



References

1. D.L. Bruhwiler, S.D. Webb. *New Algorithm for Dynamical Friction of Ions in a Magnetized Electron Beam.* AIP Conf. Proc. **1812**, 050006 (2017). <http://aip.scitation.org/doi/abs/10.1063/1.4975867>.
2. David Bruhwiler, Stephen Webb, Dan T. Abell. *A New Approach to Calculating Dynamical Friction for Magnetized Electron Cooling.* Presented at HSC Section Meeting, CERN (Hadron Synchrotron Collective effects), 24 April 2017, Geneva.