Hydraulic Components and Systems

by

Michael Rygaard Hansen
Department of Engineering
University of Agder

Torben Ole Andersen
Department of Energy Technology
Aalborg University

Preface

Hydraulics, as we consider it in this note, is the science of transmitting force and/or motion through the medium of a confined liquid. This is in one way, a rather narrow scope, because in its broadest sense, hydraulics encompasses any study of fluids in motion. It has its foundations from many thousands of years ago in ancient water works and irrigation systems. The name “hydraulics” comes from the Greek “hydros”, which means “water”.

Fluid power or oil hydraulics is an essential area of knowledge for anyone who has a technical interest in moving machinery. In for example farm tractors and implements, industrial trucks, earth-moving equipment, cars, we find applications of brute force with very precise control through hydraulic systems. Your girl friend can park your big two-ton automobile with only a slight effort at the steering wheel...the touch of a handle lifts a huge amount of load in the bucket of a loader, because the hidden giant hydraulics is there.

The purpose of this note is to put the tools of fluid power theory and practice into to the hands of the reader. The note has been prepared as an aid to basic training in hydraulic system design. In very simple language and with minimum use of mathematics, you are told the why’s and how’s of hydraulics. The note explains the fundamental principles of pressure and flow, describes the operation of the basic hydraulic components, tells how these components are combined to do their many jobs, and explores the fundamental considerations of hydraulic equipment design and use.

The note is organised in four major sections, Fundamentals (Chapter 1), Hydraulic Components (Chapter 2 and Chapter 3), Actuator Control (Chapter 4), System Design (Chapter 5) and System Analysis (Chapter 6). The fundamentals are just what the name implies, the basics of hydraulics. Note that this section also includes calculating pressure losses. The section on Hydraulic Components will give an understanding of the most important components in the system. The sections on actuator control describe approaches to speed and power of hydraulic actuators. In System Design a basic approach for arriving at a hydraulic system capable of performing a specific task is outlined. Finally, in the last section an introduction is given to both steady state and dynamic modeling of hydraulic systems.

Grimstad, August 2012

MICHAEL RYGAARD HANSEN
TORBEN OLE ANDERSEN

Contents

1
Hydraulic Principles

Hydraulic System Design

1.1 Hydrodynamics vs. Hydrostatics

Today, there are many thousands of pressure-operated machines and they are so distinct from earlier devices we must divide hydraulics into two sciences – hydrodynamics and hydrostatics. Hydrodynamics can be called the science of moving liquids under pressure. A water wheel or turbine (Figure 1.1) represents a hydrodynamic device. Energy is transmitted by the impact of a moving fluid against vanes. We are using the kinetic energy that the liquid contains.

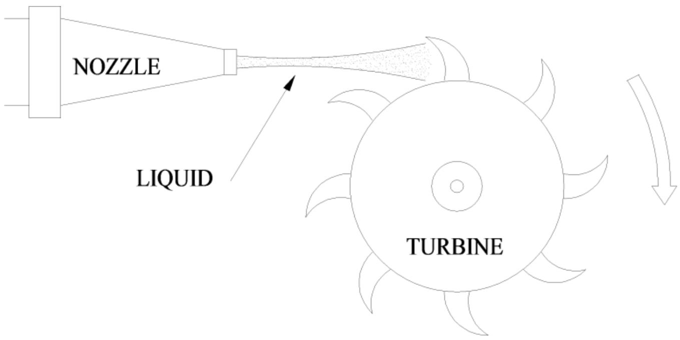


Figure 1.1 Transmission of energy using kinetic energy

In a hydrostatic device, power is transmitted by pushing on a confined liquid (Figure 1.2). The liquid must move or flow to cause motion, but the movement is incidental to the force output. A transfer of energy takes place because a quantity of liquid is subject to pressure.

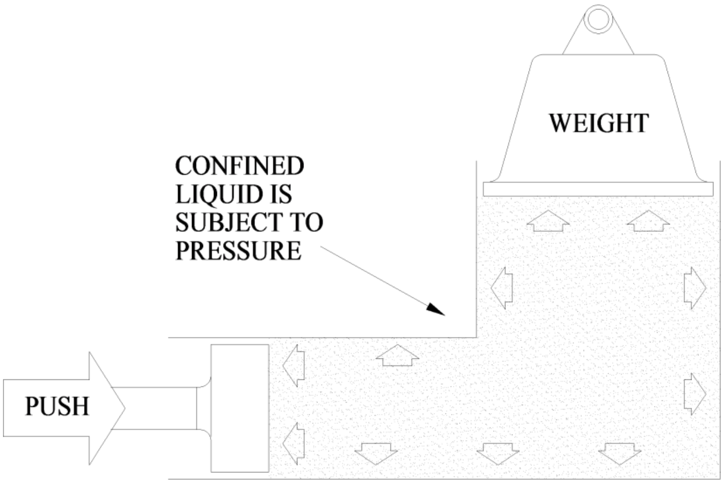


Figure 1.2 Force excerted by use of pressure

Most of the hydraulic machines in use today operate hydrostatically – that is through pressure. This note is limited to the pressure hydraulics branch, and will be using both the terms "hydraulic" and "fluid power" as is costumary in the industry.

1.2 Pressure and Flow

In studying the basic principles of hydraulics, we will be concerned with forces, energy transfer, work and power. We will relate these to two fundamental conditions or phenomena that we encounter in a hydraulic system. They are pressure and flow.

Pressure and flow, of course, must be inter-related in considering work, energy and power. On the other hand each has its own particular job to do.

- Pressure is responsible for pushing or exerting a force or torque
- Flow is responsible for making something move; for causing motion

Because these jobs are often confused, try to keep them distinct as we consider separately...and then together...the phenomena of pressure and flow.

1.2.1 Pressure provides the push

What is pressure?
To the engineer, pressure is a term used to define how much force is exerted against a specific area. The technical definition of pressure, in fact, is force per unit area.

We might say simply that pressure is a tendency to expand (or a resistance to compression) that is present in a fluid being squeezed. A fluid, by definition, is any liquid or gas. Because the term "fluid" is so widely used, then when referring to "the fluid", we mean the hydraulic liquid being used to transmit force and motion. (to satisfy the lubrication needs and other requirements in the system, they are usually specially refined and compounded petroleum oils).

Pressure in a confined liquid

If you have tried to force a stopper into a bottle that was completely full of water, you have experienced the near-incompressibility of a liquid. Each time you tried to push the stopper in, it sprang back immediately when you let it go.

When a confined liquid is compressed, there is a pressure build-up. The pressure is transmitted equally throughout the liquid container. This behaviour of a fluid is what makes it possible to transmit a push through pipes, around corners, up and down, and so on. In hydraulic systems, we use a liquid, because its near-in-incompressibility makes the action instantaneous, so long as the system is full of liquid.

Absolute and gauge pressure

It is fundamental that pressure can be created by pushing on a confined fluid *only* if there is a resistance to flow. There are two ways to push on a fluid; either by the action of some sort of mechanical pump or by the weight of the fluid itself.

The earth has an atmosphere of air extending some 80 km up. The weight of the air creates a pressure on the earth's surface. This pressure is called atmospheric pressure and any pressure condition less than atmospheric pressure is referred to as vacuum. If we measure this pressure we get

$$p_{atm}^{(abs)} = 1\, atmosphere = 760\; mmHg = 101{,}350\; Pa \tag{1.1}$$

Absolute pressure is a scale with its zero point at the complete absence of pressure, or a perfect vacuum. Gauge pressure ignores atmospheric pressure and is always measured relative to atmospheric pressure.

$$p^{(abs)} = p_{atm}^{(abs)} + p^{(gauge)} \tag{1.2}$$

In hydraulic system analysis we are merely concerned with pressure differences throughout a system, so ignoring atmospheric pressure has little effect on our analyses. Though be careful to keep track of the sign of the gauge pressure; it can be negative (vacuum), while absolute pressures are always positive.

Pressure can easily be created in a liquid with a pump, as shown in Figure 1.3. If we trap the liquid under a piston which has an area of 4.9 cm^2, and place a weight on the piston so that it pushes down with 490.5 N (= 50 $kg \cdot 9.81\; m/s^2$), we get a pressure of

$$\frac{490.5\; N}{4.9 \cdot 10^{-4}\; m^2} = 10^6\; Pa = 10\; bar \tag{1.3}$$

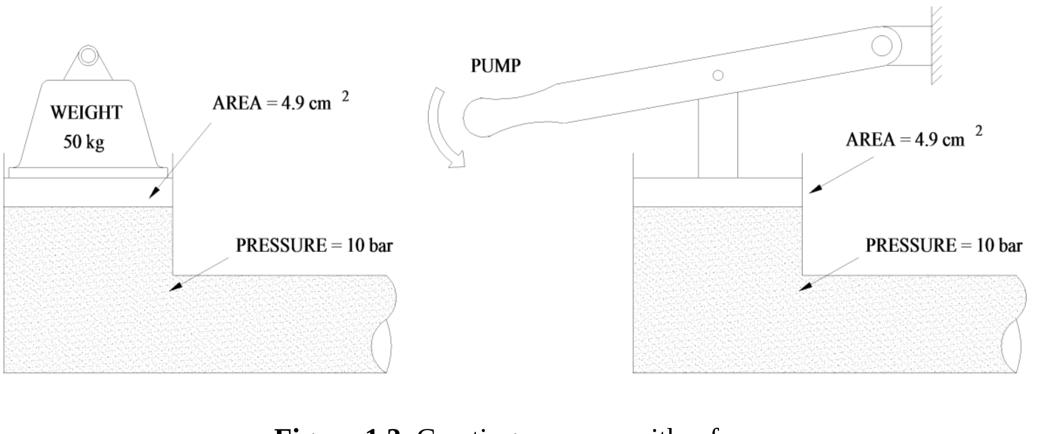


Figure 1.3 Creating pressure with a force

Now we have examined the phenomenon of pressure and how it is measured, we can go to see how it behaves in a hydraulic circuit

Pascal's law
Pascal's Law tells us that:

> *Pressure in a confined fluid is transmitted undiminished in every direction, and acts with equal force on equal areas, and at right angles to the container walls*

Pascal might have used the hydraulic lever to prove his law. He found that a small weight on a small piston will balance a larger weight on a larger piston...provided that the piston areas are in proportion to the weights.
Thus in Figure 1.4, a 20 *kg* weight on a 5 cm^2 area piston balances a 1000 *kg* weight on a 250 cm^2 piston. (ignoring the weights of the pistons themselves)

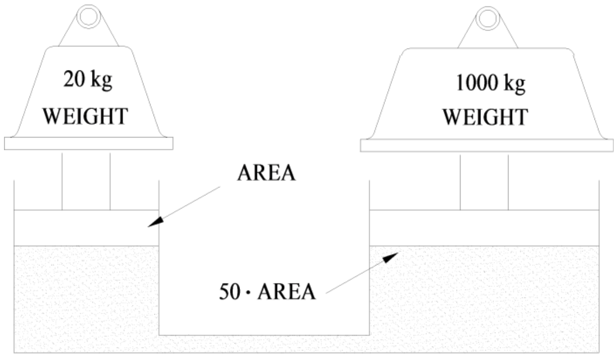


Figure 1.4 Leverage gained hydraulically

If the small piston is the pressure source, the pressure would be the weight divided by the piston area.

$$\text{Pressure} = \frac{20\ kg \cdot 9.81\ m/s^2}{5 \cdot 10^{-4}\ m^2} = 392400\ Pa \cong 3.92\ bar \tag{1.4}$$

The resulting force on the large piston is equal to this pressure multiplied by the piston area. Thus;

$$Force = 392400\ Pa \cdot 250 \cdot 10^{-4}\ m^2 = 9810\ N \tag{1.5}$$

meaning that the weight on the big piston is 1000 *kg*. We have multiplied force 50 times in this example; in other words obtained leverage of 50 to 1.

Pressure and force relationship
The example of hydraulic force has given the important relationship form Pascal's law; pressure is equal to force divided by area

$$p = F / A \quad \text{or} \quad F = p \cdot A \tag{1.6}$$

When using this relationship always use the SI units. If any other units are given, convert them to these units before solving the problem.

F (force) --- Newton (N); p (pressure) --- Pascal (Pa); A (area) --- square meters (m^2)

Let us have a look at this pressure/force/area relationship.

Back-pressure ~ series connection

If two hydraulic cylinders are connected to operate in series (Figure 1.5) the pressure required to move the second cylinder is effective against the first cylinder as a back-pressure. If each cylinder requires 50 *bar* to raise its load, the 50 *bar* of the second cylinder adds to the load of the first cylinder.

The piston areas as shown are equal, so the first cylinder would have to operate at 100 *bar*; 50 *bar* to lift its load and 50 *bar* to overcome back-pressure.

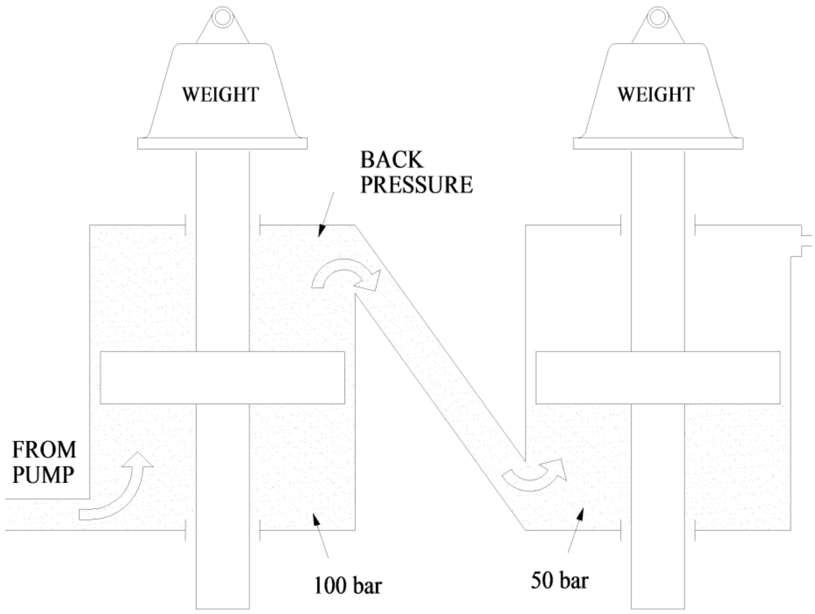


Figure 1.5 Series operation of two cylinders

Series operation is not common, but here it is used to illustrate that pressures add up in series. Anything that creates a back pressure on the device that moves the load, adds to the load, and increases the pressure requirement of the system.

Pressure in parallel connection

When several loads are connected in parallel (Figure 1.6) the oil takes the path of least resistance. Since the cylinder to the left requires the least pressure, it will move first. Furthermore, pressure won't build up beyond the needs of the left cylinder until it has reached its travel limit.

Then pressure will rise just high enough to move the cylinder in the middle (this is the situation shown in Figure 1.6). Finally when the cylinder in the middle is at its limit, pressure will rise to move the last cylinder.

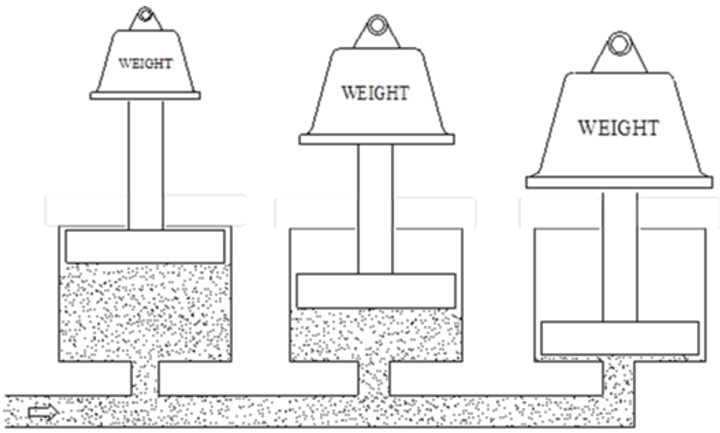


Figure 1.6 Pressure in parallel connection. The medium loaded piston is moving.

1.2.2 Flow makes it go

What is flow?
Flow is much easier to visualise than pressure. In our kitchen sink, for instance, we have atmospheric pressure. The city water works has built up a pressure in our pipes. When we open the tap, the pressure difference forces the water out. Thus, movement of the water is caused by a difference in pressure at two points.
In a hydraulic system, flow is usually produced by the action of a hydraulic pump; a device used to continuously push on the hydraulic fluid.

Velocity and flow rate
Velocity of a fluid is the average speed of its particles past a given point. It is usually measured in meter-per-second (*m/s*). Velocity is an important consideration in sizing the hydraulic lines that carry the fluid between components. A low velocity is desirable to reduce friction and turbulence in the hydraulic fluid.

Flow rate is the measure of how much volume of the liquid passes a point in a given time. It is usually measured in litre-per-minute (*l/min*). Flow rate is important in that it determines the speed at which the load moves, and therefore is important to the consideration of power.

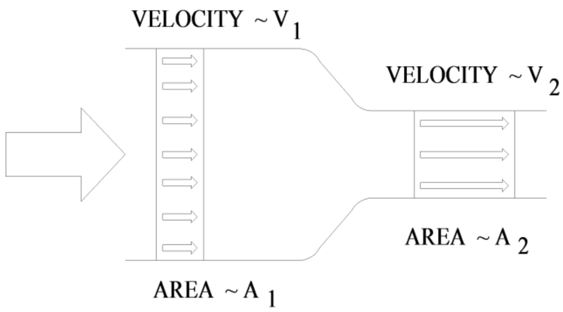


Figure 1.7 Flow through a conduit

Suppose we are pumping a constant flow rate through the conduit shown in Figure 1.7. If A_1 and A_2 are cross-sectional areas, it is obvious that a constant flow rate Q (*l/min*) will result in a lower velocity, when the diameter increases or a higher velocity when the diameter decreases. In fact, the velocity of oil in a hydraulic line is inversely proportional to the cross-sectional area (or to the diameter squared). Thus

$$\text{flow rate Q } (m^3/s) = \text{area A } (m^2) \times \text{velocity V } (m/s) = \text{constant} \tag{1.7}$$

Flow rate and speed

We can easily relate flow rate Q (*l/min*) to the speed at which the load moves, if we consider the cylinder volume we must fill and the distance the cylinder piston travels (Figure 1.8). The volume of the cylinder is simply the length of the stroke multiplied by the piston area. Suppose the left cylinder in Figure 1.8 is 20 cm long and holds one litre of oil. The right cylinder also holds one litre, but is only 10 cm long. If we pump one litre per minute into each, both pistons will move their full travel in one minute. However the left cylinder must move twice as fast because it has twice as far to go in the same amount of time.

So we see that a small diameter cylinder moves faster with an equal flow rate into it. Though, we have two ways of increasing the speed at which the load moves; decrease the size of the cylinder or increase the flow rate Q (*l/min*) to the cylinder. Conversely, we slow the load down by reducing the flow or increasing the size (diameter) of the cylinder.

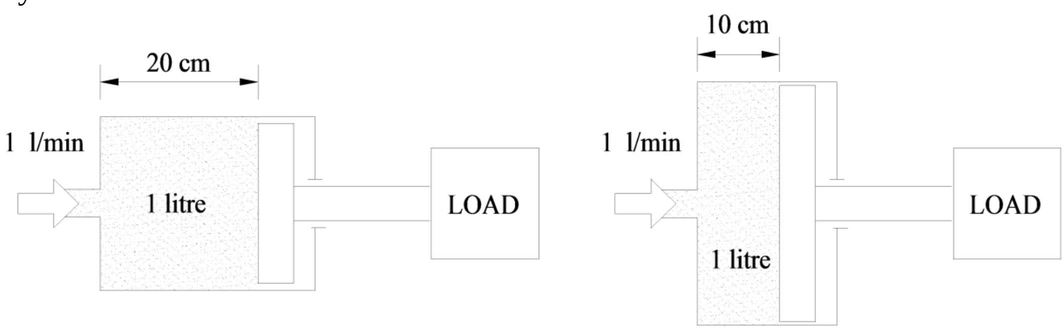


Figure 1.8 Illustrating flow rate and speed

The speed of the cylinder, then must be proportional to flow and inversely proportional to the piston area (or the diameter squared).

Flow and pressure drop

A basic rule of hydraulics is that wherever there is flow, there must be a pressure difference or pressure drop. Conversely, where there is a difference in pressure, there must be either flow or at least a difference in the level of the liquid.

The pressure difference when a liquid is flowing is used to overcome friction and to lift the fluid where necessary. When a liquid is flowing, the pressure is always highest upstream and lowest downstream. That is why we refer to the difference as “drop”.

Flow through an orifice

Pressure drop occurs to a greater degree when the flow is restricted. An orifice (Figure 1.9) is a restriction often placed in a line deliberately to create a pressure difference. There is always a pressure drop across an orifice as long as there is flow. However, if we block the flow beyond the orifice, Pascal’s Law takes over and pressure equalises on both sides.

Pressure drop also takes place when passing fluid through a valve or line. The smaller the valve passage or line the greater the pressure drop. In effect, the restrictive area acts as an orifice. Accounting for the pressure drops around a hydraulic circuit is a key factor in circuit analysis. Anytime a pressure drop occurs and no mechanical work is delivered, fluid energy is converted to heat energy. Efficiency is increased when these pressure drops are minimised.

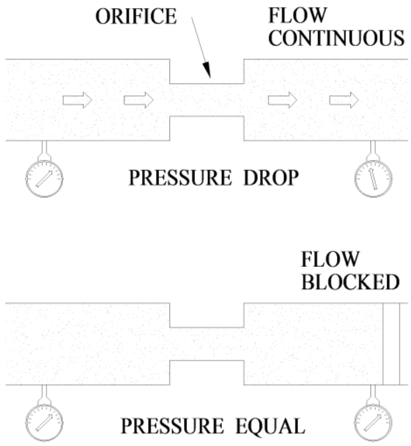


Figure 1.9 Flow through an orifice

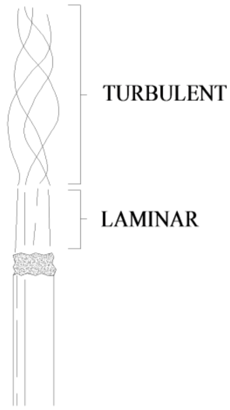


Figure 1.10 Laminar/turbulent flow

Laminar and turbulent flow
A streamlined or laminar flow (Figure 1.10) is desirable to keep friction minimised. Abrupt changes in sections, sharp turns and too high velocity all induce turbulent flow. Then instead of moving in smooth, parallel paths, the fluid particles develop cross currents. The result is a significant increase in friction and pressure drop.

Work and energy
Earlier, we developed the concepts of force and pressure, primarily as measures of effort. Work on the other hand, is a measure of accomplishment. It requires motion to make a force do work. Therefore, to do work in a hydraulic system, we must have flow.
Work is perhaps best defined as exerting a force over a definite distance. Thus if we raise 100 *kg* 10 meters we do $100 \cdot 9.81 \cdot 10 = 9810$ *Newton-meter* of work. Work is a measure of force multiplied by distance and we usually express it in Newton-meter.

$$\text{Work } (Nm) = \text{Force } (N) \times \text{Distance } (m) \tag{1.8}$$

Energy is the capacity to do work, and is expressed in the same units as work. We are familiar with several forms of energy. The load of 100 *kg* just mentioned, when it is raised, has potential energy. It is capable of doing work when it is lowered. A body in motion has kinetic energy, capable of doing work. Coal contains heat energy; a battery electrical energy; a steam boiler pressure energy.

Energy transfer in the hydraulic lever
The basic concept of fluid power is simple; mechanical energy is converted to fluid energy, which is then converted back to mechanical energy.
Let's have another look now at Pascal's hydraulic lever and see how energy is transferred there.

In Figure 1.11, we have just slightly upset the balance so that the small piston is forced down and pushes the large piston up.

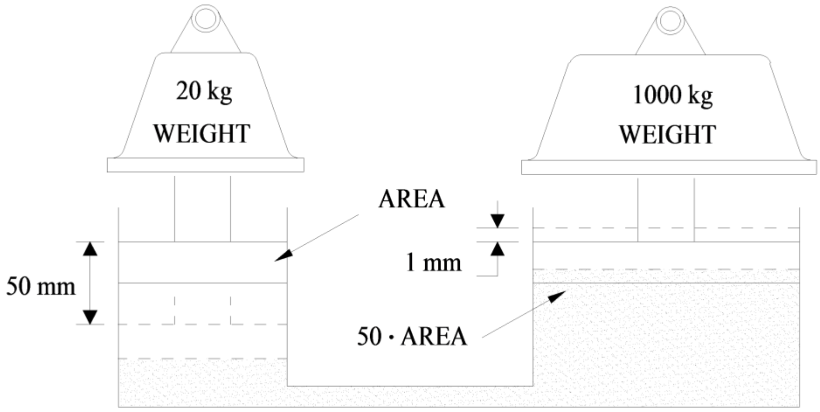


Figure 1.11 Energy transfer in a hydraulic lever

For simplicity's sake we ignore the effects of friction. At the small piston, we have moved 20 *kg* downward for 50 *mm*. In doing so, we gave up 20 · 9.81 · 0.05 = 9.81 *Nm* of potential energy. We changed the potential energy to pressure energy. The displacement flow moved the big weight up 1 *mm*. Thus, the 1000 *kg* weight received an increase of 9.81 *Nm* of potential energy. At each piston, then, we did 9.81 *Nm* of work. Thus, energy was transferred without loss from the 20 *kg* weight to the 1000 *kg* weight.

Bernouilli's principle

Bernouilli's principle tells us that the sums of pressure, potential and kinetic energy at various points in a system must be constant, if flow is constant. The potential energy term is normally negligible in hydraulic systems.

When a fluid flows through areas of different diameters (figure 1.13), there must be corresponding changes in velocity.

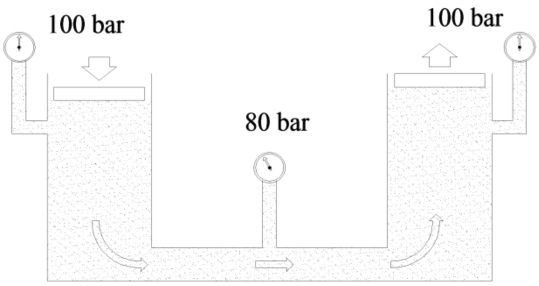


Figure 1.12 Bernoulli's principle

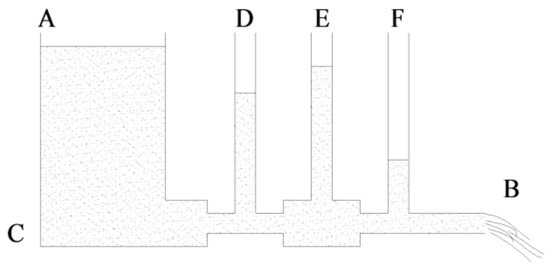


Figure 1.13 Effect of velocity and friction

At the left, the section is large so velocity is low, and pressure high. In the middle velocity must be increased because the area is smaller. Again at the right, the area increases to the original size and velocity again decreases. Bernouilli's principle states the pressure in the middle section must be less than elsewhere because velocity is greater. An increase in velocity means an increase in kinetic energy. Since energy cannot be created, kinetic energy can only increase if the static pressure decreases. In the outer sections kinetic energy is converted back to pressure. If there is no frictional loss, the pressure in the outer sections will be equal. In Figure 1.13 is shown the combined effects of friction and velocity changes. Pressure drops from maximum at C to atmospheric pressure at B. At D velocity is increased, so the pressure decreases. At E, the pressure increases while most of the kinetic energy is given up to pressure energy.

Again at F, pressure drops as velocity increases. The difference in pressure at D and F is due to friction losses.

Power
Power is the rate of doing work or the rate of energy transfer. To visualize power, think about climbing a flight of stairs. If you walk up, it's relatively easy. But if you run up, you are liable to get to the top out of breath. You did the same amount of work either way...but when you ran up, you did it at a faster rate, which required more power.

The standard unit of power is Watt...named after James Watt that related the ability of his steam engine to the pulling power of a horse. By experiments with weights, pulleys and horses, Watt decided that a horse could comfortably do 736 *Nm/s* (1 *Nm/s* = 1 *W*), hour after hour. This value has since been designated as one horsepower (*hp*).
Power, though, is force multiplied by distance divided by time:

$$\text{P (Power)} = \frac{\text{F (Force)} \times \text{D (Distance)}}{\text{T (Time)}} \tag{1.9}$$

The horsepower used in a hydraulic system can be computed if we know the flow rate and the pressure. In Figure 1.14 we have a flow rate Q into the piston chamber giving the piston a velocity V. Due to the outer force F a pressure p builds up in the piston chamber as well. While force is equal to pressure multiplied area and flow is equal to velocity multiplied area, we can write the equation that relate/convert the mechanical power to hydraulic power.

$$P_{hyd} = P_{mech} = \underline{\underline{F \cdot V}} = (p \cdot A) \cdot (Q / A) = \underline{\underline{p \cdot Q}} \tag{1.10}$$

- Pressure is required to obtain a force from a cylinder
- Flow is required to generate linear motion with a cylinder

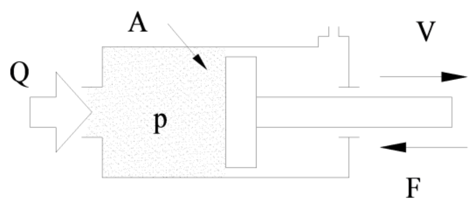


Figure 1.14 Hydraulic/mechanical power

We can see from the relationship in Equation (1.10) that an increase in either pressure or flow will increase the power. Also, if pressure or flow decreases, power decreases.

1.3 Hydraulic System Components

In the following chapters, you will be studying the components that go into hydraulic circuits, and how they are put together to do their jobs. From the beginning of Chapter 2 and in the end of this chapter we will talk about components and their interrelationships, so we will give them some basic attention now.

We have already mentioned that a pump is required to push the fluid. We have implied, too, that a cylinder is the output of the system. Besides a pump and cylinder, we also require valves to control the fluid flow; a reservoir to store the fluid and supply it to the pump; connecting lines; and various hydraulic accessories.

We will look at two very basic and simple systems now, and see how the basic components are classified.

The hydraulic jack

In Figure 1.15 is shown the basic circuit for a hydraulic jack. The difference from the hydraulic lever in Figure 1.4 is that we have added a reservoir and a system of valves to permit stroking the small cylinder or pump continuously and raising the large piston or actuator a notch with each stroke. In the top view, we see the intake stroke. The outlet check valve is closed by pressure under the load and the inlet check valve opens to allow liquid from the reservoir to fill the pumping chamber.

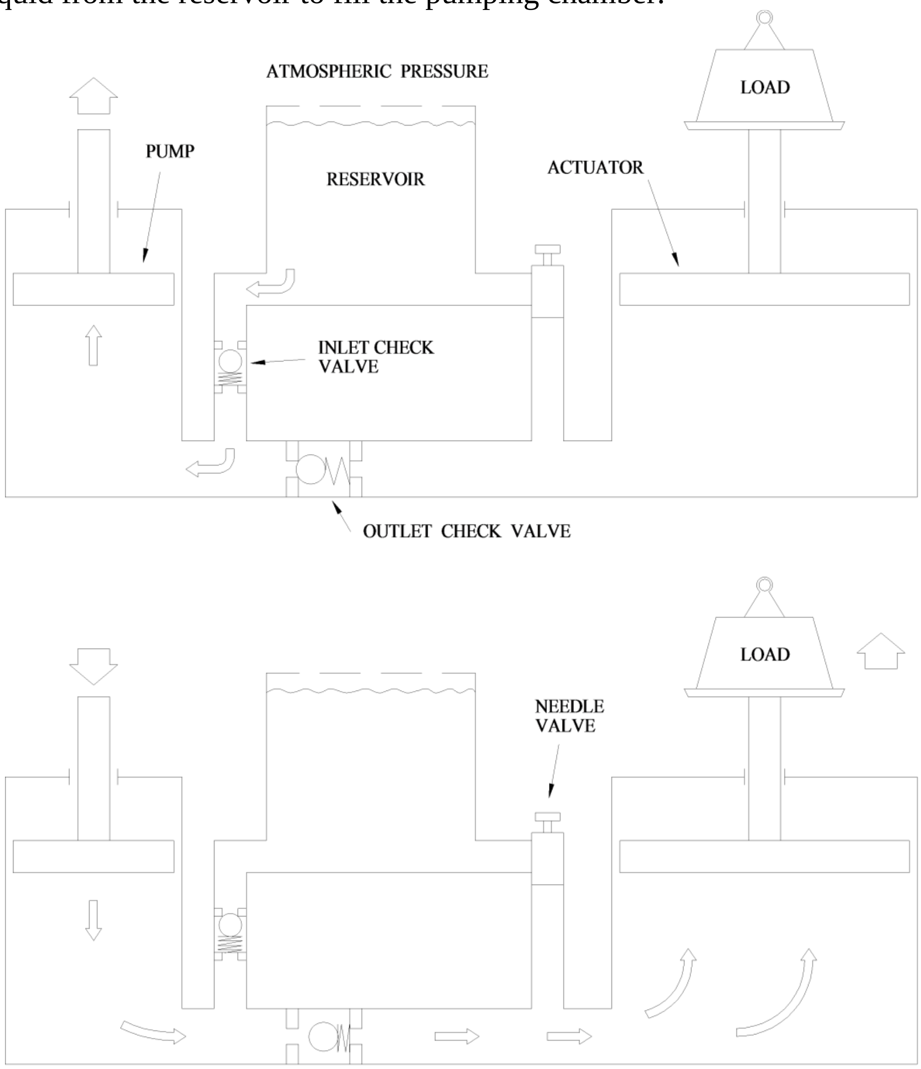


Figure 1.15 Basic circuit of a hydraulic jack

In the bottom view, the pump is stoked downward. The inlet check valve is closed by pressure and the outlet valve opens. Another “slug” of liquid is pumped under the large piston to raise it.

To lower the load, we open a third valve, a needle valve, which opens the area under the large piston to the reservoir. The load then pushes the piston down and forces the liquid into the reservoir.

Motor-reversing system

In Figure 1.16, we have an entirely different kind of system. Here, a power-driven pump operates a reversible rotary motor. A reversing valve directs fluid to either side of the motor and back to the reservoir. A relief valve protects the system against excess pressure, and can bypass pump output to the reservoir if pressure rises to high.

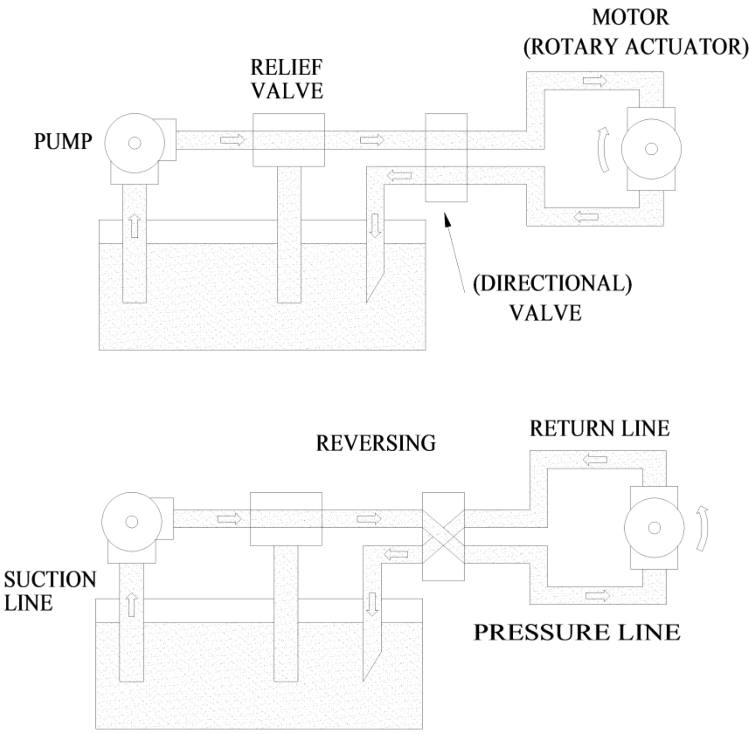


Figure 1.16 Motor-reversing system

Pump classifications

The pump in Figure 1.15 is called a reciprocating pump. Most pumps are of the rotary type as in Figure 1.16, and are driven by engines or electric motors. Rotary pumps can be constant displacement; meaning they deliver the same amount of fluid every stroke, revolution or cycle. Flow rate varies in proportion to drive speed - or by using variable displacement pumps, which can have their delivery rates changed by external controls while drive speed remains constant.

Actuator classifications

The actuator is the system's output component. It converts pressure energy to mechanical energy. A cylinder is a linear actuator. Its outputs are force and straight line motion. A motor is a rotary actuator. Its outputs are torque and rotating motion.

The large piston actuator in Figure 1.14 is a single-action cylinder. This means it is operated hydraulically in one direction only and returned by other means...in this case by gravity. A double-acting cylinder operates hydraulically in both directions.

The motor in Figure 1.15 is a reversible motor. Other motors are uni-directional or non-reversible. They can only rotate in one direction.

Valve classifications
In Chapter 3 we will study three classes of valves. They are:

1. Directional control valves
2. Pressure control valves, and
3. Flow or volume control valves

- Directional control valves tell the oil where to go by opening and closing passages. The check valves in Figure 1.14 are directional valves. They are called one-way valves, because they permit only one flow path. The reversing valve in Figure 1.15 is a four-way directional valve, because it has four flow paths.

- A pressure control valve is used to limit pressure or to control or regulate pressure in the system. The relief valve in Figure 1.15 is a pressure control valve. It limits the pressure that can build up in the circuit. Other types of pressure controls are brake valves, sequence valves, pressure reducing valves and counterbalance valves.

- Flow control valves regulate flow to control the speed of an actuator. The needle valve in Figure 1.14 has a flow control function. It restricts flow so that the load can't come down too fast.

Classification of lines
The lines which connect the components of our systems are classified according to their functions. The principal kinds of lines are:

$$Working\ lines: \begin{cases} Pressure\ lines \\ Suction\ lines \\ Return\ lines \end{cases} \qquad Non-working\ lines: \begin{cases} Drain\ lines \\ Pilot\ lines \end{cases}$$

- Working lines are lines which carry the mainstream of fluid in the system; that is, the fluid involved in the energy transfer. Starting at the reservoir, we have a suction line which carries the fluid to the pump inlet (Figure 1.15). From the pump to the actuator, is the pressure line, which carries the same fluid under pressure to do the work. After the pressure energy in the fluid is given up at the actuator, the exhaust fluid is re-routed to the reservoir through the exhaust or return line.

- Non-working lines are auxiliary lines which do not carry the main stream of flow. A drain line is used to carry leakage oil or exhaust pilot fluid back to the reservoir. A pilot line carries fluid that is used to control the operation of a component.

Circuit diagrams
We have used some very simple schematic diagrams in this chapter to illustrate hydraulics principles and the operation of components. However, outside of instructional materials we almost exclusively use a standard type of shorthand (i.e. symbols to DIN ISO 1219). Symbols for hydraulic systems are for functional interpretation. Symbols are neither dimensioned nor specified for any particular position. We will return to these diagrams in more detail in Chapter 2 and 3.

In Figure 1.17 is shown the graphical diagram for the reversible motor circuit.

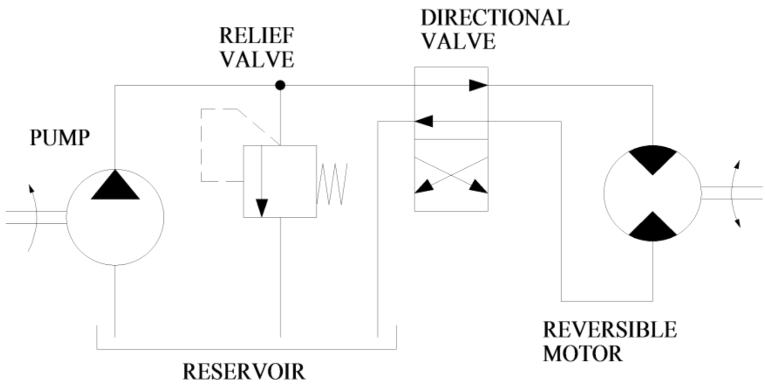


Figure 1.17 Hydraulic circuit after DIN ISO 1219

1.4 Hydraulic Losses

Kinds of energy in hydraulic system
The purpose of a hydraulic system is to transfer mechanical energy from one place to another through the medium of pressure energy. The prime source of the energy may be heat in a motor fuel or electrical energy in a battery or from power lines.
Mechanical energy driving the hydraulic pump is converted to pressure energy and kinetic energy in the fluid. This is reconverted to mechanical energy to move a load. Friction along the way causes some losses in the form of heat energy. In this section we will look at different losses that appear in hydraulic circuits.

1.4.1 Flow in lines (losses in hoses and pipes)

When a fluid flows through a line, the layer of fluid particles next to the wall have zero velocity. The velocity profile a distance away from the wall develops because of viscosity. The more viscous the fluid, the greater the change in velocity with the distance from the wall.

Viscosity
The most important of the physical properties of hydraulic fluids is the viscosity. It is a measure of the resistance of the fluid towards laminar (shearing) motion, and is normally specified to lie within a certain interval for hydraulic components in order to obtain the expected performance and lifetime. The definition of viscosities is related to the shearing stress that appear between adjacent layers, when forced to move relative (laminarly) to each other. For a Newtonian fluid this shearing stress is defined as:

$$\tau_{xy} = \mu \frac{d\dot{x}}{dy} \tag{1.11}$$

where

τ_{xy}	is the shearing stress in the fluid, [N/m^2]
μ	is the dynamic viscosity, [Ns/m^2]
$\dot{x}$	is the velocity of the fluid, [m/s]
y	is a coordinate perpendicular to the fluid velocity, [m]

In Figure 1.18 the variables associated with the definition of the dynamic viscosity are shown.

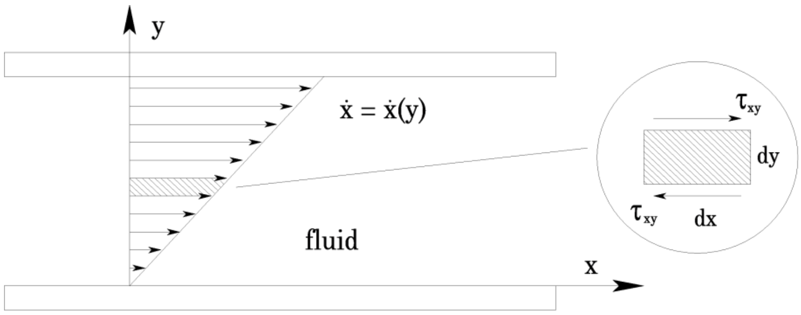


Figure 1.18 Shearing stresses in a fluid element

The usual units for the dynamic viscosity are *P* for Poise or *cP* for centipoise. Their relations to the SI-units are as follows: $1P = 100cP = 0.1Ns/m^2$.
For practical purposes, however, the dynamic viscosity is seldom used, as compared to the kinematic viscosity that is defined as follows:

$$\upsilon = \frac{\mu}{\rho} \tag{1.12}$$

where

υ	is the kinematic viscosity, [m^2/s]
μ	is the dynamic viscosity, [Ns/m^2]
ρ	is the density, [kg/m^3]

The usual unit used for υ is centistoke, *cSt*, and it relates to the SI units as follows:

$$1\ cSt = 10^{-6}\frac{m^2}{s} = 1\frac{mm^2}{s}.$$

A low viscosity corresponds to a "thin" fluid and a high viscosity corresponds to a "thick" fluid.

Reynolds number
A key issue in fluid power circuits are forces due to fluid inertia and forces due to viscosity. In general, flow dominated by viscosity forces is said to be *laminar*, and inertia dominated flow is said to be *turbulent*. In Figure 1.19 is shown the velocity profile of the two kinds of flow. Laminar flow is characterised by an orderly, smooth, parallel line motion of the fluid. Inertia dominated flow (*turbulent*) is characterised by irregular, eddylike paths of the fluid.

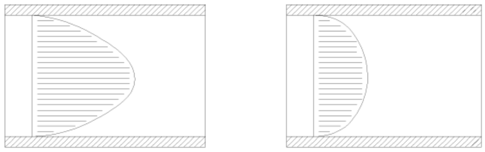


Figure 1.19 Laminar and turbulent flow profile

In fluid power it is only inertia and viscous forces that matters. Experience has shown that it is either the inertia forces or the viscous forces that dominate, giving two types of flow regimes. Osborn Reynolds performed a series of experiments in 1833 to define the transition between laminar and turbulent flow. He defined a useful quantity which describes the relative significance of these two forces in a given flow situation. The dimensionless ratio of inertia forces to viscous force is called Reynolds number and defined by

$$\mathrm{Re} = \frac{U \cdot d_h}{\upsilon} \tag{1.13}$$

where ρ is fluid mass density, υ is the kinematic viscosity, U is the average velocity of flow, and d_h is a characteristic length of the flow path. In our case d_h is taken to be the hydraulic diameter which is defined as:

$$d_h = \frac{4 \times \mathit{flow\ area}}{\mathit{flow\ perimeter}} = \frac{4 \cdot A}{C_h} \tag{1.14}$$

For a hydraulic hose or pipe it is convenient in many cases to use the following formula for Reynolds number

$$Re = \frac{V_{line} \cdot D_{line}}{\upsilon} \tag{1.15}$$

where V_{line} is the fluid velocity in the line and D_{line} is the inner diameter of the line.

Reynolds discovered the following rules with his tests:

1. If $Re < 2000$, flow is laminar
2. If $Re > 4000$, flow is turbulent

We normally use Re = 2300 as transition number for hydraulic hoses and pipes.

Darcy's equation

Friction is the main cause of loss of fluid energy as the fluid flows through a line. Because of friction, some fluid energy is converted to heat energy and exchanged into the surrounding atmosphere.

Loss along a line can be calculated as a pressure loss Δp, directly using Darcy's equation.

$$\Delta p = \lambda \cdot \frac{L}{D_{line}} \cdot \rho \cdot \frac{V_{line}^2}{2} \tag{1.16}$$

where L is the length of the line and λ is a dimensionless friction factor to be determined.

Laminar flow

For laminar flow, the friction factor can be given as

$$\lambda = 64 / Re \tag{1.17}$$

Substitution of Equation 1.17 into Darcy's equation gives the Hagen-Poiseuille equation

$$\Delta p = \frac{128 \cdot \mu \cdot L \cdot Q}{\pi \cdot D_{line}^4} \tag{1.18}$$

where Q is the flow rate through the line.

Turbulent flow

When the flow is turbulent, the friction factor is a function of Reynolds number and the relative roughness of the line, but for smooth lines and Reynolds numbers less than 100.000 the equation given by Blasius can be used to calculate the friction factor.

$$\lambda = 0.3164 \cdot \frac{1}{Re^{0.25}} \tag{1.19}$$

1.4.2 Losses in Fittings

Tests have shown that pressure losses in fittings are proportional to the square of the velocity of the fluid

$$\Delta p = \xi \cdot \frac{\rho}{2} \cdot V_{line}^2 \tag{1.20}$$

where ξ is a friction factor that must be determined by tests. Some common factors are

Fitting	**Friction factor ξ**
Standard tee	1.3
45° elbow	0.5
Return bend (U-turn)	1.5

1.4.3 The orifice equation (losses in valves)

The flow restrictions or orifices are a basic means for the control of fluid power. An orifice is a sudden restriction of short length in a flow passage and may have a fixed or variable area.

Equation 1.21 is Bernoulli's equation with negligible gravity forces.

$$\frac{p}{\rho}+\frac{u^2}{2}=constant \tag{1.21}$$

As an important case where Equation 1.21 is used consider flow through an orifice (Figure 1.20).

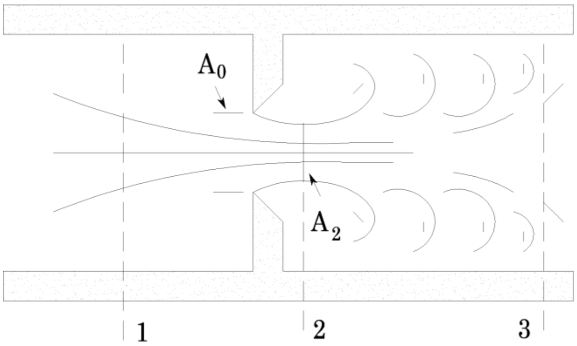


Figure 1.20 Flow through an orifice; turbulent flow

Since most orifice flow occurs at high Reynolds numbers, this region is of great importance. Experience has justified the use of Bernoulli's equation in the region between point 1 and 2. The point along the jet where the area becomes a minimum is called the *vena contracta*. The ratio between the area at vena contracta A_2 and the orifice area A_0 defines the so called contraction coefficient C_c.

$$C_c = A_2 / A_0 \tag{1.22}$$

After the fluid has passed the vena contracta there is turbulence and mixing of the jet with the fluid in the downstream region. The kinetic energy is converted into heat. Since the internal energy is not recovered the pressures p_2 and p_3 are approximately equal.

Now it is possible to use Bernoulli's equation 1.21 to calculate the relation between the upstream velocity u_1 to the velocity u_0 in vena contracta. Therefore

$$u_2^2-u_1^2=\frac{2}{\rho}\cdot(p_1-p_2) \tag{1.23}$$

Applying the continuity equation for incompressible flow yields

$$A_1\cdot u_1 = A_2\cdot u_2 = A_3\cdot u_3 \tag{1.24}$$

Combining Equation 1.23 and 1.24 and solving for u_2 gives

$$u_2=\left[1-\left(\frac{A_2}{A_1}\right)^2\right]^{-1/2}\cdot\sqrt{\frac{2}{\rho}(p_1-p_2)} \tag{1.25}$$

In the real world there will always be some viscous friction (and deviation from ideal potential flow), and therefore an empirical factor C_v is introduced to account for this discrepancy. The factor C_v is typically around 0.98. Since $Q = A_2 u_2$ the flow rate at vena contracta becomes,

$$Q = \frac{C_v A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{\frac{2}{\rho}(p_1 - p_2)} \tag{1.26}$$

Defining the discharge coefficient C_d in Equation 1.26 it is possible to express the orifice flow by the orifice area.

$$C_d = \frac{C_v C_c}{\sqrt{1-C_c^2(A_0/A_1)^2}} \tag{1.27}$$

Now, combining Equation 1.22, 1.26, and 1.27 the orifice equation (in Danish *blændeformlen*) can be written

$$Q = C_d A_0 \sqrt{\frac{2}{\rho}(p_1 - p_2)} \tag{1.28}$$

Normally A_0 is much smaller than A_1 and since $C_v \approx 1$, the discharge coefficient is approximately equal to the contraction coefficient. Different theoretical and experimental investigations has shown that a discharge coefficient of $C_d \approx 0.6$ is often assumed for all spool orifices.

At low temperatures, low orifice pressure drop, and/or small orifice openings, the Reynolds number may become sufficiently low to permit laminar flow. Although the analysis leading to Equation 1.28 is not valid at low Reynolds numbers, it is often used anyway by letting the discharge coefficient be a function of Reynolds number. For *Re < 10* experimental results show that the discharge coefficient is directly proportional to the square root of Reynolds number; that is $C_d = \delta \cdot \sqrt{\mathrm{Re}}$. A typical plot of such a result is shown in Figure 1.21.

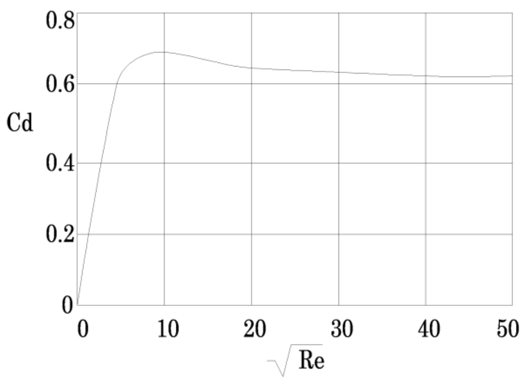


Figure 1.21 Plot of a discharge coefficient versus Reynolds number for an orifice

So the orifice equation states that the flow through an orifice is proportional to the square root of the pressure drop across the orifice

Solving for the pressure drop Δp, we obtain the orifice equation with the flow rate Q as the independent variable; a form more useful for our analysis of fluid power circuits.

$$\Delta p = k_{eq} \cdot Q^2 \quad ; \quad k_{eq} = \frac{\rho}{2} \cdot \frac{1}{C_d^2 \cdot A_0^2} \tag{1.29}$$

This relationship shows there is a quadratic relation between Δp and Q. Most valves form some type of orifice in their internal geometry, and thus have a pressure vs. flow characteristic following Equation 1.29. Knowing the flow through the valve, the designer can estimate the pressure drop. The technical data sheets supplied by almost all valve manufacturers have a pressure vs. flow curve, from which it is possible to determine k_{eq}.

1.4.4 Leakage flow

Many hydraulic components have precision made parts with very fine clearances between internal moving elements, but still, due to high pressure, they make leakage pathways.
Dryden (1956) presents the following expression for leakage flow as a function of pressure drop across a rectangular leakage pathway.

$$Q = \frac{w \cdot h^3}{12 \cdot \mu \cdot L} \cdot \Delta p \tag{1.30}$$

where w is the width of the rectangular opening and h the height of the rectangular opening (Figure 1.22)

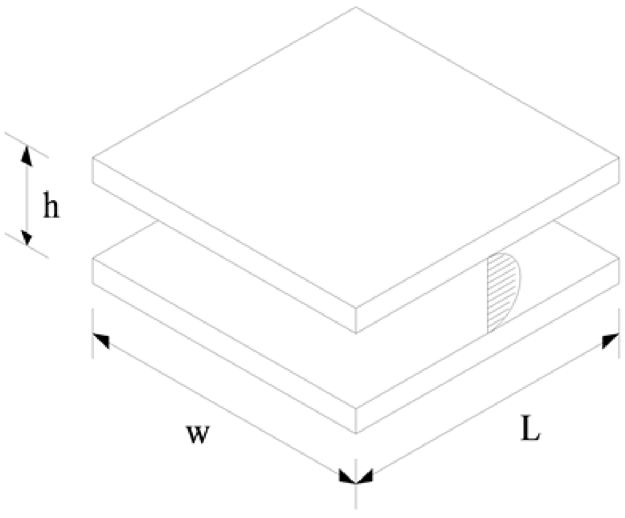


Figure 1.22 Geometry of leakage flow idealized as a thin rectangle

1.5 Summary

Now that we have studied basic principles and have an idea of how hydraulics work, lets close this chapter with some important characteristics of hydraulic systems and a short summary.

Key characteristics/advantages of fluid power are:

1. High power density (high power output per unit mass of system)
2. Operation may commence from rest under full load
3. Smooth adjustment of speed, torque, force are easily achieved
4. Simple protection against overloading (relief valve opens to protect system)

A key disadvantage is the inefficiency

In a hydraulic system mechanical energy is converted to hydraulic energy (pressure and flow), which then is converted back to mechanical energy.

1. Pressure is required to obtain torque from a motor or force from a cylinder
2. Flow is required to generate rotary motion with a motor or linear motion with a cylinder

Accounting for the pressure drop in a hydraulic system is a very important factor in circuit analysis and design. Pressure drops occurring without delivering mechanical work, means that hydraulic energy is converted to heat.
Pressure drops occurring in different types of valves, is in most hydraulic circuit the major source of losses, and dominates over the losses in the lines and fittings.

----- oo 0 oo -----

2
Characteristics of Pumps and Motors

Hydraulic System Design

2.1 Introduction to Hydrostatic Pumps and Motors

In this chapter we discuss types of positive displacement machines with particular emphasis on steady-state performance parameters and characteristics. A basic understanding of the components facilitates understanding of any type of hydraulic circuit. The basic components used in hydraulic circuits include pumps, cylinders, motors and hydraulic drives, valves and reservoirs. Chapter 3 treats valves and Chapter 4 combine valves and pumps to form basic hydraulic power elements. Chapter 5 is devoted to synthesis of hydraulic systems.

Definitions
Hydraulic pumps and motors are used to convert mechanical energy into hydraulic energy and vice versa. It is transmission of power by fluid using positive displacement pumps and motors. A *positive* displacement pump is one in which each revolution of the pump shaft is associated with a fixed quantity of fluid delivered and similarly a positive displacement motor is one in which each revolution of the motor shaft is associated with a fixed quantity of fluid accepted.
In *non-positive* pumps and motors, such as turbine and centrifugal pumps the flow is continuous from inlet to outlet and results from energy being directly imparted from the fluid stream. These machines are basically low pressure with high volume output.
Another way to classify positive and non-positive displacement machines is whether the inlet is sealed from the outlet.

- If the inlet and outlet are connected hydraulically so that the fluid can recirculate in the pump/motor when pressure builds up, the pump/motor is *non-positive* displacement
- It the inlet is sealed from the outlet, the pump will deliver fluid anytime the inlet is kept supplied and the pump is driven. Such a pump is classified as *positive* delivery or *positive* displacement, and requires a relief valve to protect it from pressure overloads.

While non-positive pumps and motors are quite ineffective and not suited for control purposes only positive displacement pumps are used in hydraulic systems.

2.1.1 Pump classifications

The pump is driven by a prime mover which is usually an electric motor or a petrol or diesel engine. The energy input from the prime mover to the pump is converted into high-pressure energy in the fluid which is transmitted through pipes and in turn is converted into rotational energy by a motor or translational energy by a cylinder.
Hydraulic pumps are classified by their design – gear, vane and piston. In all cases a moving element in a fixed container displaces fluid from an inlet to an outlet port. The pressure at the inlet port is due to the head of fluid from the reservoir and at the outlet port by the resistance imposed by the work load on the hydraulic motor/cylinder. Plus the resistance due to friction in pipes, valves, etc.

Gear-type pumps
These employ the principle of a pair of meshing gears, contained in a housing fitted with suction and discharge ports as shown in Figure 2.1

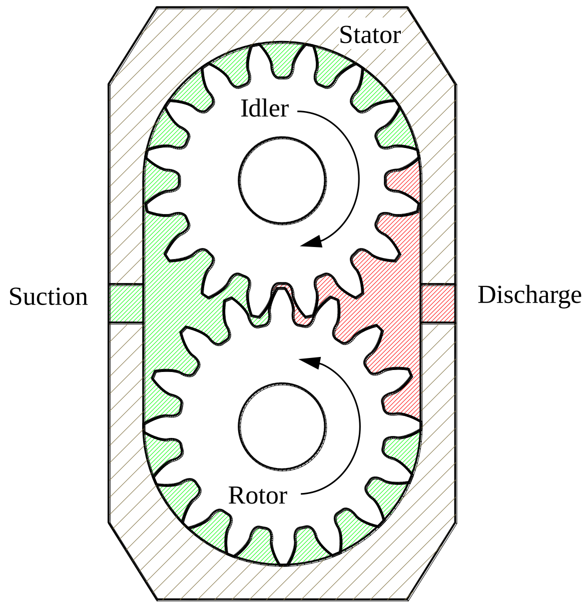


Figure 2.1 Design of a gear pump

One gear (the rotor) is driven by the prime mover and as it rotates and drives the idler gear. The fluid is displaced between the gears and the inner surface of the stator. The fluid is thus carried around from the inlet port to the outlet port, the main seal between the two being provided by the meshing gears. Some fluid is trapped between the gear teeth and the design of the pump allows for this fluid to be transferred back to the suction port.

There must obviously be clearance between the stationary casing and the tips of the moving gear. As pressure builds up on the outlet side of the pump the pressure difference across the gears results in leakage of fluid. This leakage is commonly called “slip” and is a factor present to at greater or lesser extent in all positive displacement pumps. The volumetric efficiency is reduced as the leakage increases and the mechanical (or mechanical-hydraulic) efficiency is reduced because of the power wasted in producing the slip. The performance (efficiency, losses, etc.) of pumps and motors is dealt with in detail in Section 2.2 and 2.3.
In a gear pump slip occurs not only across the gear teeth but also between the gears and the side faces. To reduce this as far as possible these side plates are hydraulically loaded to maintain close contact onto the sides of the gears.

Vane-type pumps
The construction of a vane pump is shown in Figure 2.2. Here the moving member consists of a rotor in which close fitting vanes are carried radially in slots and their tips bear against the stationary housing. The vanes are free to move in and out of the slots. As the rotor turns, the vanes are thrust against the stator to form an effective seal. Side plates are used to keep the oil confined to an area the width of the rotor and vanes. The rotor centre is eccentric to the stator centre so that the chambers between adjacent vanes expand when connected to the suction port and compress when connected to the discharge port. The pump in Figure 2.2 is equipped with a double stroke which increases the displacement and helps balance the rotor. The connection to the suction and discharge ports are via kidney-shaped connections in the side plates.

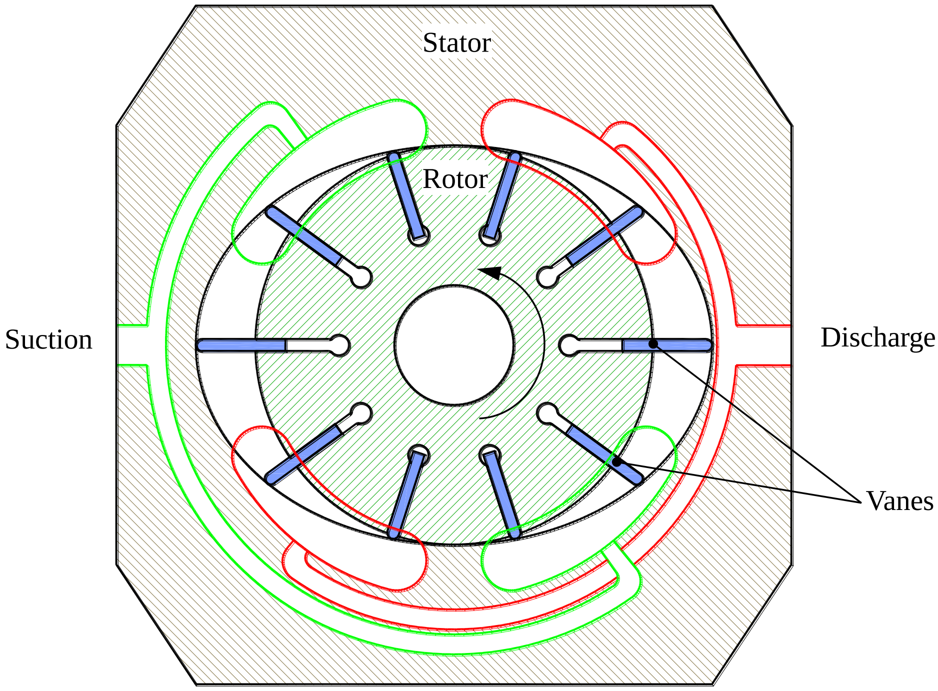


Figure 2.2 A double stroke balanced vane pump

Vane pumps can discharge a variable volume by changing the degree of eccentricity between the rotor and stator, see Figure 2.3. In that case, a ring with adjustable eccentricity relative to the rotor is added to the pump. If the rotor is dead centre with the ring there is no pumping action. At maximum eccentricity, the greatest volume of fluid will be pumped. The degree of eccentricity is adjusted by the use of suitable controls outside the casing.
The slip in a vane pump is in general less than that in a conventional gear pump and a higher volumetric and mechanical efficiency is maintained because the vane wear is compensated for by the outward movement of the vanes.

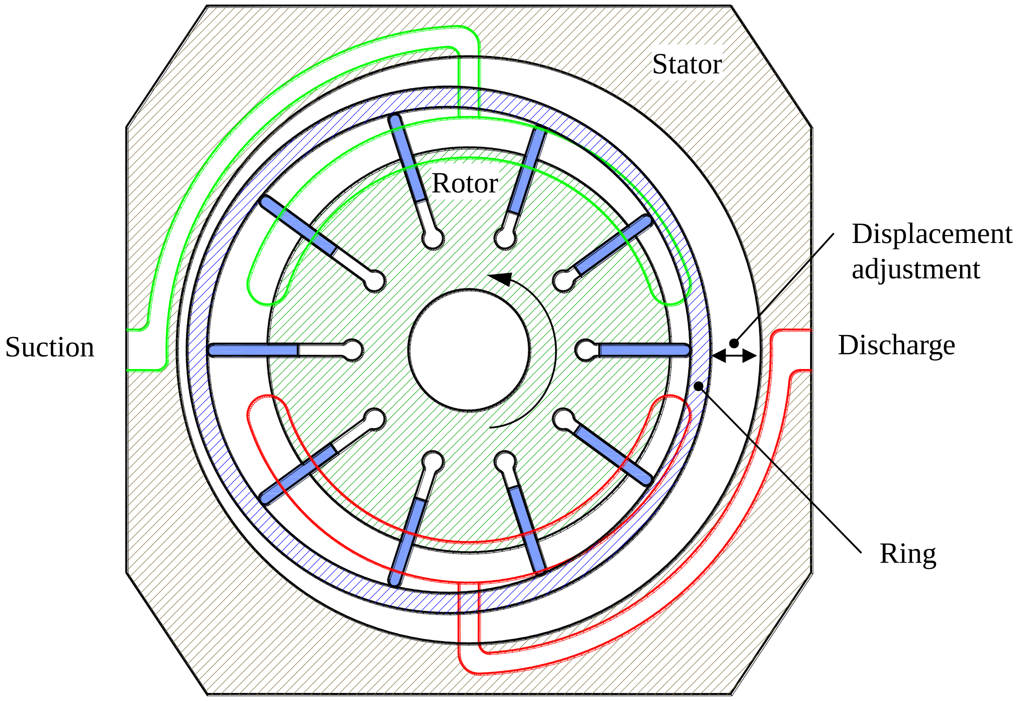


Figure 2.3 A variable displacement vane pump

Piston-type pump
In the positive displacement piston pump the rotor is a cylinder block and the moving members are pistons sliding in the cylinder bores. There are no seals on the pistons which are close-fitting in the bores. The pumping action is obtained by reciprocating the pistons relative to their bores and feeding the fluid to and from the cylinders by inlet and discharge valves.
Piston-type pumps are commonly used for applications that require high pressures and accurate control of the discharge volume. There are many different designs, but generally all designs are based on the radial piston-type or the axial piston-type.
Axial piston pumps present the widest variety in design. The axial piston pump shown in Figure 2.4, contains a cylinder block shaped rotor with pistons that are equally spaced around the cylinder block axis. The cylinder bores are parallel to the axis. The pistons reciprocate parallel to the rotor centre line. The drive shaft rotates the rotor that slides against the stationary end plate. The stationary end plate mates with the surface of the cylinder barrel to prevent leakage of fluid. The pistons are connected to the drive shaft via ball and socket joints. As the cylinder barrel block rotates in contact with the stationary valve plate, the cylinders are alternately ported to the suction (expanding chamber) and discharge (compressing chamber) connection via the kidney shaped

connections so that expanding chambers are conn. The bent axis design shown in Figure 2.4 has variable displacement because the end plate can be rotateed in the stator. When the rotor center line is aligned with the drive shaft center line there is no pumping action.

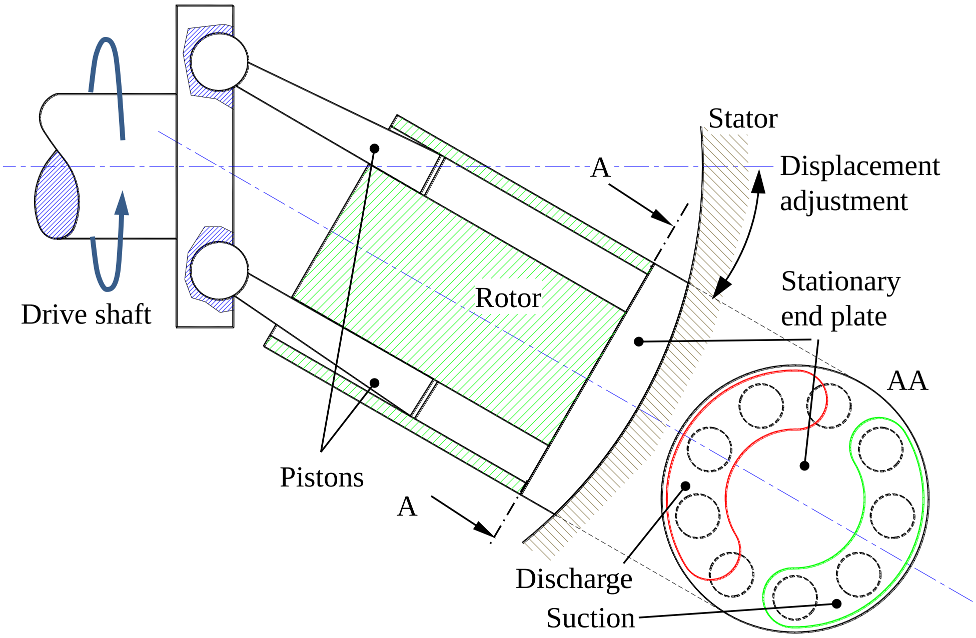


Figure 2.4 Bent axis axial piston pump with variable displacement.

As an alternative to the bent axis design we have the swash plate design, shown in Figure 2.5. Here, the rotor and drive shaft are always aligned whereas the end plate = swash plate is rotated relative to theese. The angle between the cylinder block and the swash plate causes the pistons to reciprocate generating a pumping action similar to the one described for the bent axis design.

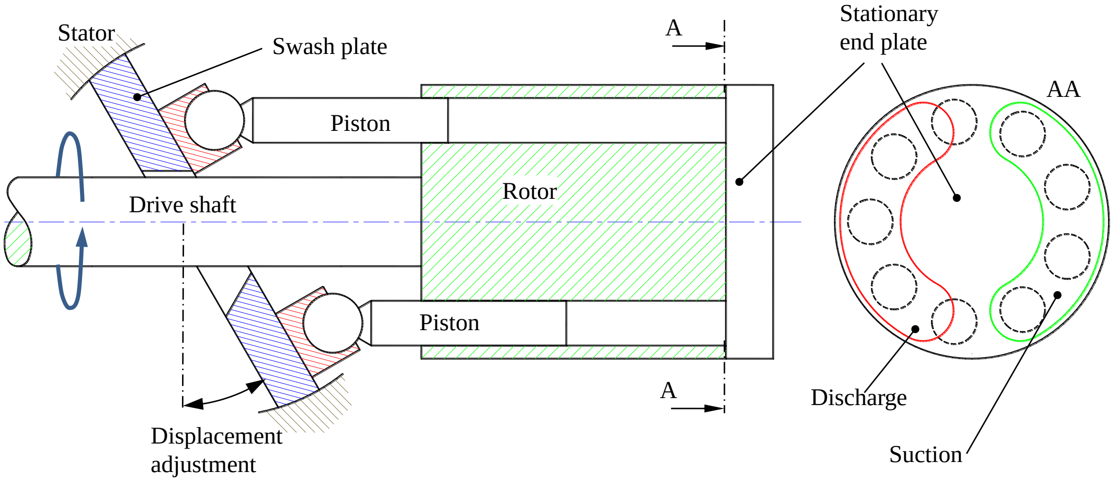


Figure 2.5 Swash plate axial piston pump with variable displacement.

The swash plate angle can relatively easily be adjusted during operation facilitating a wide range of pump control options that will be described in more details in Chapter 4. Adjusting the angle increases or decreases the piston stroke to increase or decrease the volume output. An important feature is the ability to reverse the direction of the fluid. This can be done by tilting the swash plate in the opposite direction.

2.1.2 Motor classifications

In hydraulic power transmissions technology as a whole the term “hydraulic motor” means a device which accept fluid from a source such as a pump and produces linear and rotary motion. The former is obtained from a cylinder and ram assembly and the latter from a rotary hydraulic motor. Although many pumping and control units are common to systems employing both types of motor the term generally implies the use of a rotary motor. Therefore in the following text the term “hydraulic motor” means a fluid-operated unit capable of continuous rotation.

As with positive displacement pumps, the motors follow the three basic types of design – gear, vane and piston. Apart from minor differences the motors are the same as their equivalent design of pumps. Indeed some manufacturers provide the same unit which can be used either as a pump or as a motor.

The principle of operation is simply that whereas a pump is driven by a prime mover to draw fluid in at its inlet port and pump it out from its outlet port, the hydraulic motor is feed from the inlet to the outlet port causing its shaft to rotate.

Thus, in a gear motor, hydraulic fluid enters the inlet and is trapped between the casing and the gear teeth, which causes the gear to rotate and thereby the output shaft connected to the gear. As the teeth mesh at the outlet, the fluid is discharged.

In a vane motor the fluid impinges on the vanes to turn the rotor.

In a piston motor the fluid acts on the pistons and reciprocates them to rotate the output shaft.

2.2 Hydrostatic Pumps

The purpose of this chapter is to give an overview of the governing equations for hydraulic pumps and actuators as components in a hydraulic system. Hence, the chapter does not go into much detail about the different types of pumps and actuators and when to choose one or the other.

2.2.1 Basic equations

The main parameter for a hydraulic pump is the stroke displacement, D, defined as the displacement pr. revolution of the pump. It relates to the pump flow as follows:

$$Q_{tP} = D \cdot n \tag{2.1}$$

where

Q_{tP}	theoretical pump flow, [volume/time]
D	stroke displacement, [volume/revolution]
n	rotational speed of the pump, [revolutions/time]

The rotational speed relates to angular velocity as follows:

$$n = \frac{\omega}{2 \cdot \pi} \tag{2.2}$$

where

n	rotational speed, [revolutions/time]
ω	angular velocity, [radians/time]

Defining the unit displacement, D_ω, as the displacement pr. radian leads to:

$$D = 2 \cdot \pi \cdot D_\omega \tag{2.3}$$

$$Q_{tP} = D_\omega \cdot \omega \tag{2.4}$$

where

Q_{tP}	theoretical pump flow, [volume/time]
D	stroke displacement, [volume/revolution]
D_ω	unit displacement, [volume/radian]
ω	angular velocity, [radians/time]

Some pumps have adjustable displacements. In that case the displacements may be determined from:

$$D = \alpha \cdot D_{\max} \qquad D_\omega = \alpha \cdot D_{\omega,\max} \tag{2.5}$$

where

D	stroke displacement, [volume/revolution]
α	displacement control parameter, $-1.0 \le \alpha \le 1.0$
$D_{\max}$	maximum stroke displacement, [volume/revolution]

The power supplied to the fluid by the pump may be determined based on the general expression for hydrokinetic power:

$$P = Q \cdot p \tag{2.6}$$

where

P	power, [power]
Q	flow, [volume/time]
p	pressure, [pressure]

Accordingly, the theoretical power put into the hydraulic system, e.g. the fluid, by the pump may be determined as:

$$P_{t,P\to F} = Q_{tP} \cdot \left(p_P - p_S \right) = Q_{tP} \cdot \Delta p_P \tag{2.7}$$

where

$P_{t,P\to F}$	theoretical power supplied by the pump to the fluid, [power]
Q_{tP}	theoretical pump flow, [volume/time]
p_P	pressure at the pressure side of the pump, [pressure]
p_S	pressure at the suction side of the pump, [pressure]
Δp_P	pressure rise across the pump, [pressure]

The pressure and flow variables of Equation (2.7) are shown in Figure 2.6.

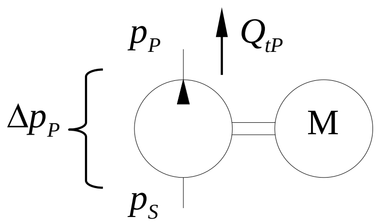


Figure 2.6 Diagram illustration of motor driven pump. The pressure and flow variables associated with the pump are shown.

The pressure rise across the pump is defined as:

$$\Delta p_P = p_P - p_S \tag{2.8}$$

The theoretical mechanical power delivered to the pump from some power source is given as:

$$P_{t,PS\rightarrow P} = M_{tP} \cdot \omega \tag{2.9}$$

where

$P_{t,PS\rightarrow P}$ theoretical power supplied to the pump by the power supply, [power]
M_{tP} theoretical input torque to the pump from the power supply, [torque]
ω angular velocity, [radians/time]

Without any losses, the power delivered to the pump by the power supply equals the power supplied by the pump to the fluid, i.e., combining Equation (2.7) and (2.9) gives:

$$Q_{tP} \cdot \Delta p_P = M_{tP} \cdot \omega \tag{2.10}$$

Inserting Equation (2.4) and (2.3) into Equation (2.10) leads to:

$$M_{tP} = \frac{Q_{tP} \cdot \Delta p_P}{\omega} = D_\omega \cdot \Delta p_P = \frac{D \cdot \Delta p_P}{2 \cdot \pi} \tag{2.11}$$

2.2.2 Efficiencies

The expressions developed for the pump flow and pump torque in the previous section correspond to an ideal pump without power losses of any kind. In reality the pump will **produce less flow** than the theoretical value and the pump will **require more torque** than the theoretical value. Hence, only a part of the power supplied to the pump will end up as hydrokinetic power, whereas the power loss will heat up the pump and its surroundings including the hydraulic fluid.
The fact that the pump delivers less flow than theoretically expected is expressed by means of a **volumetric efficiency**:

$$\eta_{vP} = \frac{Q_P}{Q_{tP}} \tag{2.12}$$

where

η_{vP} volumetric efficiency of the pump

Q_P	(actual) flow of the pump, [volume/time]
Q_{tP}	theoretical pump flow, [volume/time]

The volumetric loss is mainly due to leakage in the form of laminar clearance flow from the high pressure chambers to the low pressure chambers within the pump. This leakage flow is mainly laminar (although some models also include turbulent leakage) and therefore proportional to the pressure rise and inverse proportional to the viscosity:

$$Q_{lP} = K_{lP} \frac{\Delta p_P}{\mu} \tag{2.13}$$

where

Q_{lP}	leakage flow within the pump, [volume/time]
K_{lP}	leakage constant for the pump, [volume]
Δp_P	pressure rise across the pump, [pressure]
μ	dynamic viscosity, [pressuretime]

The volumetric constant tends to vary with the displacement for the same type of pump (larger pumps will have larger dimensions including the leakage clearances). Combining Equations (2.1), (2.12) and (2.13) gives:

$$\eta_{vP} = \frac{Q_{tP} - Q_{lP}}{Q_{tP}} = 1.0 - \frac{Q_{lP}}{Q_{tP}} = 1.0 - \frac{K_{lP} \cdot \Delta p_P}{\mu \cdot D \cdot n} \tag{2.14}$$

It is seen that the volumetric efficiency will depend on pressure rise, rotational speed and the viscosity (mainly temperature), i.e., change with the working conditions. The variation of the volumetric efficiency may be viewed graphically for 2 different situations, see Figure 2.7. The curves are based on Equation (2.14).

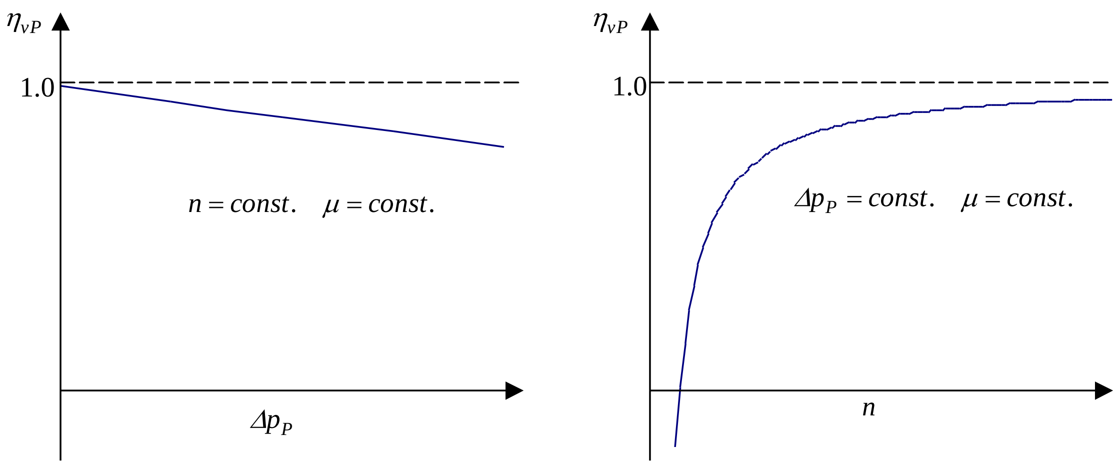


Figure 2.7 The volumetric efficiency for a hydraulic pump as function of the pressure rise across the pump and the rotational speed of the pump, respectively.

The volumetric efficiency decreases linearly with increasing pressure rise when rotational speed and viscosity is held constant. When varying the rotational speed for

fixed viscosity and pressure rise it is seen that the volumetric efficiency goes to zero and below. At zero volumetric efficiency the pump is just capable of producing enough theoretical flow to make up for its own internal leakage.
It should be noted that in case of insufficient suction pressure, the volumetric efficiency will decrease with increasing pump speed because there is not enough time for the suction chambers to be properly filled with fluid.
The fact that the pump requires more input torque than theoretically expected is expressed by means of a hydro-mechanical efficiency:

$$\eta_{hmP} = \frac{M_{tP}}{M_P} \tag{2.15}$$

where

η_{hmP}	hydro-mechanical efficiency
M_{tP}	theoretical input torque to the pump, [torque]
M_P	(actual) input torque to the pump, [torque]

In general, 4 different types of hydro-mechanical losses may be experienced:

1. Mechanical friction due to mechanical contact between parts of the pump moving relative to each other. This loss is proportional to the pressure rise.
2. Viscous (laminar) friction due to shearing of fluid films between parts of the pump moving relative to each other. This loss is proportional to the speed of the moving parts and the viscosity.
3. Hydrokinetic (turbulent) friction due to turbulent pump flow around restrictions, bends, etc. within the pump. This loss is proportional to the square of the flow.
4. Static friction due mainly to sealings. This loss is constant.

The different hydro-mechanical losses may be expressed as either pressure drops, or more conveniently, as extra input torques to the pump:

$$M_{mP} = K_{mP} \cdot \Delta p_P \tag{2.16}$$

$$M_{vP} = K_{vP} \cdot \mu \cdot n \tag{2.17}$$

$$M_{hP} = K_{hP} \cdot n^2 \tag{2.18}$$

$$M_{sP} = cst. \tag{2.19}$$

where

M_{mP}	input torque required to overcome mechanical friction, [torque]
M_{vP}	input torque required to overcome viscous friction, [torque]
M_{hP}	input torque required to overcome turbulent friction, [torque]
M_{sP}	input torque required to overcome static friction, [torque]
K_{mP}, $K_{\upsilon P}$, K_{hP}	pump dependant constants
Δp_P	pressure rise across the pump, [pressure]
n	rotational speed, [revolutions/time]
μ	dynamic viscosity, [pressuretime]

Inserting Equations (2.16)…(2.19) into Equation (2.15) gives:

$$\eta_{hmP} = \frac{M_{tP}}{M_{tP} + M_{mP} + M_{\nu P} + M_{hP} + M_{sP}} \tag{2.20}$$

Introducing Equation (2.11) in the above and rearranging leads to an expression that shows the dependency of the hydro-mechanical efficiency on pressure, speed and viscosity.

$$\eta_{hmP} = \frac{1.0}{C_0 + \frac{1}{\Delta p_P}\left(C_1 + C_2 \cdot \mu \cdot n + C_3 \cdot n^2\right)} \tag{2.21}$$

where

$C_{0..3}$ pump dependent constants

In Figure 2.8 the variation of the hydro-mechanical efficiency for constant pump speed and constant pressure rise, respectively, may be viewed graphically. The curves are based on Equation (2.21).

As seen in Figure 2.8 the hydro-mechanical efficiency goes to zero as the pressure rise goes to zero for fixed pump speed. This corresponds to the pump meeting no resistance (no demand to pressurize the fluid), but still demanding some input torque from the power supply in order to overcome the viscous, turbulent and constant friction losses.

The (total) efficiency of a pump is defined as follows:

$$\eta_P = \frac{P_{P\to F}}{P_{PS\to P}} \tag{2.22}$$

where

η_P efficiency of the pump.

$P_{P\to F}$ (actual) power delivered by the pump to the fluid, [power]

$P_{PS\to P}$ (actual) power delivered by the power supply to the pump, [power]

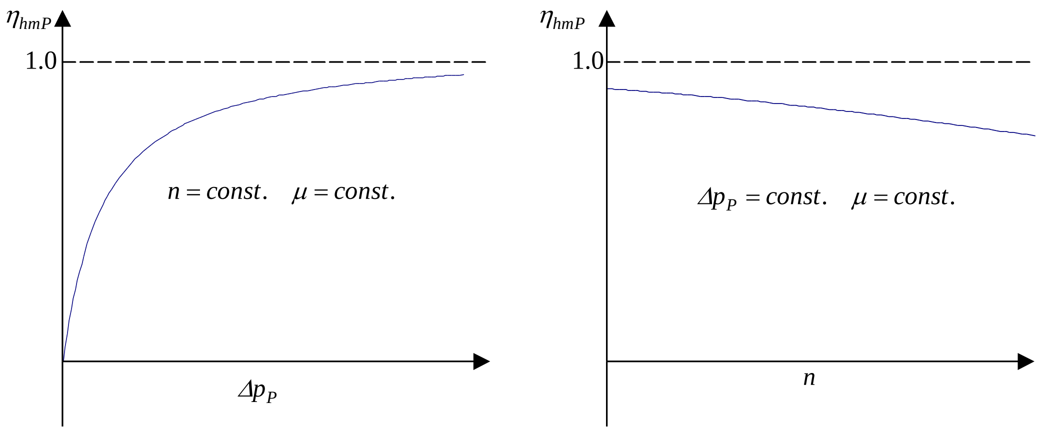


Figure 2.8 The hydro-mechanical efficiency for a hydraulic pump as function of the pressure rise across the pump and the rotational speed of the pump, respectively

The power delivered by the pump to the fluid is found by combining Equation (2.7) and (2.12)

$$P_{P \to F} = Q_P \cdot \Delta p_P = \eta_{vP} \cdot Q_{tP} \cdot \Delta p_P = \eta_{vP} \cdot \omega \cdot D_\omega \cdot \Delta p_P \tag{2.23}$$

Similarly, the power delivered by the power supply to the pump is found by combining Equation (2.9) and (2.15)

$$P_{PS \to P} = M_P \cdot \omega = \frac{M_{tP} \cdot \omega}{\eta_{hmP}} = \frac{D_\omega \cdot \Delta p_P \cdot \omega}{\eta_{hmP}} \tag{2.24}$$

Inserting Equation (2.23) and (2.24) into Equation (2.22) yields the following expression for the pump efficiency

$$\eta_P = \eta_{vP} \cdot \eta_{hmP} \tag{2.25}$$

where

η_P	efficiency of the pump.
η_{vP}	volumetric efficiency of the pump
η_{hmP}	hydro-mechanical efficiency

The pump flow and required drive torque become:

$$Q_P = \eta_{vP} \cdot n \cdot D = \eta_{vP} \cdot \omega \cdot D_\omega \tag{2.26}$$

$$M_P = \frac{1}{\eta_{hmP}} \cdot \frac{D \cdot \Delta p_P}{2 \cdot \pi} = \frac{1}{\eta_{hmP}} \cdot D_\omega \cdot \Delta p_P \tag{2.27}$$

The dependency of the total efficiency on the pressure rise across the pump, the speed of revolution and the viscosity is complex, especially as **the volumetric efficiency is best at low pressure and high speed**, whereas the **hydro-mechanical efficiency is best at high pressure and low speed**.

2.3 Hydrostatic Motors

The main purpose of the hydraulic motor is to transform hydraulic power into mechanical power by means of a rotating output shaft. Hydraulic motors are exclusively made as positive displacement motors, where the driven volumes are separated in pressurized and relieved (depressurised) chambers. By means of a flow to the motor, typically provided by some hydraulic pump, these chambers will vary in such a way that the output shaft of the motor is rotated. Depending on the load on the shaft from the driven mechanical system the hydraulic motor will demand a certain pressure drop, i.e., a certain pressure level at its intake, in order to move. The hydraulic motor may be thought of as an inverted hydraulic pump.

2.3.1 Basic equations

As for the hydraulic pump, the main parameter for a hydraulic motor is the stroke displacement, D, defined as the displacement pr. revolution of the motor shaft. It relates to the motor flow demand as follows:

$$Q_{tM} = D \cdot n = D_{\omega} \cdot \omega \tag{2.28}$$

where

Q_{tM}	theoretical motor flow demand, [volume/time]
D	stroke displacement, [volume/revolution]
n	rotational speed of the motor, [revolutions/time]
D_{ω}	unit displacement, [volume/radian]
ω	angular velocity of the motor, [radians/time]

Notice that the flow through the motor is referred to as a **flow demand**, i.e., the flow required in order for the motor to meet a certain shaft speed.
Some motors have adjustable displacements. In that case the displacements may be determined from:

$$D = \alpha \cdot D_{\max} \qquad D_{\omega} = \alpha \cdot D_{\omega,\max} \tag{2.29}$$

where

D	stroke displacement, [volume/revolution]
α	displacement control parameter, $\alpha_{\min} < \alpha \le 1.0$
$D_{\max}$	maximum stroke displacement, [volume/revolution]

The displacement control parameter must be kept above some minimum vale, $\alpha_{\min}$, as the motor shaft otherwise will rotate with infinite speed for any flow.
The theoretical power put into the motor from the hydraulic system, e.g. the fluid, may be determined as:

$$P_{t,F\to M} = Q_{tM} \cdot \left(p_i - p_o\right) = Q_{tM} \cdot \Delta p_M \tag{2.30}$$

where

$P_{t,F\to M}$	theoretical power supplied to the motor by the fluid, [power]
Q_{tM}	theoretical motor flow demand, [volume/time]
p_i	pressure at the inlet side of the motor, [pressure]
p_o	pressure at the outlet side of the motor, [pressure]
Δp_M	pressure drop across the motor, [pressure]

The pressure and flow variables of Equation (2.30) are shown in Figure 2.9.

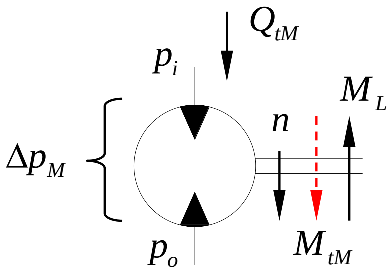


Figure 2.9 Diagram illustration of a hydraulic motor driving a load. The pressure and flow variables as well as the torques associated with the motor are shown. The motor is subjected to a positive load.

The theoretical mechanical power supplied by the motor to the load is given as

$$P_{t,M\rightarrow L} = M_{tM} \cdot \omega \tag{2.31}$$

where

$P_{t,M\rightarrow L}$	theoretical power supplied by the motor to the load, [power]
M_{tM}	theoretical output torque from the motor, [torque]
ω	angular velocity, [radians/time]

Without any losses, the power delivered to the motor by the fluid equals the power supplied by the motor to the load, i.e., combining Equation (2.30) and (2.31) gives

$$Q_{tM} \cdot \Delta p_M = M_{tM} \cdot \omega \tag{2.32}$$

Inserting Equation (2.28) into (2.32) and rearranging leads to

$$M_{tM} = \frac{Q_{tM} \cdot \Delta p_M}{\omega} = D_{\omega} \cdot \Delta p_M = \frac{D \cdot \Delta p_M}{2 \cdot \pi} \tag{2.33}$$

In steady state conditions (constant shaft speed), the torque delivered to the output shaft is opposite in direction but in size to the torque load on the shaft.

$$M_{tM} = M_L \tag{2.34}$$

In Figure 2.9 the load torque, M_L, is opposite to the direction of the speed, n. Whenever this is the case we have a **positive load**, and the motor is said to be motoring. As an example, this corresponds to the motor driving a wire drum that is hoisting a load. For a positive load the pressure drop is, by definition:

$$\Delta p_M = p_i - p_o \tag{2.35}$$

Alternatively, we have a **negative load** whenever the load torque, M_L, is in the same direction as the speed, n, see Figure 2.10.

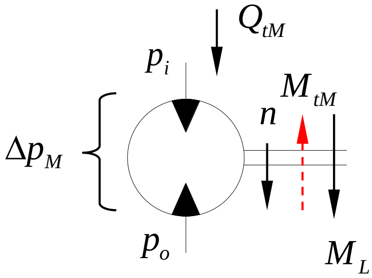


Figure 2.10 Diagram illustration of a hydraulic motor driving a load. The pressure and flow variables as well as the torques associated with the motor are shown. The motor is subjected to a negative load.

As an example, this corresponds to the motor driving a wire drum that is lowering a load. In this case the return pressure must be the highest, and the motor is being driven as a pump by the external load. The governing equations become:

$$P_{t,F\to M} = Q_{tM}\cdot\left(p_i - p_o\right) = -Q_{tM}\cdot\left(p_o - p_i\right) = -Q_{tM}\cdot\Delta p_M \tag{2.36}$$

Where

$P_{t,F\to M}$	theoretical power supplied to the motor by the fluid, [power]
Q_{tM}	theoretical motor flow demand, [volume/time]
p_i	pressure at the inlet side of the motor, [pressure]
p_o	pressure at the outlet side of the motor, [pressure]
Δp_M	pressure drop across the motor, [pressure]

The pressure drop for a negative load is, by definition, given as:

$$\Delta p_M = p_o - p_i \tag{2.37}$$

From (2.36) it is seen that the power supplied to the motor by the fluid will be a negative number which underlines the fact that the motor is driven as a pump and therefore supplies power to the fluid. Similarly, we get a negative theoretical mechanical power supplied by the motor to the load

$$P_{t,M\to L} = -M_{tM}\cdot\omega \tag{2.38}$$

where

$P_{t,M\to L}$	theoretical power supplied by the motor to the load, [power]
M_{tM}	theoretical output torque from the motor, [torque]
ω	angular velocity, [radians/time]

Eqs. (2.32) and (2.33) are the same for both positive and negative loads although it should be remembered that the sign of Δp_M in (2.33) changes with the type of loading according to (2.35) and (2.37).

2.3.2 Efficiencies

The expressions developed for the motor flow demand and motor torque in the previous section correspond to an ideal motor without power losses of any kind. In reality the motor **will require more flow** than the theoretical value and the motor will **deliver less torque** than the theoretical value. Hence, only a part of the hydrokinetic power supplied to the pump will end up as mechanical power, whereas the power loss will heat up the motor and its surroundings including the hydraulic fluid.
The influence of the efficiencies depend on whether the motor is subjected to a positive load or a negative load. First, we will investigate the influence of the efficiencies on a motor subjected to **positive load** and later this will be supplemented with the similar equations relating th **negative load.**
The fact that a motor subjected to a positive load requires more flow in order to reach a certain shaft speed, n, than theoretically expected is expressed by means of a **volumetric efficiency**:

$$\eta_{vM} = \frac{Q_{tM}}{Q_M} \tag{2.39}$$

where

η_{vM}	volumetric efficiency of the motor
Q_{tM}	theoretical flow demand of the motor, [volume/time]
Q_M	(actual) flow demand of the motor, [volume/time]

As for pumps the volumetric loss is mainly due to leakage in the form of laminar clearance flow from the high pressure chambers to the low pressure chambers within the motor. This way some of the flow will pass through the motor without helping to rotate the motor shaft. The clearance flow is proportional to the pressure drop and inverse proportional to the viscosity

$$Q_{lM} = K_{lM} \frac{\Delta p_M}{\mu} \tag{2.40}$$

where

Q_{lM}	leakage flow across the motor, [volume/time]
K_{lM}	leakage constant for the motor, [volume]
Δp_M	pressure drop across the motor, [pressure]
μ	dynamic viscosity, [pressuretime]

The volumetric constant tends to vary with the displacement for the same type of motor (larger motors will have larger dimensions including the leakage clearances). Combining Equation (2.28), (2.39), and (2.40) gives

$$\eta_{vM} = \frac{Q_{tM}}{Q_{tM} + Q_{lM}} = \frac{1.0}{1.0 + \frac{Q_{lM}}{Q_{tM}}} = \frac{1.0}{1.0 + \frac{K_{lM} \cdot \Delta p_M}{D \cdot n \cdot \mu}} \tag{2.41}$$

It is seen that the volumetric efficiency will depend on pressure drop, motor speed and the viscosity (mainly temperature) under working conditions. The variation of the volumetric efficiency may be viewed graphically for 2 different situations, see Figure 2.11. The curves are based on Equation (2.41).

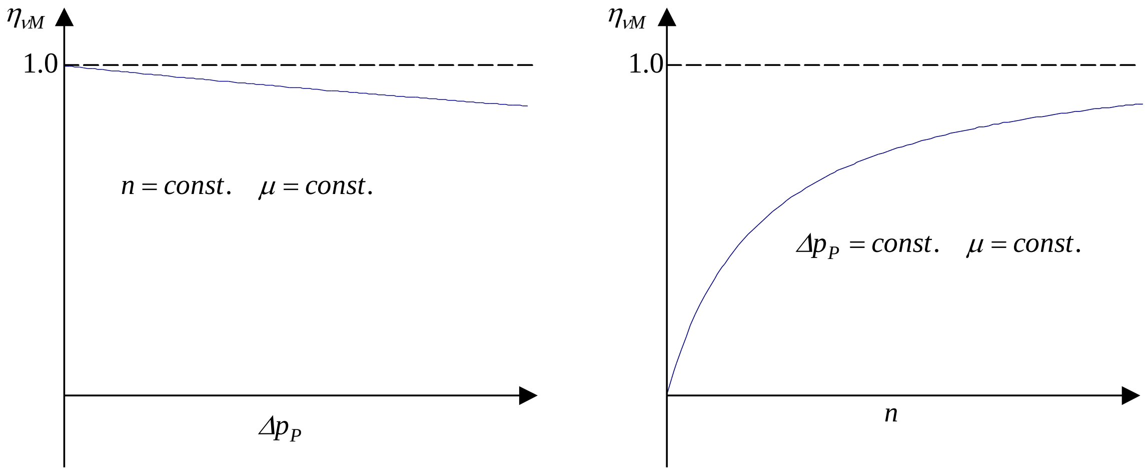


Figure 2.11 The volumetric efficiency for a hydraulic motor as function of the pressure drop across the motor and the rotational speed of the motor shaft, respectively.

When the motor is stalled (n = 0) at a certain pressure and, subsequently, no power is supplied to the load, the motor still receives power via the leakage flow, i.e., the efficiency is effectively zero, as can be seen in Figure 2.11 to the right.
The fact that a motor subjected to a positive load delivers less output torque than theoretically expected is expressed by means of a hydro-mechanical efficiency:

$$\eta_{hmM} = \frac{M_M}{M_{tM}} \tag{2.42}$$

where

η_{hmM}	hydro-mechanical efficiency of the motor
M_{tM}	theoretical output torque from the motor, [torque]
M_M	(actual) output torque from the motor, [torque]

In general, the 4 different types of hydro-mechanical losses explained in the previous section on hydraulic pumps also appear in hydraulic motors. When inserted in Equation (2.42) the following expression is obtained:

$$\eta_{hmM} = \frac{M_{tM} - M_{mM} - M_{vM} - M_{hM} - M_{sM}}{M_{tM}} \tag{2.43}$$

where

η_{hmM} hydro-mechanical efficiency of the motor

M_{tM} theoretical output torque from the motor, [torque]
M_{mM} loss in output torque due to mechanical friction, [torque]
$M_{\upsilon M}$ loss in output torque due to viscous friction, [torque]
M_{hM} loss in output torque due to turbulent friction, [torque]
M_{sM} loss in output torque due to static friction, [torque]

Employing the dependencies introduced in Equation (2.16)...(2.19) in the above and rearranging leads to an expression that shows the dependency of the hydro-mechanical efficiency on pressure, speed and viscosity.

$$\eta_{hmM} = D_0 - \frac{1}{\Delta p_M}\left(D_1 + D_2 \cdot \mu \cdot n + D_3 \cdot n^2\right) \tag{2.44}$$

where

$D_{0..3}$ motor dependant constants
Δp_M pressure drop across the motor, [pressure]
n motor speed of revolution, [revolutions/time]
μ dynamic viscosity, [pressuretime]

In Figure 2.12 the variation of the hydro-mechanical efficiency for constant motor speed and constant pressure drop, respectively, may be viewed graphically. The curves are based on Equation (2.44).
When the motor runs with a constant speed, the hydro-mechanical efficiency will be zero, when the pressure drop has a value where it is just capable of maintaining equilibrium with the internal torque losses. This leaves nothing to rotate any load and the motor is on the border line of acting like a hydraulic pump driven by the load.

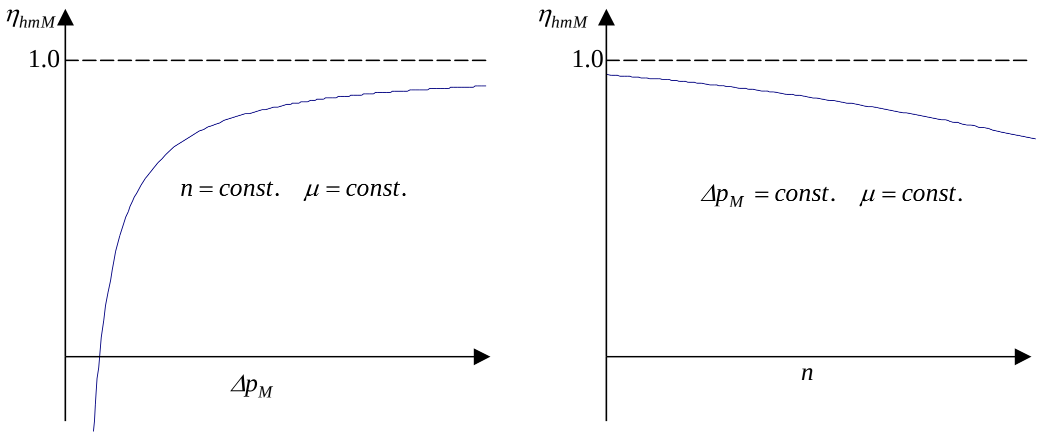


Figure 2.12 The hydro-mechanical efficiency for a hydraulic motor as function of the pressure drop across the motor and the rotational speed of the motor, respectively.

In the following equations we look at the influence of the efficiencies on the motor equations. The total efficicency for a motor subjected to a positive load is:

$$\eta_M = \frac{P_{M \to L}}{P_{F \to M}} \tag{2.45}$$

where

η_M total efficiency of the motor.

$P_{M \to L}$ (actual) power delivered by the motor to the load, [power]

$P_{F \to M}$ (actual) power delivered by the fluid to the motor, [power]

The power delivered by the fluid to the motor is found by combining Equation (2.39) and (2.28)

$$P_{F \to M} = Q_M \cdot \Delta p_M = \frac{Q_{tM} \cdot \Delta p_M}{\eta_{vM}} = \frac{D_\omega \cdot \omega \cdot \Delta p_M}{\eta_{vM}} \qquad (2.46)$$

Similarly, the power delivered by the motor to the load is found by combining Equation (2.42) and (2.33)

$$P_{M \to L} = M_M \cdot \omega = \eta_{hmM} \cdot M_{tM} \cdot \omega = \eta_{hmM} \cdot D_\omega \cdot \Delta p_M \cdot \omega \qquad (2.47)$$

Inserting Equation (2.46) and (2.47) into Equation (2.45) yields the following expression for the motor efficiency:

$$\eta_M = \eta_{vM} \cdot \eta_{hmM} \qquad (2.48)$$

where

η_M total efficiency of the motor.

η_{vM} volumetric efficiency of the motor

η_{hmM} hydro-mechanical efficiency of the motor

Further, expressions for the flow demand and motor torque can be readily computed as:

$$Q_M = \frac{1}{\eta_{vM}} \cdot D \cdot n = \frac{1}{\eta_{vM}} \cdot D_\omega \cdot \omega \qquad (2.49)$$

$$M_M = \eta_{hmM} \cdot \frac{D \cdot (p_i - p_o)}{2 \cdot \pi} = \mu_{hmM} \cdot D_\omega \cdot (p_i - p_o) = M_L \qquad (2.50)$$

Again, steady state conditions are assumed:

$$M_M = M_L \qquad (2.51)$$

For a **negative load** the definition of the efficiencies are reversed:

$$\eta_{vM} = \frac{Q_M}{Q_{tM}} \qquad (2.52)$$

$$\eta_{hmM} = \frac{M_{tM}}{M_M} \qquad (2.53)$$

$$\eta_M = \frac{P_{M\to F}}{P_{L\to M}} = \frac{-P_{F\to M}}{-P_{M\to L}} = \frac{\eta_{vM}\cdot n\cdot D\cdot \Delta p_M}{\frac{1}{\eta_{hmM}}\cdot\frac{D\cdot\Delta p_M}{2\cdot\pi}\cdot\omega} = \eta_{vM}\cdot\eta_{hmM} \tag{2.54}$$

where

η_M	total efficiency of the motor.
$P_{M\to F}$	(actual) power delivered by the motor to the fluid, [power]
$P_{L\to M}$	(actual) power delivered by the load to the motor, [power]

The flow demand and motor torque for the negative load become similar to those for the pump (see Eqs. 2.26 and 2.27)

$$Q_M = \eta_{vM}\cdot D\cdot n = \eta_{vM}\cdot D_\omega\cdot\omega \tag{2.55}$$

$$M_M = \frac{1}{\eta_{hmM}}\cdot\frac{D\cdot(p_o - p_i)}{2\cdot\pi} = \frac{1}{\eta_{hmM}}\cdot D_\omega\cdot(p_o - p_i) \tag{2.56}$$

As in the case with hydraulic pumps the dependency of the total efficiency on the pressure drop across the motor, the motor speed and the viscosity is complex, with **the volumetric efficiency best at low pressure and high speed** and the **hydro-mechanical efficiency best at high pressure and low speed**.

2.4 Hydraulic Cylinders

The main purpose of the hydraulic cylinder is to transform hydraulic power into mechanical power by means of a translating piston rod. In general, a piston attached to the piston rod, uses the cylinder housing to seal of 2 pressure chambers. Each chamber is connected to the remaining hydraulic system. When hydraulic flow is led to either one of the pressure chambers, the piston and piston rod moves in the corresponding direction.

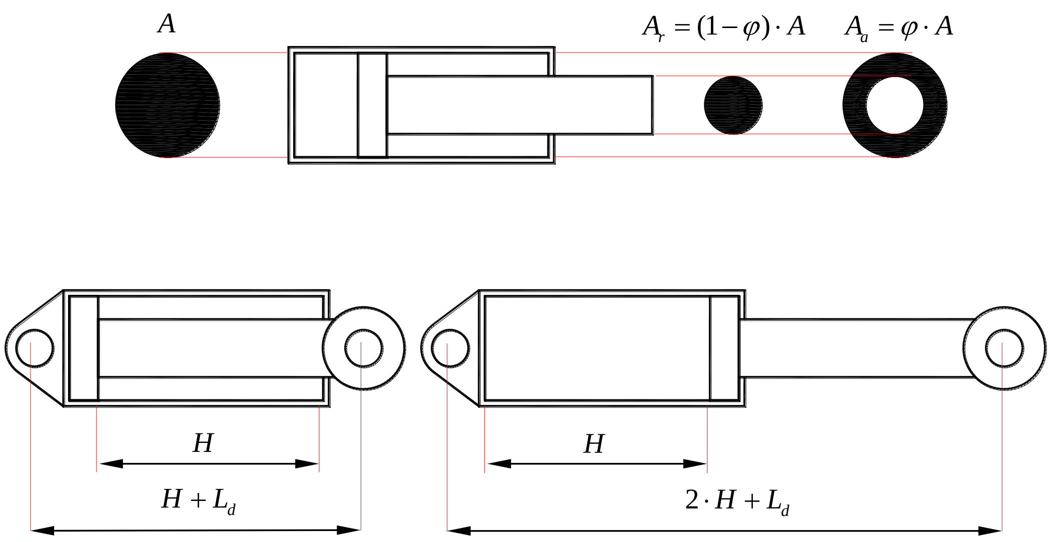


Figure 2.13 Diagram illustration of main cylinder parameters.

Depending on the load on the piston rod from the driven mechanical system the hydraulic cylinder will demand a certain pressure drop across its piston, in order to move.

2.4.1 Basic equations

The main parameters for a hydraulic cylinder are:

1. The stroke, H, [length]
2. The piston area, A, [area]
3. The area ratio, φ
4. Dead length, L_d, [length]

The above parameters are all shown in Figure 2.13. The area ratio is the ratio between the annulus area (piston area - piston rod area) and the piston area.

$$\varphi = \frac{A_a}{A} = \frac{A - A_r}{A} \tag{2.57}$$

Hence, $0 \leq \varphi \leq 1.0$, with the 2 extreme cases shown in Figure 2.14.

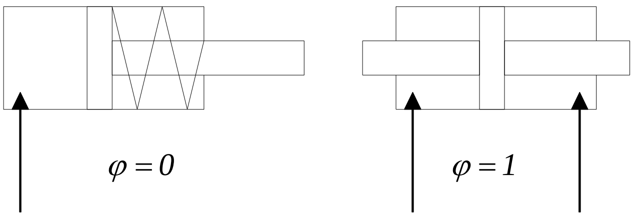


Figure 2.14 Diagram illustration of cylinders corresponding to extreme values of φ

In order to set up equations for the flow demand of the cylinder and the relationship between load, cylinder force and pressures it is necessary to discuss the possible modes of operation of a cylinder. As in the case of the motor, a cylinder may be subjected to both positive and negative loads. Further, since a cylinder, in general, is assymetric it is also necessary to distinguish between the direction of motion that is divided into out-stroke and in-stroke. This gives a total number of four combinations that are shown in Figure 2.15.

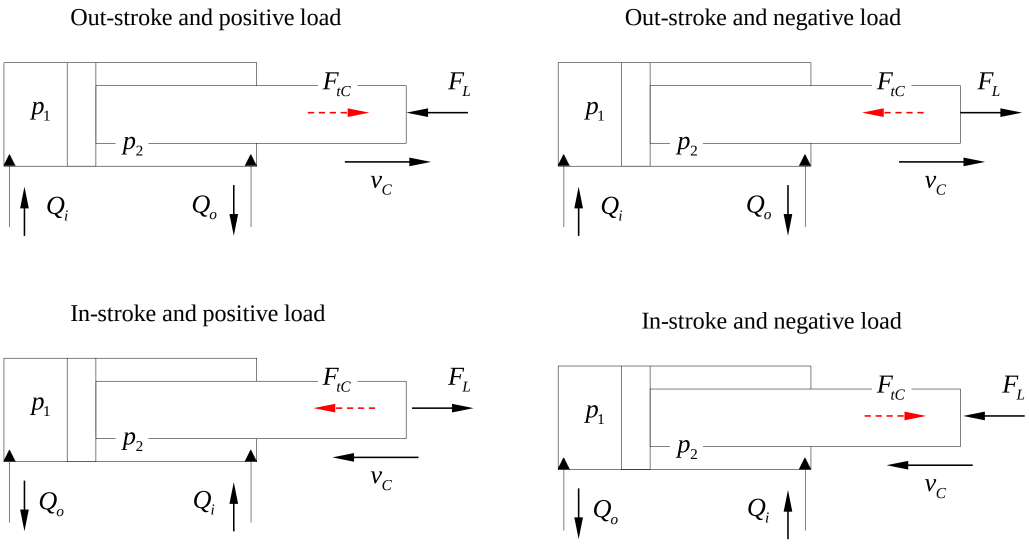


Figure 2.15 Ideal cylinders shown in all four modes of operation.

As always, a positive load is acting against the direction of motion and vice versa. The governing equations for all four modes of operation are as follows:

Out-stroke

$$Q_i = v_C \cdot A \tag{2.58}$$

$$Q_o = \varphi \cdot A \cdot v_C \tag{2.59}$$

$$F_{tC} = p_1 \cdot A - p_2 \cdot \varphi \cdot A = F_L \quad \text{(positive load)} \tag{2.60}$$

$$F_{tC} = p_2 \cdot \varphi \cdot A - p_1 \cdot A = F_L \quad \text{(negative load)} \tag{2.61}$$

In-stroke

$$Q_i = \varphi \cdot v_C \cdot A \tag{2.62}$$

$$Q_o = A \cdot v_C \tag{2.63}$$

$$F_{tC} = p_2 \cdot \varphi \cdot A - p_1 \cdot A = F_L \quad \text{(positive load)} \tag{2.64}$$

$$F_{tC} = p_1 \cdot A - p_2 \cdot \varphi \cdot A = F_L \quad \text{(negative load)} \tag{2.65}$$

In Eqs. (2.60), (2.61), (2.64) and (2.65) definitions of the theoretical cylinder force are introduced that ensure a positive values. Also, steady state conditions are assumed which means:

$$F_{tC} = F_L \tag{2.66}$$

2.4.2 Efficiencies

The equations developed in the previous section are all based on ideal conditions without any power loss in the hydraulic cylinder. The hydraulic cylinder has, just like the hydraulic pump and hydraulic motor, both a volumetric and a hydro-mechanical efficiency.

The volumetric efficiency means that the cylinder will require more flow than theoretically expected, in order to meet a certain velocity. The volumetric loss is caused

by leakage across the sealings between the piston and the cylinder housing and the sealings between the piston rod and the housing. These losses are, however, so small for a typical cylinder that they may be ignored. This means that the volumetric efficiency approximately is 100% (or 1.0).
The hydro-mechanical efficiency, however, cannot be disregarded. It is almost exclusively caused by mechanical friction between the piston seals and the cylinder housing and the piston rod and the cylinder housing seals. There exist several definitions but the most common for a cylinder subjected to a positive load is:

$$\eta_{hmC} = \frac{F_C}{F_{tC}} \tag{2.67}$$

where

η_{hmC}	hydro-mechanical efficiency of the cylinder
F_C	(actual) force applied by the cylinder on the piston, [force]
F_{tC}	theoretical force applied by the cylinder on the piston, [force]

Equation (2.67) can be used to derive a general expression for the friction force

$$\eta_{hmC} = \frac{F_C}{F_{tC}} = \frac{F_{tC} - F_{mC}}{F_{tC}} \Rightarrow F_{mC} = (1 - \eta_{hmC}) \cdot F_{tC} \tag{2.68}$$

where

η_{hmC}	hydro-mechanical efficiency of the cylinder
F_C	(actual) force applied by the cylinder on the piston, [force]
F_{mC}	mechanical friction force, [force]

In these notes Equation (2.,68) is used as the correlation between the theoretical cylinder force (Eqs. (2.60), (2.61), (2.64) and (2.65)) and the friction force.
Disregarding the volumetric loss the total efficiency of the cylinder corresponds to the hydro-mechanical efficiency, i.e.

$$\eta_C = \eta_{hmC} \tag{2.69}$$

where

η_C	total efficiency of the cylinder
η_{hmC}	hydro-mechanical efficiency of the cylinder.

As an approximation the mechanical friction force, F_{mC}, is proportional to the force applied by the cylinder on the load. Inserting this in Equation (2.48) and (2.49) reveals that for a cylinder the variation in efficiencies can typically be disregarded:

$$\eta_C = \eta_{hmC} \approx const. \tag{2.70}$$

The four modes of operation when taking into account the mechanical friction are shown in Figure 2.16. The figure corresponds to Figure 2.15 except that the friction force has been added.

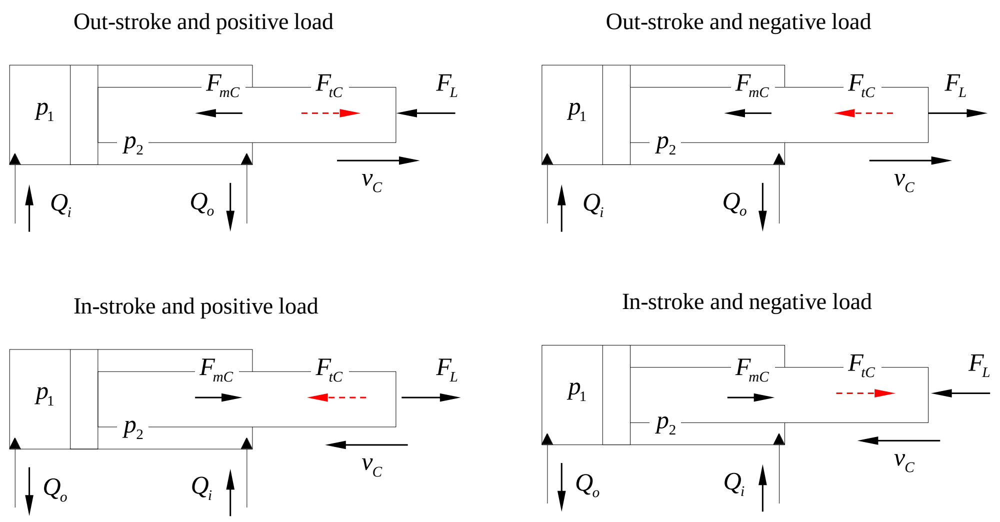


Figure 2.16 Cylinders shown in all four modes of operation including the friction force.

The equations relating flows and velocities are the same, i.e., Eqs. (2.58), (2.59), (2.62) and (2.63) also applies for a non-ideal cylinder. The equations relating the pressures in the cylinder chambers and the external load change:

Out-stroke

$$F_C = F_{tC} - F_{mC} = F_L \Rightarrow F_{tC} = p_1 \cdot A - p_2 \cdot \varphi \cdot A = \frac{F_L}{\eta_{hmC}} \quad \text{(positive load)} \tag{2.71}$$

$$F_C = F_{tC} + F_{mC} = F_L \Rightarrow F_{tC} = p_2 \cdot \varphi \cdot A - p_1 \cdot A = \frac{F_L}{2 - \eta_{hmC}} \quad \text{(negative load)} \tag{2.72}$$

In-stroke

$$F_C = F_{tC} - F_{mC} = F_L \Rightarrow F_{tC} = p_2 \cdot \varphi \cdot A - p_1 \cdot A = \frac{F_L}{\eta_{hmC}} \quad \text{(positive load)} \tag{2.73}$$

$$F_C = F_{tC} + F_{mC} = F_L \Rightarrow F_{tC} = p_1 \cdot A - p_2 \cdot \varphi \cdot A = \frac{F_L}{2 - \eta_{hmC}} \quad \text{(negative load)} \tag{2.74}$$

Steady state conditions are assumed which means:

$$F_C = F_L \tag{2.75}$$

----- oo 0 oo -----

3
Characteristics of Valve Operation

Hydraulic System Design

3.1 Introduction

Valves are used in our hydraulic systems to control the operation of the actuators. Very often, in fact, we find the valves referred to as the “control”, particular where a number of them are built into a single assembly.
The valves assert their authority in the circuit by regulating pressure; by creating special pressure conditions; by deciding how much oil will flow in portions of the circuit; and by telling the oil where to go.
We group hydraulic valves into three general categories: directional controls, pressure controls, and flow controls. Some valves, however, have multiple functions that fall into more than one of these categories.
Valves are rated by their size, pressure capabilities and pressure drop vs. flow. They are usually named for their functions, but may be named according to their design as well.

This chapter gives an overview of the most common valve types and describe their characteristics.

3.2 Flow Force Equation

In general fluid passing through orifices will try to close the orifices. For fixed orifices this is not a problem, but for orifices with variable discharge areas, the two bodies generating the orifice geometry will be pulled together, normally affecting the expected behaviour of the valve or increasing the actuation demand. In Figure 3.1, two typical variable orifices are shown.

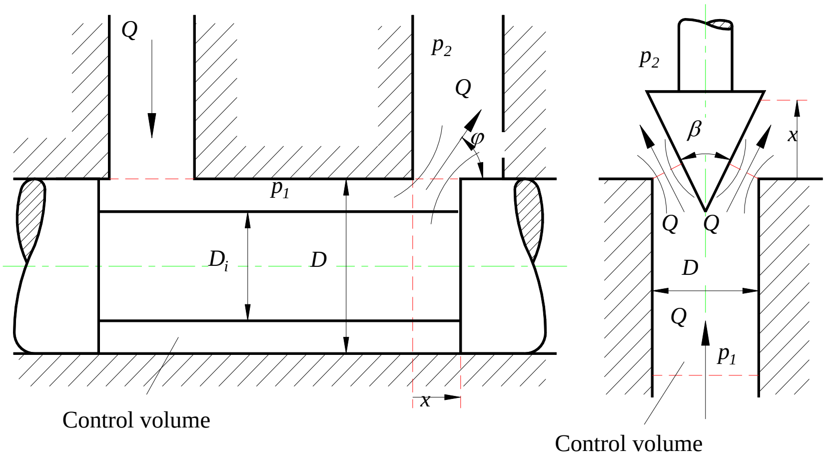


Figure 3.1 Typical variable orifices in hydraulic systems. To the left a spool-drill orifice and to the right a poppet-seat orifice.

To the left, a spool moves within a drilled valve house generating a band-shaped orifice with discharge area: $A = \pi \cdot D \cdot x$. To the right, a poppet is moved relative to a valve house seat, generating a conical band-shaped orifice with discharge area: $A \approx \pi \cdot D \cdot x \cdot \sin(\beta / 2)$.
For both cases the basic momentum conservation is utilized. It states that the sum of external forces on a system corresponds to the time derivative of the linear momentum of the system:

$$\sum F = \frac{d\left(\sum m \cdot v\right)}{dt} \tag{3.1}$$

where

$\sum F$	sum of external forces on a system, [force]
$\sum m \cdot v$	sum of linear momentum of the system, [mass·velocity]

Employing Equation (3.1) on a control volume containing steady state flow it may be reformulated to:

$$\sum F = \sum \dot{m} \cdot v \tag{3.2}$$

Equation (3.2) states that the sum of external forces on a control volume corresponds to the sum of mass flow times velocity **leaving** the control volume.
Defining a control volume, *V*, as a ring with inner diameter D_i and outer diameter *D*, see Figure 3.1, it is seen that flow enters perpendicular and leaves through the band-shaped orifice at some angle, φ, relative to the spool centre axis.
The required resulting force on the ring of fluid is determined from:

$$F_{x,fl} = \dot{m} \cdot v_x = \rho \cdot Q \cdot \frac{Q}{C_C \cdot A} \cdot \cos\varphi \approx \frac{\rho \cdot Q^2 \cdot \cos\varphi}{C_D \cdot A} = \sqrt{2 \cdot \rho} \cdot \cos\varphi \cdot Q \cdot \sqrt{p_1 - p_2} \tag{3.3}$$

where

$F_{x,fl}$	required axial force on the control volume from the spool, [force]
$\dot{m}$	mass flow, [kg/time]
v_x	exit flow velocity in axial direction, [length/time]
ρ	mass density, [mass/volume]
Q	flow, [volume/time]
C_C	contraction coefficient
A	discharge area of the band-shaped orifice, [area]
φ	angle between the spool axis and the orifice flow direction
C_D	discharge coefficient
p_1-p_2	pressure drop across the orifice, [pressure]

From Figure 3.1 it is seen that the control ring volume is acted upon by an axial force to the right. Only the spool can supply this force, hence, the ring acts upon the spool with an equal and opposite force, trying to move the spool to the left, i.e., closing it. The same result would be obtained if the direction of flow was reversed, because the sum of the mass flow times the reversed velocity would now be **entering** the control volume, still requiring a force to the right to be supplied to the control volume. An analytical solution to a problem of similar nature suggests $\varphi = 69°$ but in general the angle may vary from 0 - $90°$, depending on the opening, the radial clearance and any variation in spool and valve house geometry relative to a perfect sharp edge.

Basically, the same expression as Equation (3.3) is obtained when, applying Equation (3.2) to the poppet valve in Figure 3.1 to the right. Defining a rotational control volume with an outer diameter, *D*, and embracing the poppet as shown, the required resulting force on the cylinder of fluid is determined from:

$$F_{x,fl} = \dot{m} \cdot v_x = \rho \cdot Q \cdot \frac{Q}{C_C \cdot A} \cdot \cos\frac{\beta}{2} \approx \frac{\rho \cdot Q^2 \cdot \cos\frac{\beta}{2}}{C_D \cdot A} = \sqrt{2 \cdot \rho} \cdot \cos\frac{\beta}{2} \cdot Q \cdot \sqrt{p_1 - p_2} \qquad (3.4)$$

As in the case with the spool, only the poppet can act upon the control volume in axial direction, hence, the poppet is pulled towards its seat.
For the poppet it seems that the jet angle is more easily obtained as simply half the poppet angle. However, the jet angle may vary substantially, especially at small openings, and specifically, the jet will have a tendency to jump from one configuration to another at a certain combination of flow and opening.

3.3 Directional Control Valves

Directional control valves belong to the group of valves controlling flow direction. Their purpose is to direct pump flow to an actuator as well as allow return flow from the same actuator to the reservoir. They are classified according to the number of service ports and number of possible configurations (positions). Hence, the directional control valves shown in Figure 3.2 are referred to as a 2/2-way, a 3/2-way and a 4/3-way directional valve.

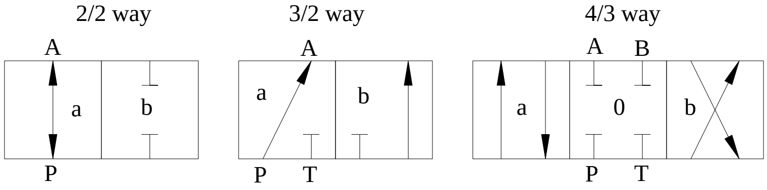


Figure 3.2 Basic diagrams of directional control valves.

By convention, the designation of the service ports are almost always referred to as P (pump connection), T (tank connection), A and B (actuator connections). Similarly, the designation of the position of the valve is referred to as a-b (2 positions) and a-0-b (3 positions). The port connections shown for the 4/3-way valve in Figure 3.1 are typical for the 3 positions a, 0, and b, respectively. There exist, however, several different port connections for both 2-, 3-, and 4-working ports, see Figure 3.3, and they may be combined to yield directional control valves with almost any thinkable functionality.

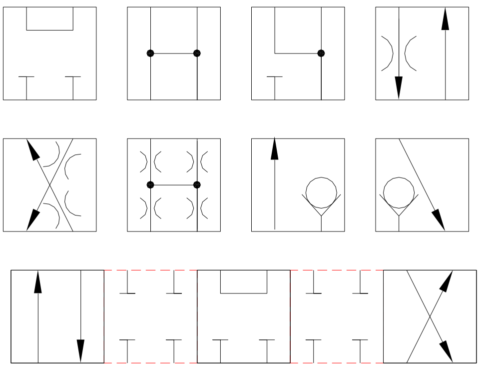


Figure 3.3 A few of the building blocks for directional control valves. Also a 4/3-way valve with specification of intermediate positions is shown.

By distinguishing between the port connections while shifting from one position to another the variety is even further multiplied. An intermediate position is displayed by means of hidden lines, see Figure 3.3.

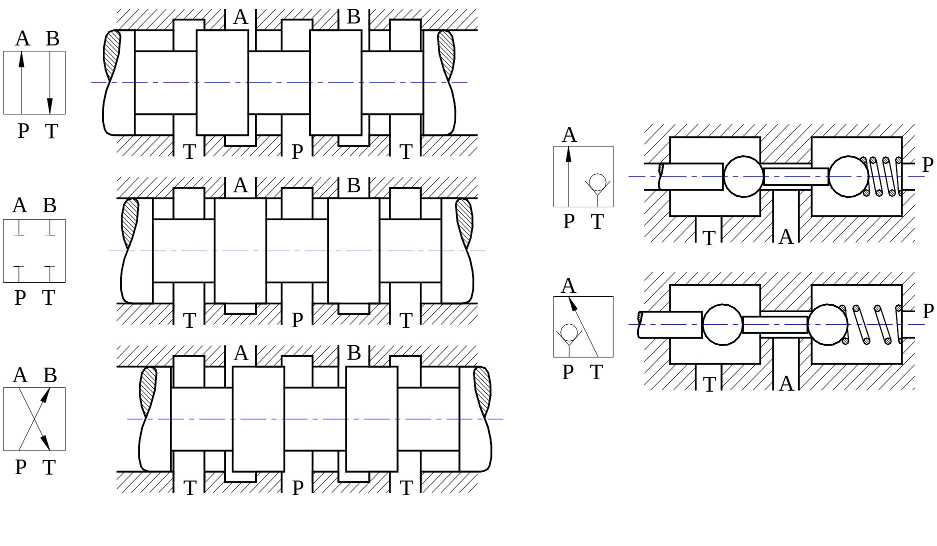


Figure 3.4 Schematic drawings of a spool directional control valve and a ball directional control valve in the different working positions.

Another way of classifying the directional control valve is by the type of moving body generating the different flow paths. The type of body is either a ball/poppet or a spool, see Figure 3.4.
The ball/poppet type valve is leak proof but unsuitable for high power flow, because the actuation is along the same line as the main flow. Contrary to this, the main flow is perpendicular to the actuation, for spool type valves. However, in order for the spool to move, a certain clearance between spool and valve housing is required causing leakage.
Directional control valves may be actuated in a great variety of ways. Normally the moving body is kept in a neutral position by a centering spring. Hence, the actuator must overcome the spring force in order to activate the valve. The types of actuation vary from manual, electric, pneumatic, hydraulic and electro-hydraulic. In Figure 3.5 the standardized symbols used for different types of actuation are shown alone and applied to a 4/3-way directional control valve.

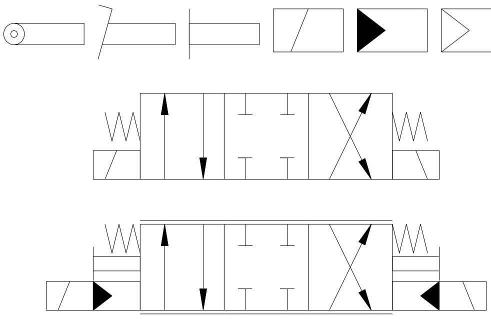


Figure 3.5 Above is shown the standardized symbols for 3 different types of manual actuation (roller, pedal and general) together with electrical, hydraulic and pneumatic actuation.

In the middle is shown an electrically actuated directional control valves with a centering spring. At the bottom is an electro-hydraulic actuated proportional directional control valve with manual override.
Also the actuation is used to distinguish between directional control valves. So-called proportional directional valves and servo directional valves are spool type valves, where the spool may be positioned proportional to some electrical input signal. In fact, this means that the proportional/servo directional control valve has an infinite number of possible positions making it a combination of a pure directional valve and a flow control valve with adjustable restrictions as functions of spool travel.

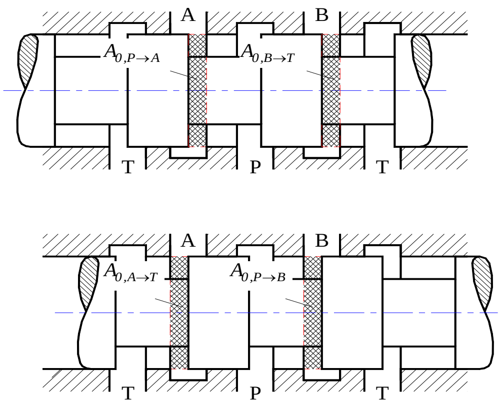


Figure 3.6 Spool of directional control valve shown in a-position and b-position. The band shaped discharge areas are shown as double-hatched rectangles.

To distinguish proportional/servo directional control valves from directional control valves an extra set of lines is added to their symbol as shown in Figure 3.5, bottom.
The governing equations for a directional control valve concerns the flow from/to the service ports as well as the pressure drop(s) across the valve.

In Figure 3.6 the spool is shown in the a- and b-position. In general, the flow will be turbulent after passing through the discharge areas, $A_{0,P\rightarrow A}$ and $A_{0,B\rightarrow T}$ or $A_{0,P\rightarrow B}$ and $A_{0,A\rightarrow T}$. Thus, the orifice equation may be used to describe the pressure drop:

$$Q = C_D \cdot A \cdot \sqrt{\frac{2}{\rho} \cdot \Delta p} \quad \Leftrightarrow \quad \Delta p = \frac{\rho}{2 \cdot C_D^2 \cdot A^2} \cdot Q^2 \tag{3.5}$$

where

Q	flow through the orifice, [volume/time],
C_D	discharge coefficient,
A	discharge area, [area]
ρ	mass density, [mass/volume]
Δp	pressure drop across the orifice, [pressure]

For the type of orifices normally encountered in directional control valves the discharge coefficient will lie around 0.55-0.65.

For an ordinary directional control valve with everything closed in neutral position the governing equations for flow and pressure drop become:

Position (The different indices refer to the service ports)

a	$Q_P = Q_A = Q_{P\rightarrow A}$ $Q_B = Q_T = Q_{B\rightarrow T}$	$Q_{P\rightarrow A} = K_{P\rightarrow A} \cdot \sqrt{p_P - p_A}$ $Q_{B\rightarrow T} = K_{B\rightarrow T} \cdot \sqrt{p_B - p_T}$
0	$Q_P = Q_A = Q_B = Q_T = 0$	
b	$Q_P = Q_B = Q_{P\rightarrow B}$ $Q_A = Q_T = Q_{A\rightarrow T}$	$Q_{P\rightarrow B} = K_{P\rightarrow B} \cdot \sqrt{p_P - p_B}$ $Q_{A\rightarrow T} = K_{A\rightarrow T} \cdot \sqrt{p_A - p_T}$

(3.6)

where

K	coefficient that mainly depends on geometry, [volume/(time·pressure^{½})]
p	pressure, [pressure]
Q	flow, [volume/time]

The different constants in Equation (3.6) vary from valve to valve, as they almost exclusively depend on the geometry of the orifices, i.e., spool geometry, spool stroke as well as house geometry. A typical chart for a directional control valve is a Q-Δp curve for each flow passages, see Figure 3.7. The valve in the figure is symmetrical in the sense, that the P → A port and the P → B port are identical, and the A → T port and the B → T port are identical.

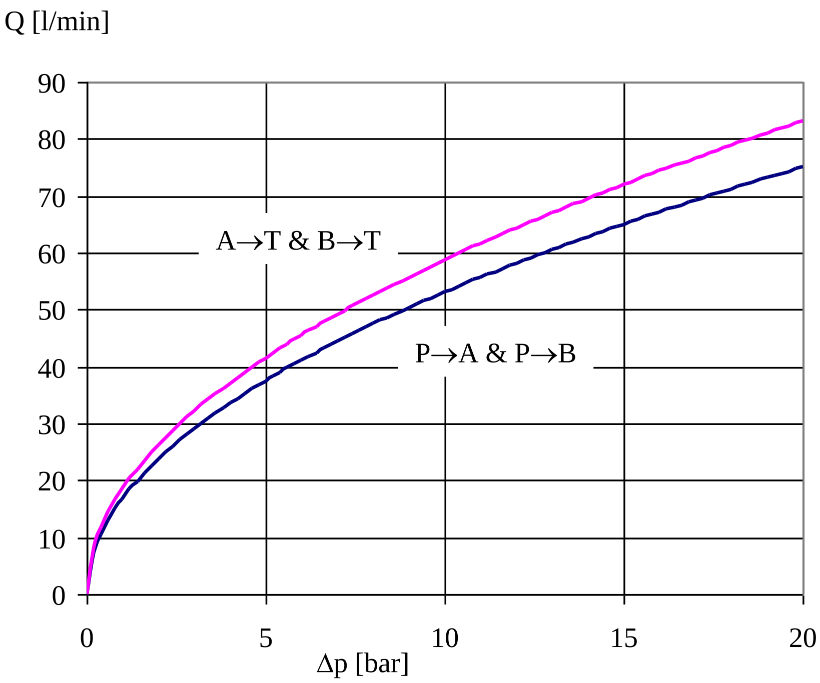


Figure 3.7 Computed Q-Δp curves for a directional valve.

For a proportional/servo directional control valve the discharge area is not a constant but varies with spool travel, hence, the governing equations will include the spool travel, x, as a variable:

Position (The different indices refer to the service port connections)

a	$Q_P = Q_A = Q_{P \to A}$ $Q_B = Q_T = Q_{B \to T}$	$Q_{P \to A} = K_{P \to A} \cdot \dfrac{A_{P \to A}(x)}{A_{0,P \to A}} \cdot \sqrt{p_P - p_A}$ $Q_{B \to T} = K_{B \to T} \cdot \dfrac{A_{B \to T}(x)}{A_{0,B \to T}} \cdot \sqrt{p_B - p_T}$
0	$Q_P = Q_A = Q_B = Q_T = 0$	
b	$Q_P = Q_B = Q_{P \to B}$ $Q_A = Q_T = Q_{A \to T}$	$Q_{P \to B} = K_{P \to B} \cdot \dfrac{A_{P \to B}(x)}{A_{0,P \to B}} \cdot \sqrt{p_P - p_B}$ $Q_{A \to T} = K_{A \to T} \cdot \dfrac{A_{A \to T}(x)}{A_{0,A \to T}} \cdot \sqrt{p_A - p_T}$

(3.7)

where

K	coefficient that mainly depends on geometry, [volume/(time·pressure^{½})]
A	spool travel dependant discharge area, [area]
A_0	maximum value for a discharge area (full spool travel), [area]
x	spool travel, [length]
p	pressure, [pressure]
Q	flow, [volume/time]

Often, the spool is machined to obtain a tailor-made flow dependency on the spool travel, i.e., linear, progressive or regressive. A typical chart for a proportional directional valve is a Q-x curve for each flow passage and for different pressure drops, see Figure 3.8.

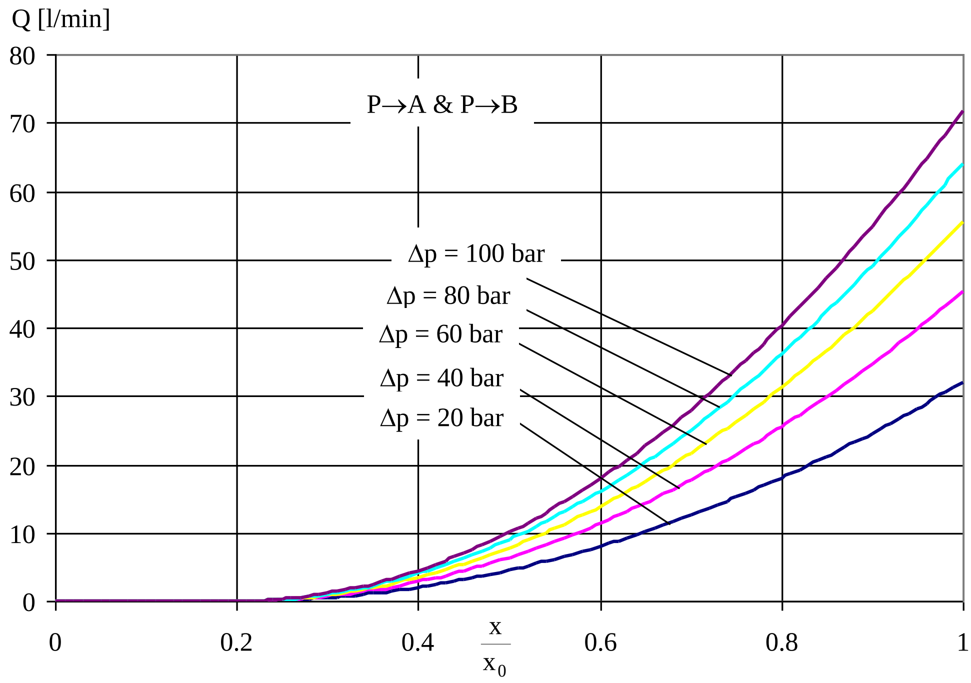


Figure 3.8 Computed discharge flow from a proportional directional control valve as a function of the relative spool travel. The value x_0 represents the full spool travel (stroke).

In Figure 3.8 there is no flow until the relative spool travel is 0.2, indicating that directional control valves normally have a certain dead band, in order to keep leakage down.
A major problem for the actuation of directional control valves are the flow forces. As mentioned in section 3.2, the flow force will always try to close any orifice through which fluid is flowing. In the case of spool valves this means that the actuation has to overcome an extra force beside the force from the usual centering spring.

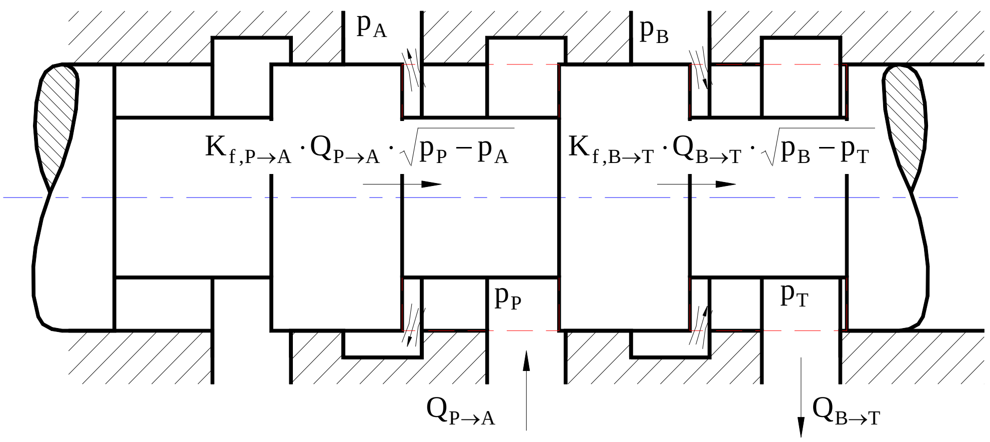


Figure 3.9 The flow forces acting on a 4/3-way spool in the a-position.

Employing Equation (3.3) the flow force acting on a spool of a 4/3-way directional control valve can be set up:

Position (The different indices refer to the service port connections)

a	$F_f = K_{f,P\to A} \cdot Q_{P\to A} \cdot \sqrt{p_P - p_A} + K_{f,B\to T} \cdot Q_{B\to T} \cdot \sqrt{p_B - p_T}$
0	$F_f = 0$
b	$F_f = K_{f,P\to B} \cdot Q_{P\to B} \cdot \sqrt{p_P - p_B} + K_{f,A\to T} \cdot Q_{A\to T} \cdot \sqrt{p_A - p_T}$

(3.8)

where

F_f flow force on the spool, [force]
K_f geometry and flow dependant coefficient, [(force·time)/(volume·pressure^{½})]
p pressure, [pressure]
Q flow, [volume/time]

Notice how the flow forces from both throttlings work in the same direction, against actuation. Information on the flow force coefficients, K_f, is not given as part of standard catalogue information unlike the discharge flow coefficients.

Check valve

Check valves belong to the group of valves controlling flow direction. They act as rectifiers in a hydraulic system, allowing (almost) free flow in one direction and preventing flow in the opposite direction. When the pressure in the inlet port, port A, reaches a certain value, crack pressure, the ball is lifted from its seat, compressing the spring and allowing flow to port B. (see Figure 3.10)

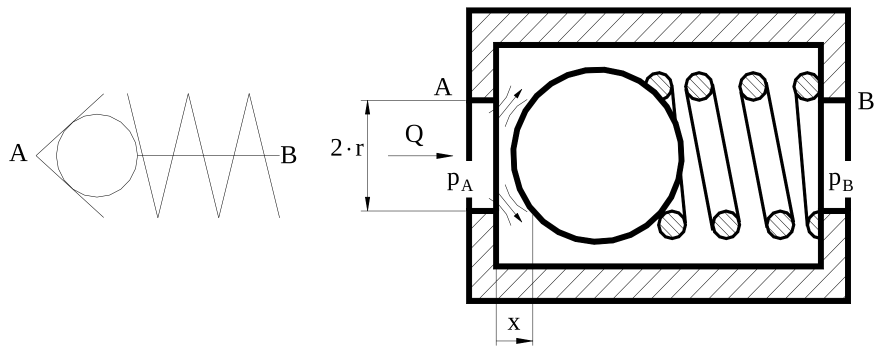


Figure 3.10 The standard symbol and a schematic drawing of a check valve.

Check valves are used extensively in hydraulic systems, typically to protect components, hold loads or allow bypass. The check valve is made with either a ball or a poppet eliminating any leakage flow. The ball/poppet may be either spring loaded or simply gravity loaded (in that case the valve must be mounted with the ball/poppet above the seat). In general, the flow pattern around a ball is more unpredictable and more unstable as compared to a poppet. For check valves instability is typically not a problem, hence, the relatively inexpensive solution with a spring loaded ball is often used.

The governing equations for a check valve are based on the orifice equation and static equilibrium:

$$Q = C_D \cdot A(x) \cdot \sqrt{\frac{2}{\rho} \cdot (p_A - p_B)} \tag{3.9}$$

$$(p_A - p_B) \cdot \pi \cdot r^2 = k_{sp} \cdot (x + x_{ic}) \tag{3.10}$$

$$p_{cr} = \frac{k_{sp} \cdot x_{ic}}{\pi \cdot r^2} + p_B \tag{3.11}$$

where

Q	flow across the valve, [volume/time]
C_D	discharge coefficient
A	discharge area, [area]
ρ	mass density, [mass/volume]
p_A	inlet pressure, [pressure]
p_B	outlet pressure, [pressure]
r	inlet radius, [length]
k_{sp}	spring stiffness, [force/length]
x	ball/poppet travel, [length]
x_{ic}	initial compression of the spring, [length]
p_{cr}	crack pressure required to open the valve, [pressure]

Equation (3.9) and Equation (3.10) constitutes 2 equations with 3 variables Q, x and p_A - p_B. Hence, knowing the flow through the valve, the pressure drop and the position of the ball/poppet may be determined. In the above static equilibrium, Equation (3.10), the flow force is not included, as it may be neglected for check valves because of the small pressure drops. The variation of the discharge area depends on the configuration: ball or poppet, see Figure 3.11.

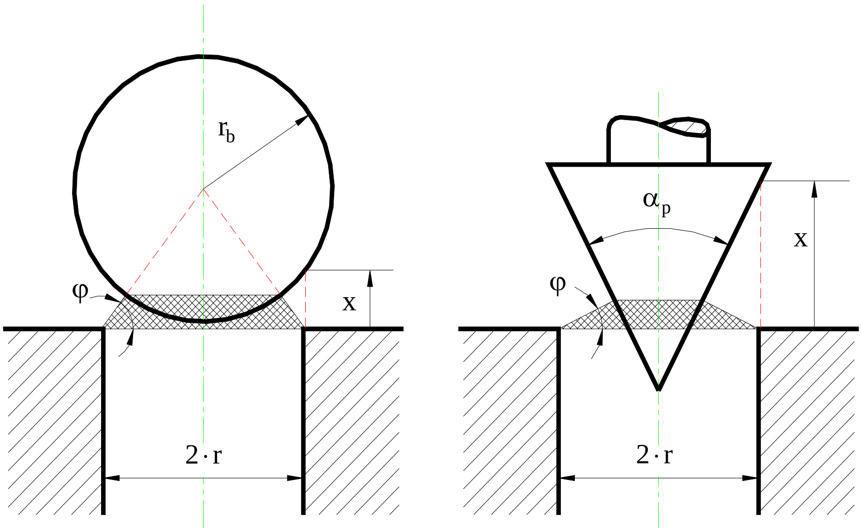


Figure 3.11 Discharge areas for a ball and poppet valve.

An expression for the discharge areas is: $A(x) = \pi \cdot 2 \cdot r \cdot x \cdot \sin\varphi - \pi \cdot x^2 \cdot \sin^2\varphi \cdot \cos\varphi$.

However, as long as the ball/poppet travel, x, is small compared to the inlet radius, r, then the discharge areas may approximately be determined as:

$$A(x) = \pi \cdot 2 \cdot r \cdot x \cdot \sin\varphi \qquad x \ll r \tag{3.12}$$

where

Poppet $$\varphi = \frac{\alpha_p}{2} \tag{3.13}$$

Ball $$\varphi = g^{-1}\left[\frac{\sqrt{r_b^2 - r^2} + x}{r}\right] \tag{3.14}$$

where

r	seat radius, [length]
x	ball/poppet travel, [length]
φ	discharge area projection angle
α_p	poppet angle
r_b	ball radius

Equation (3.12) only holds for x smaller than some maximum value x_0 where the poppet is blocked by the valve house. After that the check valve acts a simple restrictor (orifice with constant discharge area).

For a simplified analysis the ball/poppet travel is disregarded, and the pressure drop is set equal to the crack pressure, independent of the flow.

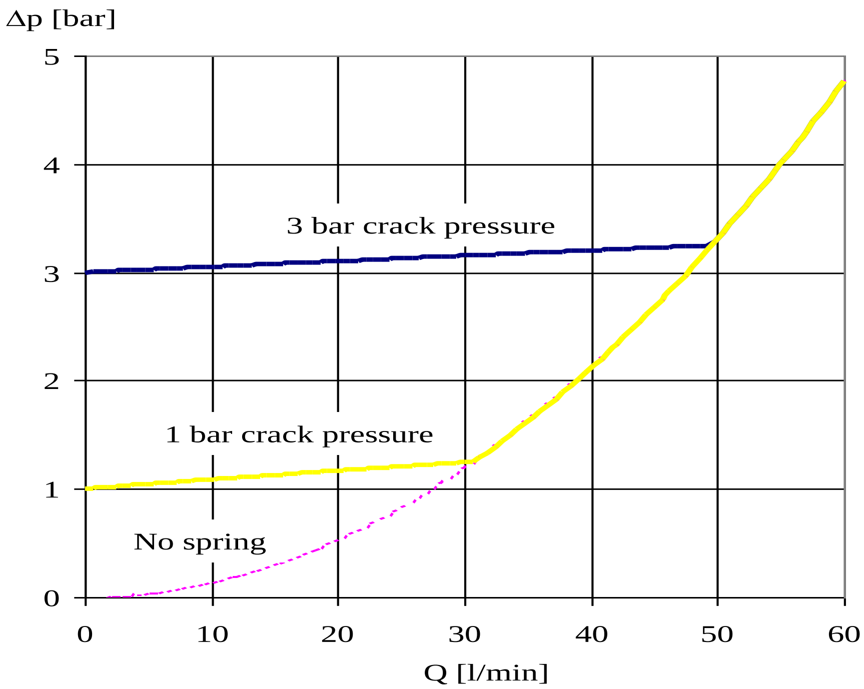


Figure 3.12 Computed Q-Δp curves for a ball check valve with different springs, i.e., crack pressures. Notice how the curves join, as the ball reaches its full travel.

A pilot operated check valve allows flow in the opposite direction when activated by a pilot piston, see Figure 3.13. This feature is very useful when lowering loads that were held by the check valve functionality.

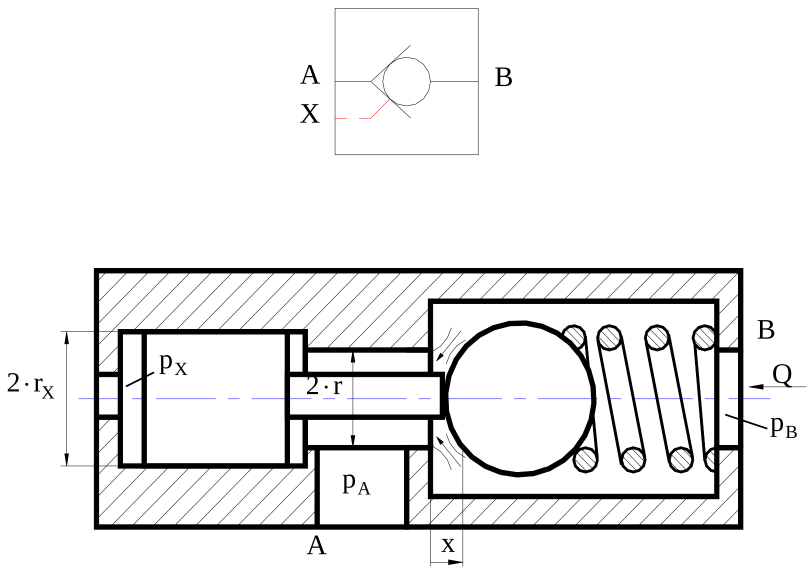


Figure 3.13 Standard symbol and schematic drawing of a pilot operated check valve. The valve is piloted open.

The governing equations for the pilot operated check valve correspond to those of the check valve except for the static equilibrium:

$$(p_x - p_A) \cdot \pi \cdot r_x^2 = (p_B - p_A) \cdot \pi \cdot r^2 + k_{sp} \cdot (x + x_{ic}) \tag{3.15}$$

$$p_{cr,x} = (p_B - p_A) \cdot \frac{r^2}{r_x^2} + \frac{k_{sp} \cdot (x + x_{ic})}{\pi \cdot r_x^2} + p_A \tag{3.16}$$

where

$p_{cr,x}$	required crack pressure in the pilot line, [pressure]
r_x	radius of the pilot piston
p_B	inlet pressure, [pressure]
p_A	outlet pressure, [pressure]
r	inlet radius, [length]
k_{sp}	spring stiffness, [force/length]
x	ball/poppet travel, [length]
x_{ic}	initial compression of the spring, [length]

Normally, the pilot piston will open the check valve to its maximum value, and it will work as a simple restrictor.

3.4 Pressure Control Valves

A pressure control valve may have the job of limiting or otherwise regulating pressure or creating a particular pressure condition required for control.

All pure pressure control valves operate in a condition approaching hydraulic balance. Usually the balance is very simple: Pressure is effective on one side or end of a ball, poppet or spool – and is opposed by a spring. In operation, the valve takes a position where the hydraulic pressure exactly balances the spring force.

3.4.1 Pressure relief valve

The function of any relief valve is to protect the hydraulic system from excessive pressure if the pressure increases above a predetermined maximum. A relief valve is an automatic relieving device that is actuated by the static pressure upstream of the valve.
Relief valves are designed to return the hydraulic fluid directly to the reservoir. A relief valve is normally closed until the system pressure approaches a reset value, called the cracking pressure. As system pressure continues to increase, the amount of flow through a properly sized relief valve will increase until the entire pump output passes through the valve. When system pressure decreases, the valve closes smoothly and quietly.

The pressure relief valve acts like a spring loaded check valve, however, the spring load and the poppet design are very different, as the relief valve typically is supposed to throttle pressure drops up to several 100 bars.

In Figure 3.11 is shown a typical relief valve in an open position.

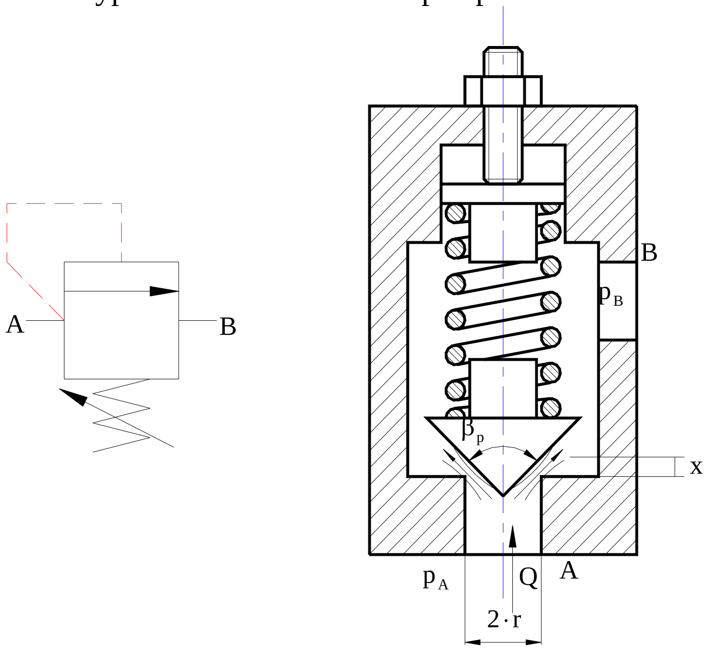


Figure 3.14 Standard symbol and schematic drawing of pressure relief/safety valve. The valve is open (blowing), i.e., the inlet pressure, p_A, is above the crack pressure.

When the pressure in port A reaches a certain value, crack pressure, the poppet is lifted from the seat, compressing the spring and allowing flow to port B. As long as flow is send across the relief valve the pressure in port A will stay at, approximately, the crack pressure. The crack pressure can be varied because the initial compression of the spring is adjustable. A pressure safety valve is designed in exactly the same way as a pressure relief valve. They differ in functionality, as the safety valve is only expected to bleed of flow, when the system pressure accidentally moves above a preset value.
In general, both relief and safety valves are made with either a spring loaded ball or a spring loaded poppet.

The governing equations for a pressure relief valve corresponds rather closely to those of a spring loaded check valve, see Equations (3.9)...(3.11). The only difference in the mathematical expressions is the inclusion of flow forces in the static equilibrium for the ball/poppet. There is a distinct difference, however, in the higher crack pressure levels, and the stiffer springs.

$$Q = C_D \cdot A(x) \cdot \sqrt{\frac{2}{\rho} \cdot (p_A - p_B)} \tag{3.17}$$

$$(p_A - p_B) \cdot \pi \cdot r^2 - K_f \cdot Q \cdot \sqrt{p_A - p_B} = k_{sp} \cdot (x + x_{ic}) \tag{3.18}$$

$$p_{cr} = \frac{k_{sp} \cdot x_{ic}}{\pi \cdot r^2} + p_B \tag{3.19}$$

where

Q	flow across the valve, [volume/time]
C_D	discharge coefficient
A	discharge area, [area]
ρ	mass density, [mass/volume]
p_A	inlet pressure, [pressure]
p_B	outlet pressure, [pressure]
r	seat radius, [length]
K_f	geometry and flow dependant coefficient, [(force·time)/(volume·pressure½)]
k_{sp}	spring stiffness, [force/length]
x	ball/poppet travel, [length]
x_{ic}	initial compression of the spring, [length]
p_{cr}	crack pressure required to open the valve, [pressure]

Equations (3.17) and Equation (3,.18) constitutes 2 equations with 3 variables Q, x and p_A-p_B. Hence, knowing the flow through the valve, the pressure drop and the position of the ball/poppet may be determined. The variation of the discharge area with the ball/poppet travel is described in Equation (3.12). According to Equation (3.2) the flow coefficient can be expressed as:

$$K_f = \sqrt{2 \cdot \rho} \cdot \cos\frac{\beta_p}{2} \tag{3.20}$$

where

K_f	geometry and flow dependant coefficient, [(force·time)/(volume·pressure½)]
ρ	mass density, [mass/volume]
β_p	poppet angle

There are a number of complex phenomena related to the flow across the ball/poppet that reduces the validity of Equation (3.4). It does, however, in most cases give a reasonable estimate of the actual flow forces.

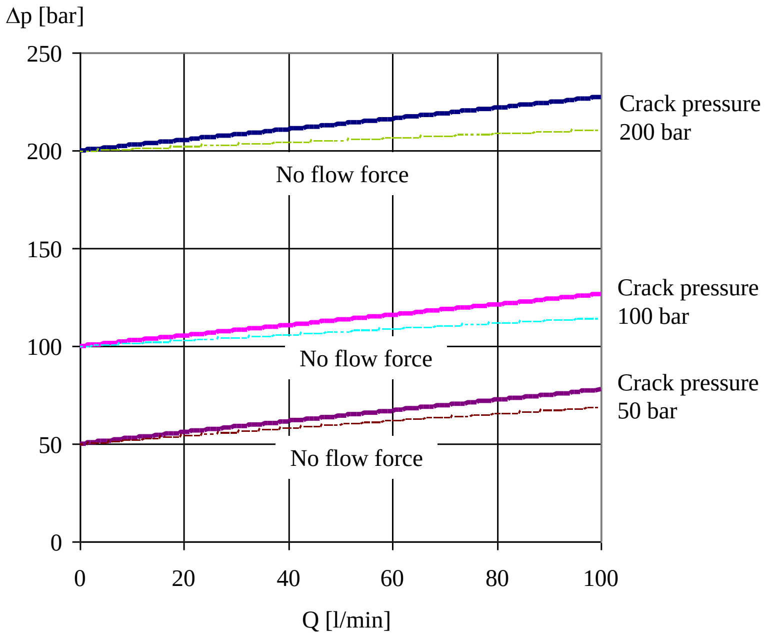


Figure 3.15 Computed Δp-Q curves for 3 relief valves with different crack pressure settings. The influence of the flow force is illustrated by not including it in the thin line curves.

In Figure 3.15, Δp-Q curves are shown for a pressure relief valve. As may be seen the presence of a flow force increases the slope of the Δp-Q curve, especially at high pressure drops. It corresponds approximately to having an extra spring. This *flow induced force* is a hydraulic reaction force acting as a result of accelerating fluid through an orifice. The pressure relief valve has 2 inherent problems: The slope of the Δp-Q curve is unwanted from a static functionality point of view. However, if the slope is decreased (weaker spring, flow force compensation), the result is often instability. A solution to this basic antagonism is the pilot operated relief valve.

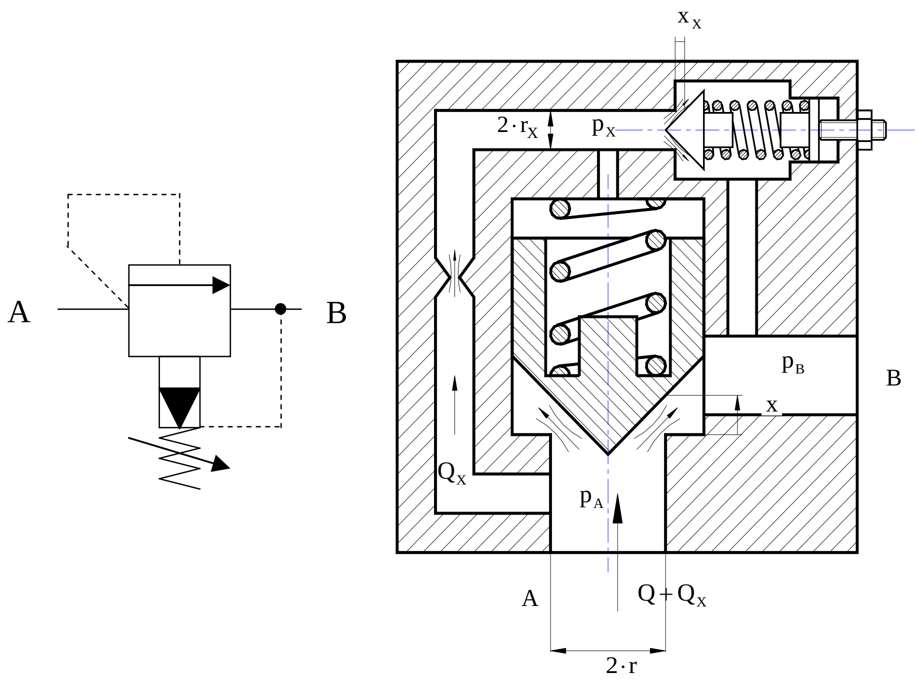


Figure 3.16 Standard symbol and schematic drawing of pilot operated pressure relief valve. The valve is piloted open, i.e., the inlet pressure, p_A, is above the pilot crack pressure, $p_{cr,X}$.

The pilot operated relief valve works as follows, see Figure 3.16: The main poppet is subjected to the same pressure on both sides, i.e., the light spring closes the main poppet effectively. The crack pressure is set by adjusting the spring load on the pilot poppet. When the pilot flow (in this case also the inlet flow) reaches the set value, a flow across the pilot poppet and subsequently across the fixed orifice, O, is introduced causing a pressure drop from p_A to p_X. The pressure drop is flow dependent, and as the flow across the pilot poppet is increased the pressure drop across O, will finally become larger than the crack pressure of the light main spring. The main poppet is lifted from its seat, and only flow forces prevent the valve, see Figure 3.17, from having a perfect flat Δp-Q curve, without any of the stability problems mentioned earlier.

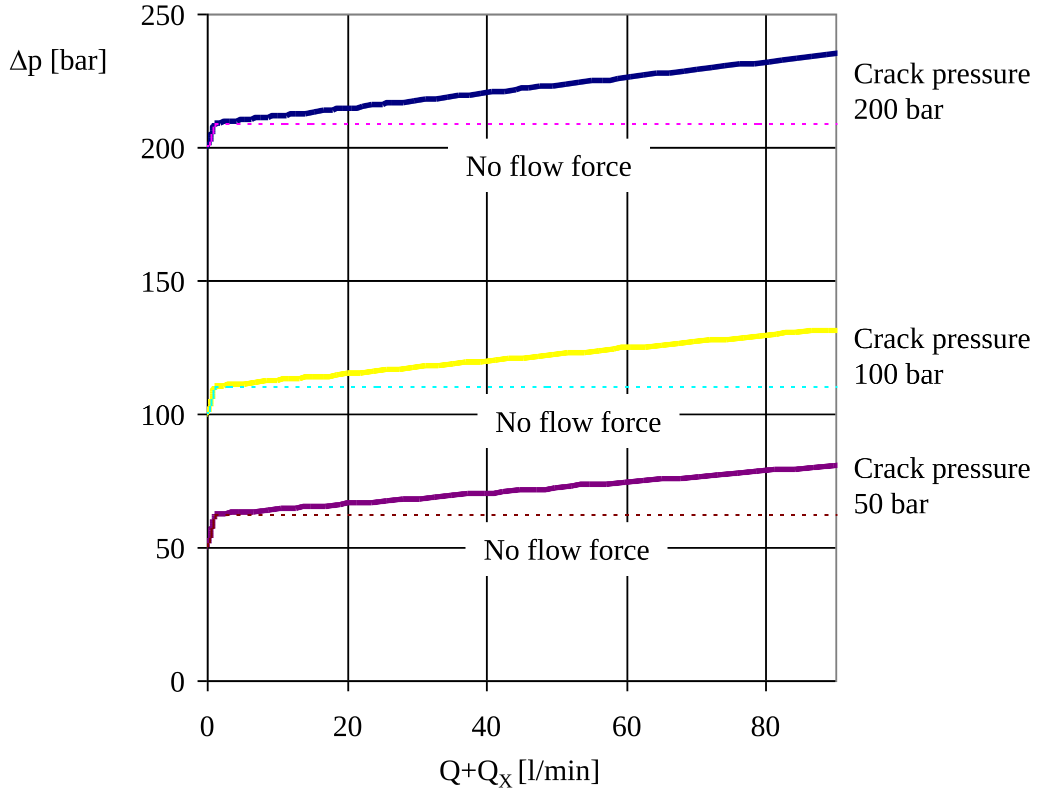


Figure 3.17 Computed Δp-Q curves for 3 pilot operated relief valves with different pilot crack pressure setting. The influence of the flow force is illustrated by the thin line curves

The curves in Figure 3.17 are based on the following governing equations for a pilot operated pressure relief valve:

$$Q = C_D \cdot A(x) \cdot \sqrt{\frac{2}{\rho} \cdot (p_A - p_B)} \tag{3.21}$$

$$(p_A - p_X) \cdot \pi \cdot r^2 - K_f \cdot Q \cdot \sqrt{p_A - p_B} = k_{sp} \cdot (x + x_{ic}) \tag{3.22}$$

$$Q_X = C_{D0} \cdot A_0 \cdot \sqrt{\frac{2}{\rho} \cdot (p_A - p_X)} \tag{3.23}$$

$$Q_X = C_{DX} \cdot A_X(x_X) \cdot \sqrt{\frac{2}{\rho} \cdot (p_X - p_B)} \qquad (3.24$$

$$(p_X - p_B) \cdot \pi \cdot r_X^2 = k_{sp,X} \cdot (x_X + x_{ic,X}) \tag{3.25}$$

$$p_{cr} = \frac{k_{sp} \cdot x_{ic}}{\pi \cdot r^2} + p_X \tag{3.26}$$

$$p_{cr,X} = \frac{k_{sp,X} \cdot x_{ic,X}}{\pi \cdot r_X^2} + p_B \tag{3.27}$$

where

Q	flow across the main poppet, [volume/time]
C_D	discharge coefficient for the main poppet orifice
A	discharge area of the main poppet orifice, [area]
ρ	mass density, [mass/volume]
p_A	inlet pressure, [pressure]
p_B	outlet pressure, [pressure]
p_X	pilot pressure, [pressure]
r	seat radius of the main poppet, [length]
K_f	geometry and flow dependant coefficient, [(force·time)/(volume·pressure^{½})] concerning the flow forces around the main poppet orifice
k_{sp}	spring stiffness of the main poppet spring, [force/length]
x	main poppet travel, [length]
x_{ic}	initial compression of the main poppet spring, [length]
C_{D0}	discharge coefficient for the fixed orifice
A_0	discharge area for the fixed orifice
Q_X	pilot flow, [volume/time]
C_{DX}	discharge coefficient for the pilot poppet orifice
A_X	discharge area of the pilot poppet orifice, [area]
r_X	pilot poppet seat radius, [length]
$k_{sp,X}$	pilot spring, [force/length]
x_X	pilot poppet travel, [length]
$x_{ic,X}$	initial compression of the pilot poppet spring, [length]
p_{cr}	crack pressure required to lift the main poppet, [pressure]
$p_{cr,X}$	crack pressure required to lift the pilot poppet, [pressure]

The pilot flow is, normally, only a few l/min, and the crack pressure of the main poppet is around 5 bar. As for the directly operated pressure relief valve the crack pressure can be varied by adjusting the initial compression of the pilot spring.

For a simplified analysis of both types of pressure relief valves, the inlet pressure is constant and equal to the crack pressure as soon as there is any flow across the valve.

3.4.2 Pressure reducing valve

A pressure reducing valve is used to limit pressure level from the normal operating pressure of the primary hydraulic system to the required pressure of a secondary hydraulic circuit
The purpose of pressure reducing valves is to maintain a desired pressure downstream of the valve, independently of (but lower than) the upstream pressure. In Figure 3.15 a schematic drawing is shown together with the standard symbol for a pressure reducing valve with bypass.
If the pressure in the regulated port, p_B, goes up, the spool moves to the right against the adjustable spring and the reservoir pressure, p_T. This tends to close the connection between ports A and B, thereby increasing the throttling and decreasing p_B. If pressure

peaks appear in the regulated line the spool will travel even further to the right and first close the connection between ports A and B. Next, after passing a certain dead band, x_D, the connection between B and T is opened. As the pressure in port T is quite low, this is in fact a pressure relief function. It is build into almost all pressure reduction valves.

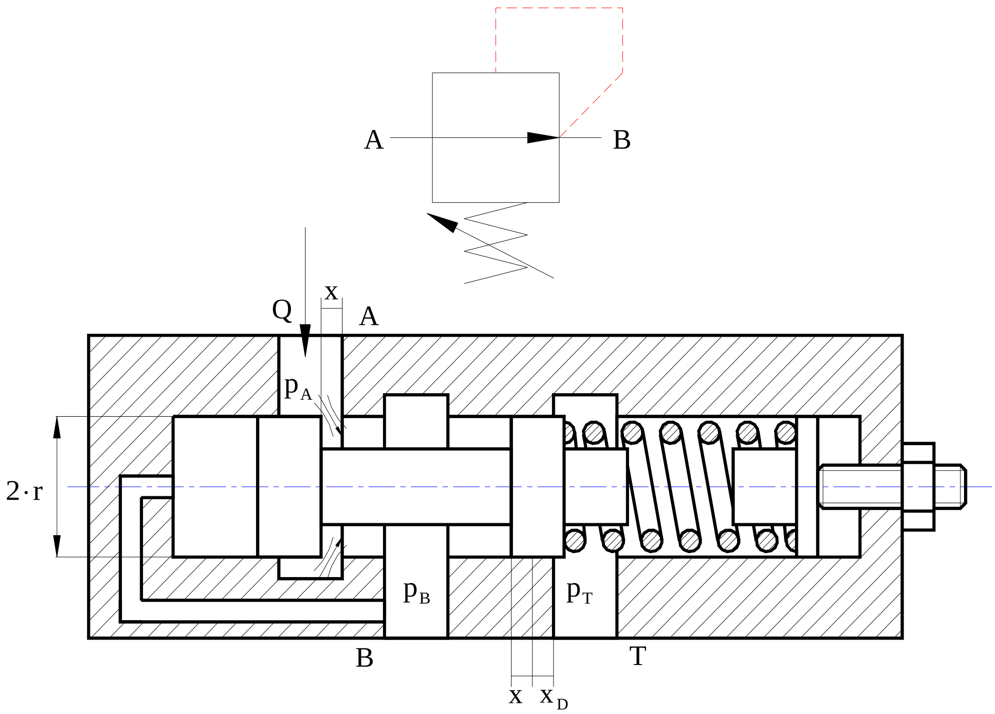


Figure 3.18 Standard symbol and schematic drawing of pressure reducing valve. The valve is throttling, i.e., the pressure in port A is reduced to the desired pressure level (set by adjusting initial compression of spring) in port B.

The governing equations for a pressure reducing valve are:
For $x > 0$*:*

$$Q = C_D \cdot A(x) \cdot \sqrt{\frac{2}{\rho} \cdot (p_A - p_B)} \tag{3.28}$$

$$(p_B - p_T) \cdot \pi \cdot r^2 + K_f \cdot Q \cdot \sqrt{p_A - p_B} = k_{sp} \cdot (x_{ic} - x) \tag{3.29}$$

$$p_{cl} = \frac{k_{sp} \cdot x_{ic}}{\pi \cdot r^2} + p_T \tag{3.30}$$

For $x < -x_D$*:*

$$Q_T = C_{D,T} \cdot A_T(x) \cdot \sqrt{\frac{2}{\rho} \cdot (p_B - p_T)} \tag{3.31}$$

$$(p_B - p_T) \cdot \pi \cdot r^2 - K_{f,T} \cdot Q_T \cdot \sqrt{(p_B - p_T)} = k_{sp} \cdot (x_{ic} - x) \tag{3.32}$$

$$p_{cr,T} = \frac{k_{sp} \cdot (x_{ic} + x_D)}{\pi \cdot r^2} + p_T \tag{3.33}$$

where

Q	flow from A to B, [volume/time]
C_D	discharge coefficient for the A to B orifice
A	discharge area of the A to B orifice, [area]
ρ	mass density, [mass/volume]
p_A	upstream pressure, [pressure]
p_B	downstream pressure, [pressure]
p_T	reservoir pressure, [pressure]
r	radius of the spool, [length]
K_f	flow force coefficient, [(force·time)/(volume·pressure^{½})] concerning the flow forces around the A to B orifice
k_{sp}	spring stiffness of the spring, [force/length]
x_{ic}	initial compression of the spring, [length]
x	spool travel, [length]
p_{cl}	downstream pressure where the A to B connection is closed, [pressure]
Q_T	flow from B to T, [volume/time]
C_T	discharge coefficient for the B to T orifice
A_T	discharge area for the B to T orifice
$K_{f,T}$	geometry and flow dependant coefficient, [(force·time)/(volume·pressure^{½})] concerning the flow forces around the B to T orifice
x_D	dead band, [length]
$p_{cr,T}$	crack pressure required downstream to open the B to T connection, [pressure]

The Δp-Q curve for the pressure relief function (B to T) corresponds to those discussed in Section 3.4.1. For the pressure reducing part ($x > 0$), some typical curves are shown in Figure 3.19, where the reservoir pressure, P_T , has been set to nil.

The influence of the upstream pressure is complex, as increased pressure will tend to push more flow through the A to B orifice, however this increase the valve closing flow force and downstream pressure. In general, catalogue material will not clearly indicate at what upstream pressure curves, like the ones shown in Figure 3.19, are determined.

The closing pressure can be varied because the initial compression of the spring is adjustable.

Just like pressure relief valves, pressure reducing valves may be pilot operated gaining some of the same advantages, i.e., the slope of Δp-Q curve is reduced without loss of stability.

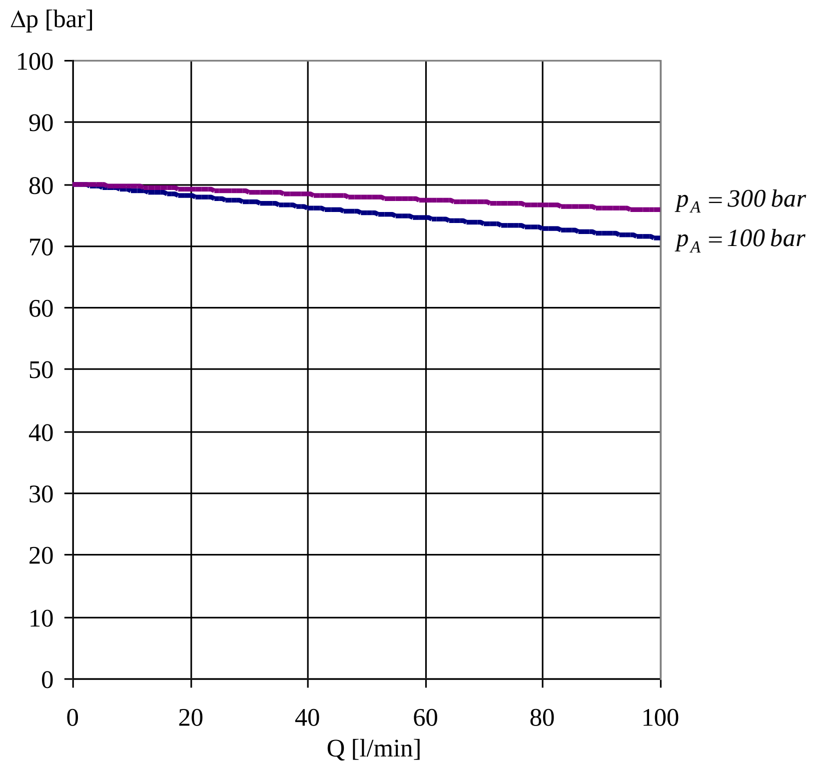


Figure 3.19 Computed Δp-Q curves for a pressure reducing valve at different inlet pressures. The desired downstream pressure, p_{cl} , is set to 80 bar.

3.5 Flow Control Valves

Flow control valves provide volume control in hydraulic circuits. Flow is controlled by either throttling or diverting the flow. Throttling the flow involves decreasing the size of an opening until all of the flow cannot pass through the orifice. Bypassing the flow involves routing part of the flow around the circuit so that the actuator device receives only the portion of flow needed to perform its task.
Hydraulic circuits that use flow control devices are called metered circuits. If an actuator has the inlet flow controlled, the circuit is a “meter-in” circuit. If an actuator has the outlet flow controlled, the circuit is a “meter-out” circuit. Flow control circuits can either be non-compensated or compensated circuits.
Non-compensated flow controls are simple valves that meter flow by throttling. The amount of flow that passes through the valve is determined by the position of the valve. As the valve is closed, flow decreases. One of the most common non-compensated valves is the adjustable needle valve, considered in section 3.5.1. Compensated flow control valves are considered in the following sections.

3.5.1 Restrictor valve

Its purpose is to act as a simple orifice, generating a pressure drop large enough, for some pressure relief valve further upstream to continuously bypass flow to tank. In that case the restricting valve will work in parallel with the pressure relief valve controlling the speed of the downstream actuator. The speed control will, however, be load dependant.

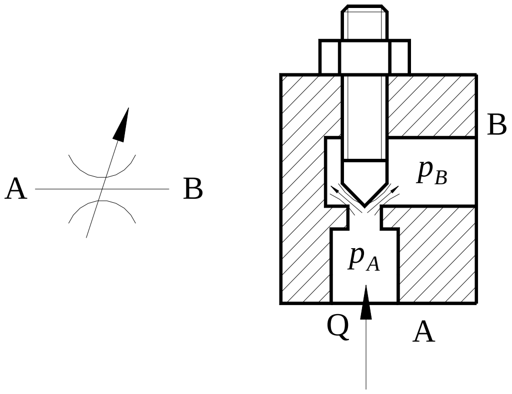


Figure 3.20 Standard symbol and schematic drawing of a restrictor valve with adjustable orifice.

Often, the discharge area of a restrictor valve can be varied, and in Figure 3.21 the Q-Δp curve for a restrictor valve with different settings of the discharge area is shown.

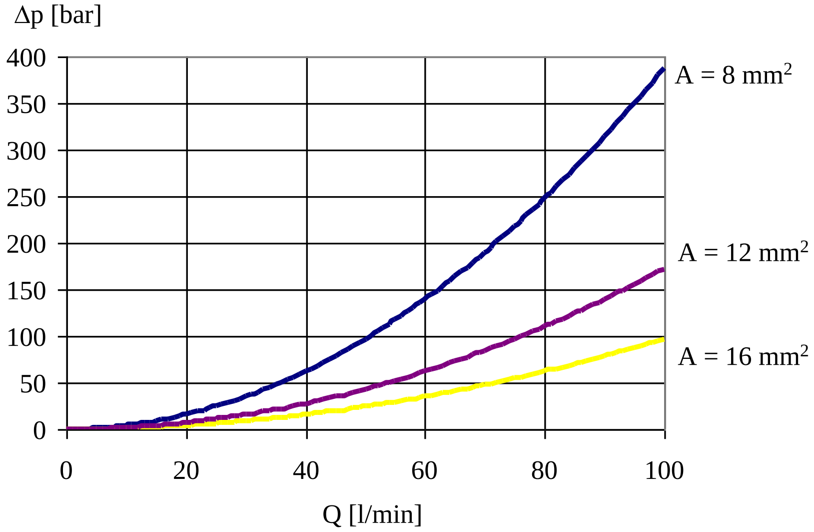


Figure 3.21 Computed Q-Δp curve for a restrictor valve with different settings of the adjustable discharge area.

The curves simply reflect the basic orifice equation, which is the only governing equation of the restrictor valve:

$$Q = C_D \cdot A \cdot \sqrt{p_A - p_B} \tag{3.34}$$

where

Q	flow, [volume/time]
C_D	discharge coefficient
A	restrictor area, [area]
p_A	upstream pressure, [pressure]
p_B	downstream pressure, [pressure]

3.5.2 2 way flow control valve

The 2-way flow control valve belongs to the group of flow rate controlling valves. Its purpose is to provide a constant flow independent of downstream pressure, i.e., actuator load. This is also referred to as a pressure compensated flow valve.

The pressure independent flow is obtained by means of a differential pressure controller, see Figure 3.19, positioned before a fixed orifice. The differential pressure controller is, essentially, a spool subjected to the pressure before and after the fixed orifice, on each end. The smaller pressure (after the orifice) works together with a spring against the higher pressure (before the orifice). This way a pressure drop, approximately equal to the initial compression of the spring, is maintained across the fixed orifice, yielding an approximately constant flow. If the intermediate pressure, p_I , increases, the flow will go up, but simultaneously the spool will tend to close, thereby reducing the flow. If the opposite happens, i.e., the downstream pressure, p_B, increases, the flow will go down, but simultaneously the spool will tend to open, thereby increasing the flow.

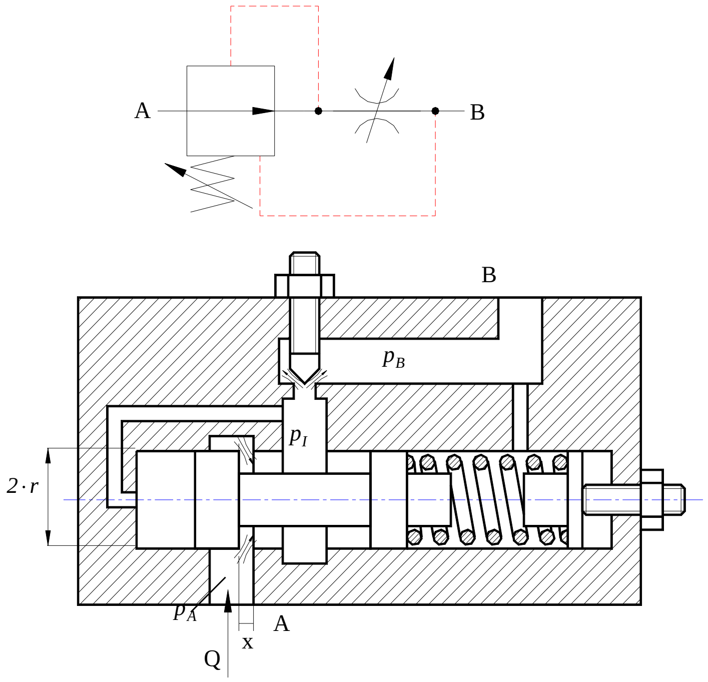


Figure 3.22 Standard symbol and schematic drawing of a 2-way flow control valve. The valve is shown working as it regulates the pressure drop across the fixed orifice by throttling the flow before the main (fixed) orifice.

The governing equations for the 2-way flow control valve are:

$$Q = C_D \cdot A(x) \cdot \sqrt{\frac{2}{\rho}(p_A - p_I)} \tag{3.35}$$

$$Q = C_{Df} \cdot A_f \cdot \sqrt{\frac{2}{\rho}(p_I - p_B)} \tag{3.36}$$

$$k_{sp} \cdot (x_{ic} - x) - (p_I - p_B) \cdot \pi \cdot r^2 - K_f \cdot Q \cdot \sqrt{p_A - p_I} = 0 \tag{3.37}$$

where

Q	flow, [volume/time]
C_D	discharge coefficient of the variable orifice
A	discharge area of the variable orifice, [area]
ρ	mass density, [mass/volume]
p_A	upstream pressure, [pressure]
p_I	intermediate pressure, [pressure]
C_{Df}	discharge coefficient of the fixed orifice
A_f	discharge area of the fixed orifice, [area]
p_B	downstream pressure, [pressure]
k_{sp}	spring stiffness, [force/length]
x_{ic}	initial compression of the spring, [length]
x	spool travel, [length]
r	spool radius, [length]
K_f	flow force coefficient, [force·time/volume·pressure^{½}]

In Figure 3.23 some typical curves Q-p_A for a 2-way flow control valve are shown for different p_B values.

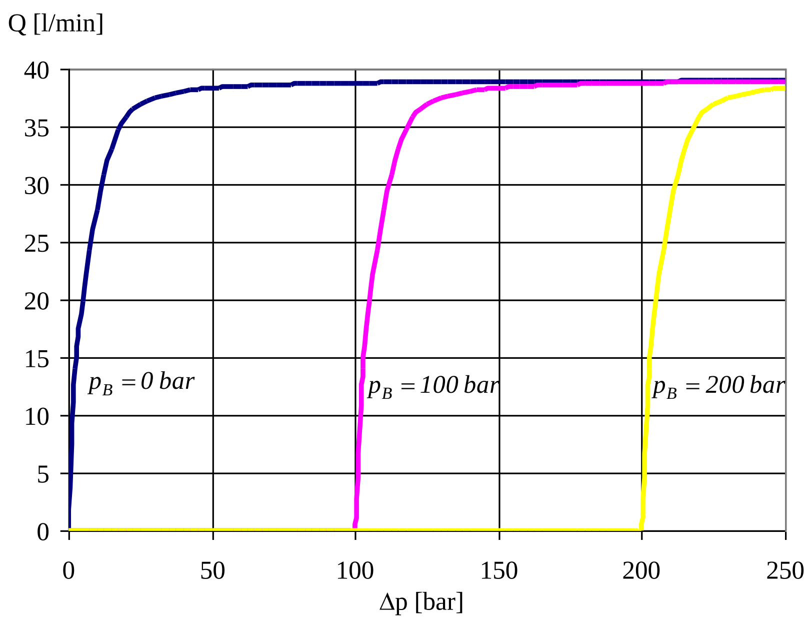


Figure 3.23 Some Q-Δp curves for a 2-way flow control valve set to deliver 40 l/min. The curves differ in load/downstream pressure, p_B.

Despite minor variations due to spring stiffness and flow forces, constant flow (and thereby constant actuator speed) is obtained. Both the area of the so-called fixed orifice as well as the initial compression of the spring can, in general, be adjusted.
It should be noted that the 2-way flow control valve *only works* when it is subjected to *a prescribed input pressure*, p_A, and not a prescribed flow, Q. Hence, it should work together with a pressure relief valve, continuously bleeding of flow and keeping system pressure approximately constant.

3.5.3 3 way flow control valve

The 3-way flow control valve belongs to the group of flow rate controlling valves. Its purpose is to provide a constant flow independent of downstream pressure, i.e., actuator load. Just like the 2-way flow control valve is also referred to as a pressure compensated flow valve.
The pressure independent flow is obtained by means of a differential pressure controller, see Figure 3.24. The differential pressure controller is, essentially, a spool subjected to the pressure before and after the fixed orifice, on each end. The smaller pressure (after the orifice) works together with a spring against the higher pressure (before the orifice). The controller measures the flow via the pressure drop across the fixed orifice. If there is too much flow the pressure drop will increase and the spool will move to the right increasing the opening of a passage from the inlet to the reservoir. Contrary, if there is too little flow the pressure drop will decrease moving the spool in the opposite direction.

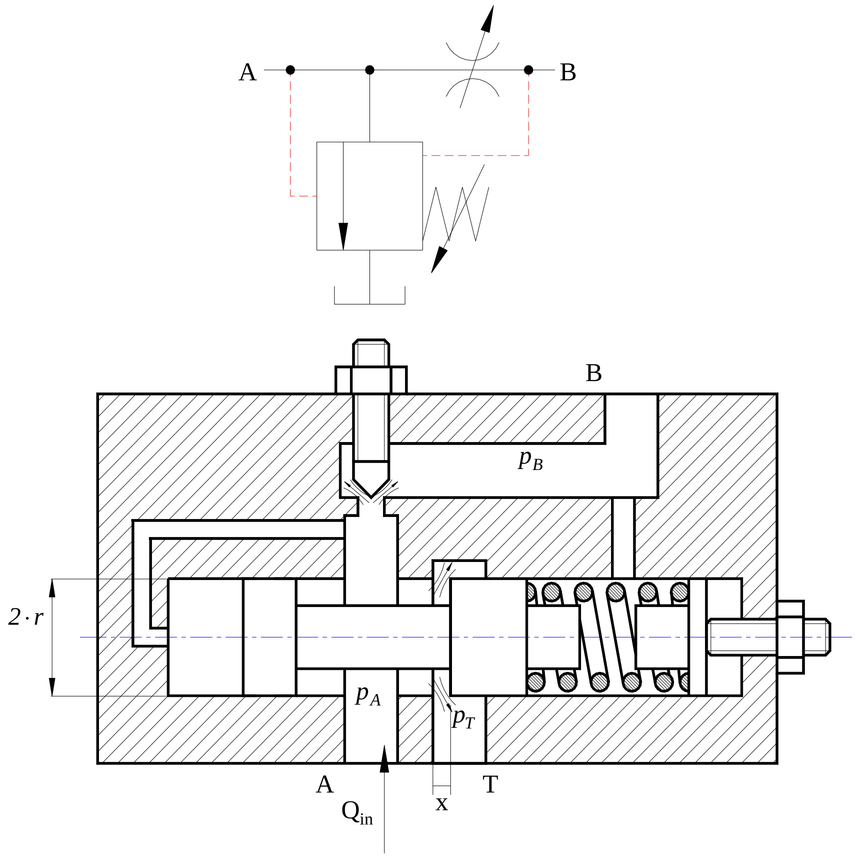


Figure 3.24 Standard symbol and schematic drawing of 3-way flow control valve. The valve is shown working, as it regulates the pressure drop across the fixed orifice bypassing excess flow to tank.

The governing equations for the 3-way flow control valve are:

$$Q = C_{Df} \cdot A_f \cdot \sqrt{\frac{2}{\rho}(p_A - p_B)} \tag{3.38}$$

$$Q_T = C_D \cdot A(x) \cdot \sqrt{\frac{2}{\rho}(p_A - p_T)} \tag{3.39}$$

$$(p_A - p_B)\cdot \pi \cdot r^2 - k_{sp}\cdot (x_{ic} + x) - K_f \cdot Q_T \cdot \sqrt{p_A - p_T} = 0 \tag{3.40}$$

$$Q_{in} = Q + Q_T \tag{3.41}$$

where

Q	flow used by the remaining system (actuators), [volume/time]
C_{Df}	discharge coefficient of the fixed orifice
A_f	discharge area of the fixed orifice, [area]
ρ	mass density, [mass/volume]
p_A	upstream pressure, [pressure]
p_B	downstream pressure, [pressure]
C_D	discharge coefficient of the variable orifice
A	discharge area of the variable orifice, [area]
p_T	reservoir pressure, [pressure]
r	spool radius, [length]
k_{sp}	spring stiffness, [force/length]
x_{ic}	initial compression of the spring, [length]
x	spool travel, [length]
K_f	flow force coefficient, [force·time/volume·pressure^{½}]
Q_{in}	total flow sent into the valve, [volume/time]

A couple of Q-Q_{in} curves for a 3-way flow control valve are shown in Figure 3.25 for 2 different values of p_B. The reservoir pressure, P_T, has been set to zero.

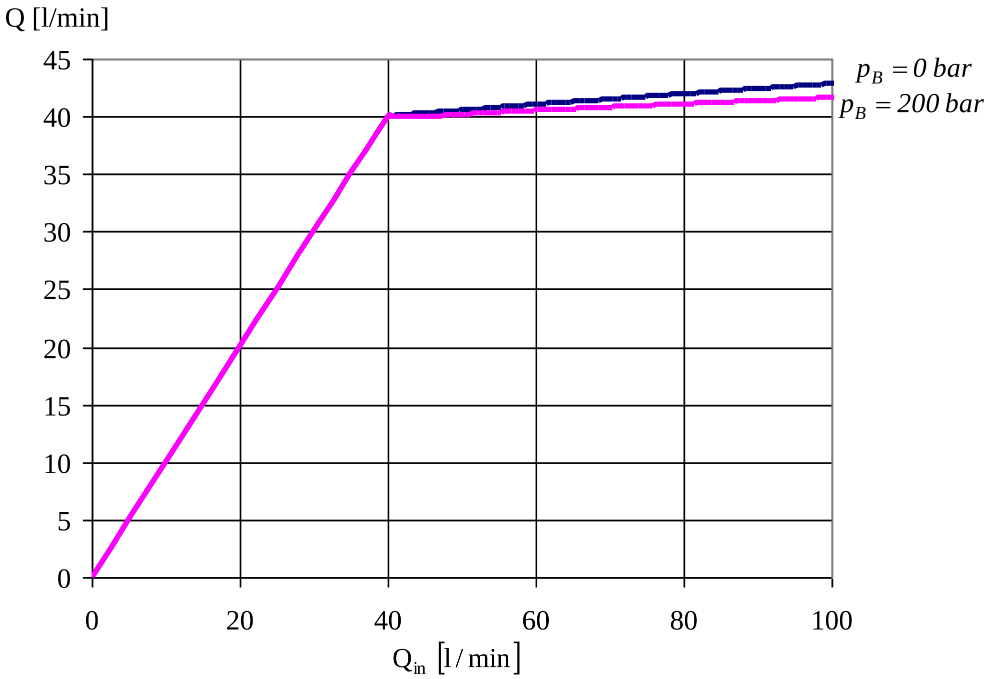


Figure 3.25 *Computed Q-Q_{in} curves for a 3-way flow control valve. The curves differ in load/downstream pressure, p_B*

Despite minor variations due to spring stiffness and flow forces, constant flow (and thereby constant actuator speed) is obtained. Both the area of the so-called fixed orifice as well as the initial compression of the spring can, in general, be adjusted.

If compared to the 2-way flow control valve, it is clear that the bypassed flow to port T (reservoir) is not used to drive any actuator. The 2-way flow control valve utilizes all flow that it receives. However, the 2-way flow controller also throttles the working fluid twice, whereas the 3-way flow controller only throttles the working flow once keeping down the temperature of the fluid.

Another basic difference is the fact that the 3-way flow controller *only works* when subjected to a *prescribed input flow* $Q_{in} = Q + Q_T$ and not if subjected to a prescribed pressure, p_A. Hence, it should work together with a pump delivering constant or controlled flow.

----- oo 0 oo -----

4
Actuator Control

Hydraulic System Design

4.1 Introduction

In general, the flow, the pressure or the power, i.e., pressure times flow, that is directed to each individual actuator may be controlled in a number of ways, depending on the type of system. In the following some classic approaches to speed, pressure and power control of hydraulic actuators is described.

Direct speed control

The simplest way of speed control is obtained by transporting the flow from a constant displacement pump directly to the actuator(s), see Figure 4.1.

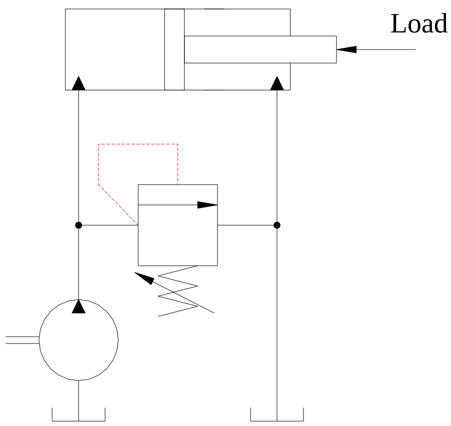


Figure 4.1 Direct speed control of a hydraulic cylinder

A safety valve with crack pressure a certain percentage above the highest load pressure is necessary to avoid pressure overload. There are no power losses associated with this system, i.e., all the power put into the pump is directed to the actuator. Hence, the system is simple = inexpensive and efficient. There are, however, some major drawbacks. Direct control is only applicable to systems with one actuator working at the time and always at the same speed. Also the rotational speed of the primary mover (the motor driving the pump) must be insensitive to moment variations in order for the speed control to be properly pressure compensated/load independent. Finally, using AC-motors with rotational speeds fixed at certain discrete values it may be a bit of a puzzle to find a pump/actuator combination that gives the desired actuator speed.

4.2 Pump Speed Control

In this type of systems the speed control is accomplished by the delivered pump flow rate. In the case where the output actuator is a cylinder or a fixed displacement motor, variation of the speed is only possibly by variation of the flow, thus we must use a variable displacement pump. The higher cost, in comparison to fixed displacement pumps, can to some degree be accepted due to a higher operating efficiencies

4.2.1 Pump pressure control

With a variable displacement pump, the pump pressure may be controlled by adjusting the pump displacement. However, it requires a downstream load that increases with increasing flow and decreases with decreasing flow.
In Figure 4.2 a diagram corresponding to direct pump pressure control is shown including a fixed orifice, Q_L, that represents the flow dependant load.

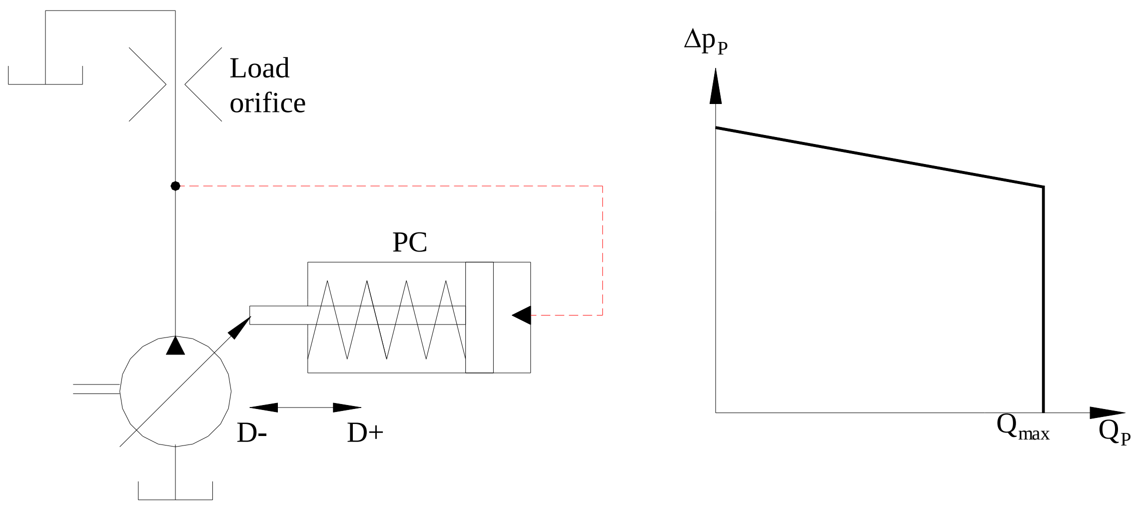


Figure 4.2 Direct pump pressure control applied to a load orifice

Direct pressure control works as follows: If pump pressure, p_P, increases, the piston of the positioning cylinder, PC, will move to the left. This reduces the pump displacement and, subsequently, the pump flow. As the pump flow is reduced, the pump pressure (pressure drop across load orifice) will decrease. Contrary, a decrease in pump pressure will lead to an increase in pump displacement and, subsequently, an adjusting pump pressure increase. The spring of the positioning cylinder is compressed corresponding to the desired pump pressure level.

The simple direct control has no loss associated with it but is not well suited for normal hydraulic applications because the spring of the positioning cylinder needs to be very stiff as it is acting directly against the system pressure. This will lead to a noticeable slope in a Δp_P-Q_P curve, see Figure 4.2 to the right, because the stiff spring is compressed corresponding to the travel required to change the pump displacement.

A much more usual pressure control is the pilot operated pump pressure control, see Figure 4.3.

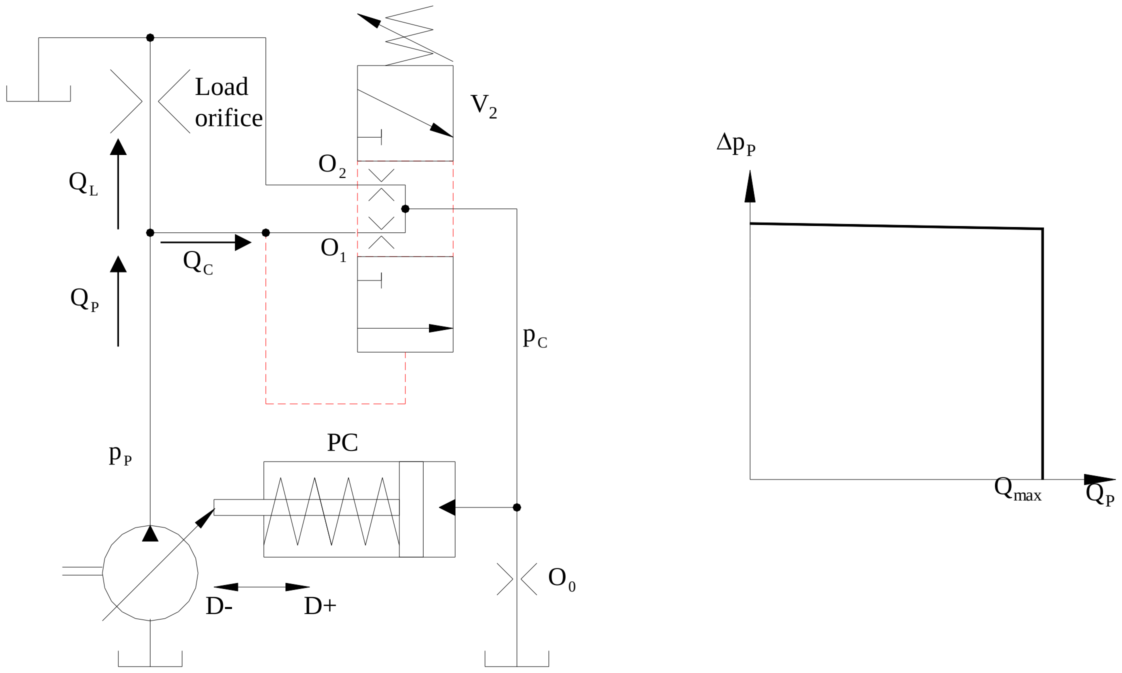


Figure 4.3 Pilot operated pump pressure control applied to a load orifice

It works as follows: When pump pressure increases, the spool of the 3/2-way proportional directional valve, V_2, will move up. This will widen the orifice, O_1, that connects the pump pressure, p_P, and the control pressure, p_C, and narrow the orifice, O_2, which connects the control pressure to the tank reservoir. These variations in orifice areas will increase the control pressure and move the piston of the control cylinder to the left, reducing pump displacement, pump flow and pump pressure. Contrary, a decrease in pump pressure will move the control spool down narrowing O_1 and widening O_2. This will lead to a decrease in control pressure and, subsequently, an increase in pump displacement, pump flow and pump pressure. The spring of the proportional directional control valve is compressed to give the desired pump pressure level. The spring of the positioning cylinder is rather weak, set to work against the control pressure that is reduced by means of the fixed bleed off orifice O_0 .

For the pilot operated pump pressure control, the control pressure can be reduced significantly; hence a much weaker spring can be employed in the positioning cylinder. In comparison, the stiff spring acting on the control valve is only compressed a small fraction, hence the pump pressure is almost constant in a Δp_P-Q_L curve, see Figure 4.3 to the right.

The efficiency of the pilot operated pressure control is:

$$\eta = \frac{P_L}{P_P} = \frac{Q_L \cdot \Delta p_L}{Q_P \cdot \Delta p_P} = \frac{(Q_P - Q_C) \cdot \Delta p_P}{Q_P \cdot \Delta p_P} = 1.0 - \frac{Q_C}{Q_P} \tag{4.1}$$

where

η	efficiency of the pump pressure control
P_L	power delivered to the load, [power]
P_P	power delivered to the pump, [power]
Q_L	flow across the load orifice, [volume/time]
Δp_L	pressure drop across the load orifice = pump pressure, [pressure]
Δp_P	pressure rise across the pump = pump pressure, [pressure]
Q_P	pump flow, [volume/time]
Q_C	control flow, [volume/time]

As indicated by Equation 4.1 the control flow should be minimized, hence the discharge areas of the different orifices, $O_{0..2}$, are kept as small as possible. This also means that the flow through the orifices will tend to be of a more laminar nature. It also means that the control operation may become temperature (viscosity) dependant. Together with the unwanted transition from turbulent to laminar flow also the sensitivity to contamination sets a lower bound on the size of the orifices and thereby the control flow. Typical values for control flow are 2-4 l/min.

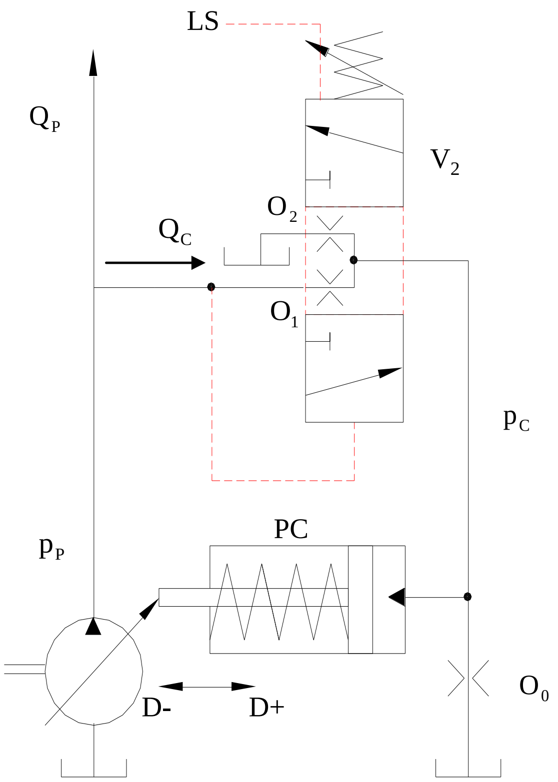


Figure 4.4 Pilot operated pump pressure control based on load sensing signal from actuator(s).

Clearly pump pressure control is not well suited for more than one actuator unless extra valve control is added. Also, it is obvious that if the pump pressure control is applied to a load = downstream pressure that is insensitive to variations in flow then the pump displacement will either be zero (load pressure above set pressure) or maximum (load pressure below set pressure).

A popular version of pump pressure control is so-called load sensing (LS) where the highest pressure level in the actuators of the system is used as reference pressure on the control spool rather than the tank reservoir pressure. The corresponding diagram is shown in Figure 4.4.

Typically, a valve arrangement involving proportional directional control valves are build in, between the pump pressure and the actuators delivering the LS pressure signal. They will act as the load orifice. The spring of the control spool is set to a value ensuring that pump pressure is kept 10-20 bar higher than the LS pressure. The efficiency of such an LS-system is potentially very high.

4.2.2 Pump flow control

With a variable displacement pump the pump flow may be controlled by controlling the pressure drop across a fixed orifice. This can be achieved by means of direct control or pilot operated control. Due to the same conditions as described for pump pressure control, see section 4.3, this is, however, almost exclusively carried out as pilot operated control and only this configuration will be gone through here. The set up may look as shown in Figure 4.5.

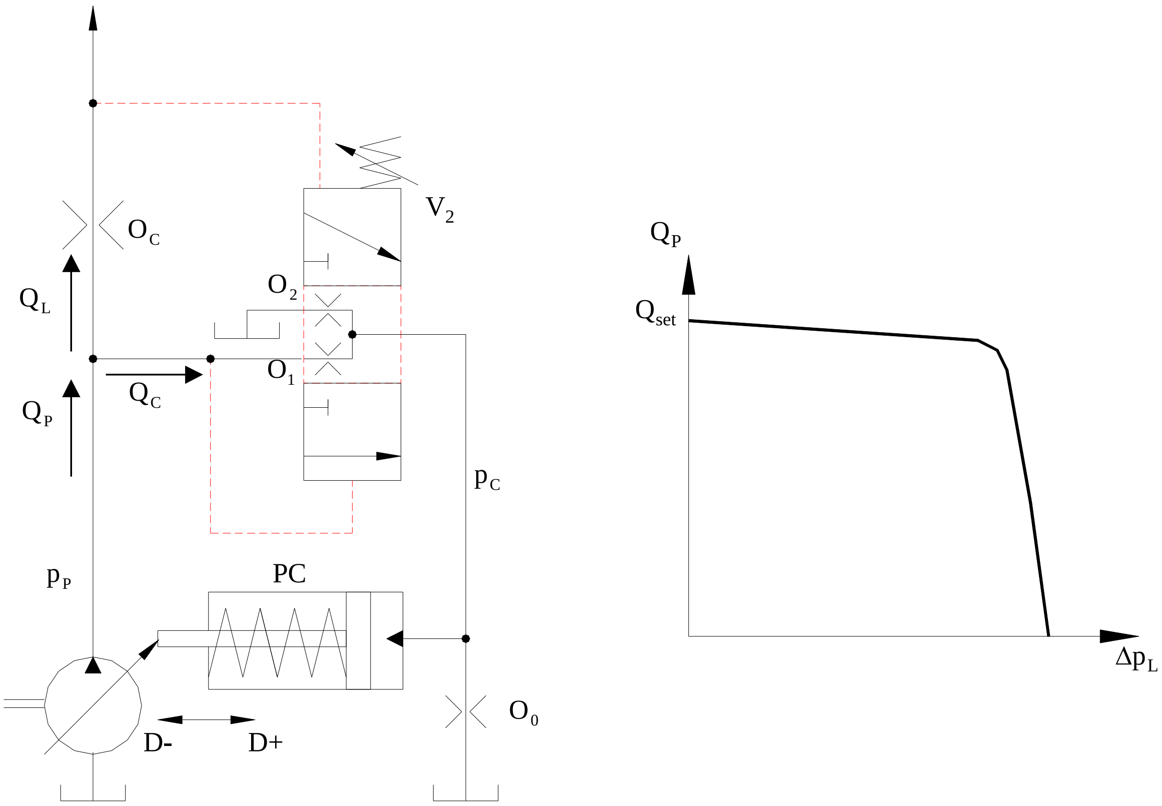


Figure 4.5 Pilot operated pump flow control

It works as follows: If the load flow, Q_L, increases, the pressure drop across the control orifice, O_C, increases. This causes the control spool of the 3/2-way proportional directional control valve, V_2, to move up widening the orifice connecting the pump

pressure and control pressure, O_1, while narrowing the orifice connecting the control pressure to tank, O_2. As a result the control pressure will increase thereby reducing the pump displacement and, thus, readjust the load flow. Contrary if the load flow decrease the control spool will move down, reducing the control pressure. This will cause an increase in pump displacement and thereby a readjustment of the load flow.
For the pilot operated pump flow control, the control pressure can be reduced significantly as compared to the system pressure, hence a relatively weak spring can be employed in the positioning cylinder. As a result the load flow is almost constant in a Q_L-Δp_L curve, see Figure 4.5 to the right. The slope of the curve is caused by increased throttling from pump pressure to control pressure until, finally, the spool generated orifices loose the control ability (the connection between pump pressure and control pressure almost closed off).
The efficiency of the pilot operated flow control is:

$$\eta = \frac{P_L}{P_P} = \frac{Q_L \cdot \Delta p_L}{Q_P \cdot \Delta p_P} = \left(1.0 - \frac{\Delta p_C}{\Delta p_P}\right) \cdot \left(1.0 - \frac{Q_C}{Q_P}\right) \tag{4.2}$$

where

η	efficiency of the pump flow control
P_L	power delivered to the load, [power]
P_P	power delivered to the pump, [power]
Q_L	flow across the load orifice, [volume/time]
Δp_L	pressure drop across the load, [pressure]
Δp_P	pressure rise across the pump = pump pressure, [pressure]
Δp_C	pressure drop across the control orifice, [pressure]
Q_P	pump flow, [volume/time]
Q_C	control flow, [volume/time]

Similar to pump pressure control the control flow should be kept at a minimum. For pump flow control, however, also the pressure drop across the control orifice reduces the efficiency, hence the pressure drop across it should be kept at a minimum without compromising functionality. The lower limit is determined by the minimum spring stiffness of the control spool required in order to avoid instability. Typical values for pressure drops across control orifice are 10-20 bar.

4.2.3 Pump power control

In this strategy, using a variable displacement pump, the pump power, i.e., the power delivered by the pump, is kept constant. There are 2 different ways of obtaining this; an approximate approach and an exact approach.
The approximate approach corresponds in principle to pressure control in its build up, see Figure 4.6.
The difference is the extra spring on the control spool. This rather stiff spring is not activated until the control spool has travelled a certain distance. The effect on the Δp_P-Q_{load} curve is an increase in slope as shown in Figure 4.6 to the right.
In the curve is also shown the exact hyperbola corresponding to constant pump power. By inserting an extra spring and thereby forcing a jump in slope, this hyperbola is approximately met. In some cases up to three springs are used.

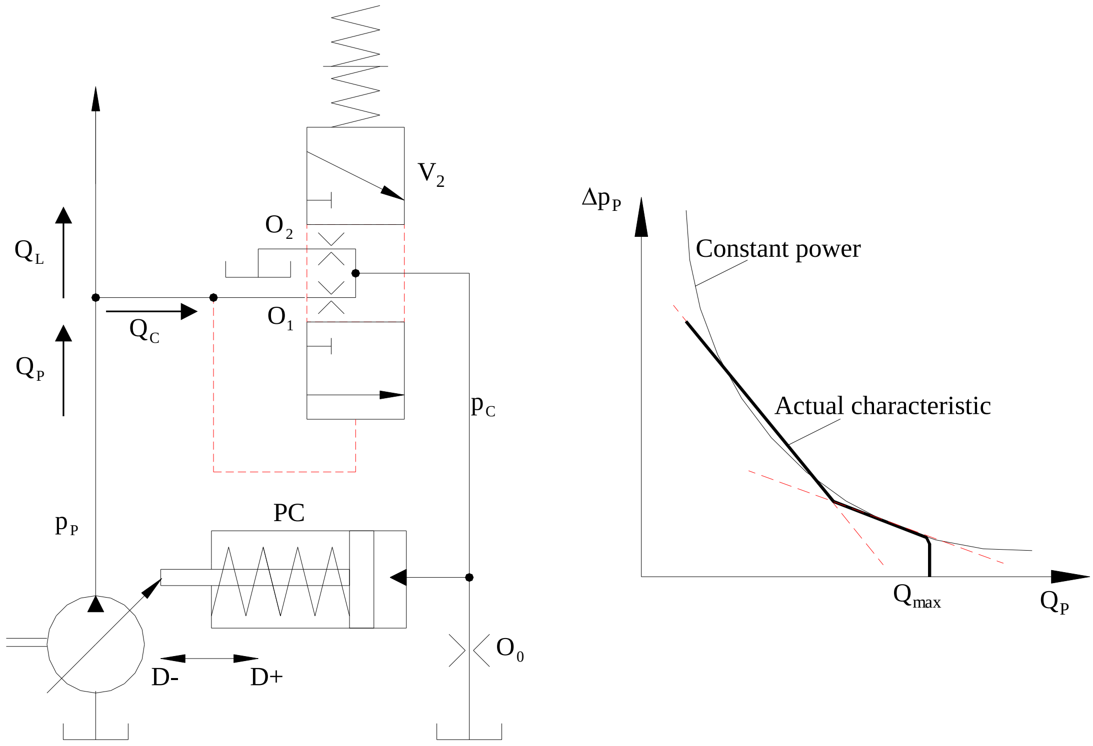


Figure 4.6 Pilot operated pump power control by means of extra spring(s).

Exact power control is achieved if the set up in Figure 4.7 is used. In this case the desired increase in control spool spring stiffness is obtained by means of gearing. As the pump is de-stroked the measuring piston, MP, is moved towards the pivot point of the lever arm, hence, requiring more pump pressure to keep the control spool open. In other words, as pump flow goes down the pump pressure increases in the same ratio, maintaining constant pump power.

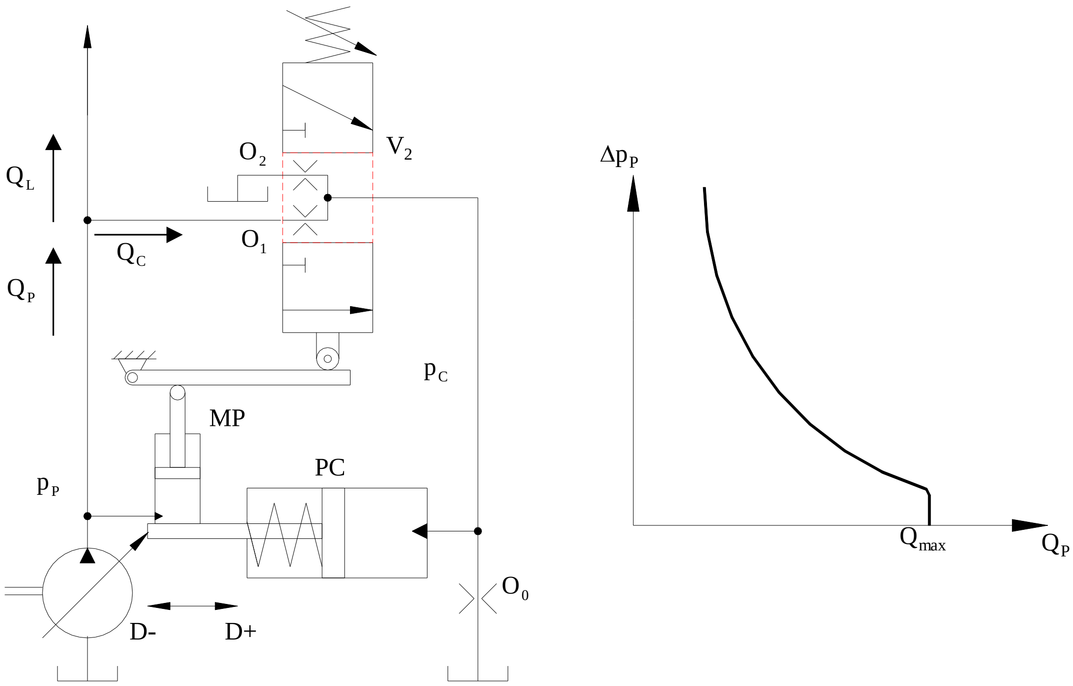


Figure 4.7 Pilot operated pump power control by means of a measuring piston.

The Δp_P-Q_{load} curve which corresponds to the exact hyperbola is shown in Figure 4.7 to the right. The curve is limited by the maximum flow of the pump whereas a separate pressure overload is required.
The expression for the efficiency of a pump power control corresponds to that of a pump pressure control, see Equation 4.1. The difference is that the power control is much better suited for applications with a wide range in actuator speed and load. The cost is a more complicated system.

4.3 Valve Speed Control

Valve flow/speed control of hydraulic systems is based on variation of resistance to flow in the system. This can be achieved by changing the flow resistance in the delivery line or the return line of the actuator or by by-passing the flow not needed for speed control to the reservoir.

4.3.1 Meter-in speed control

Meter-in speed control is obtained by inserting a 2-way flow control valve upstream relative to the actuator, see Figure 4.8.

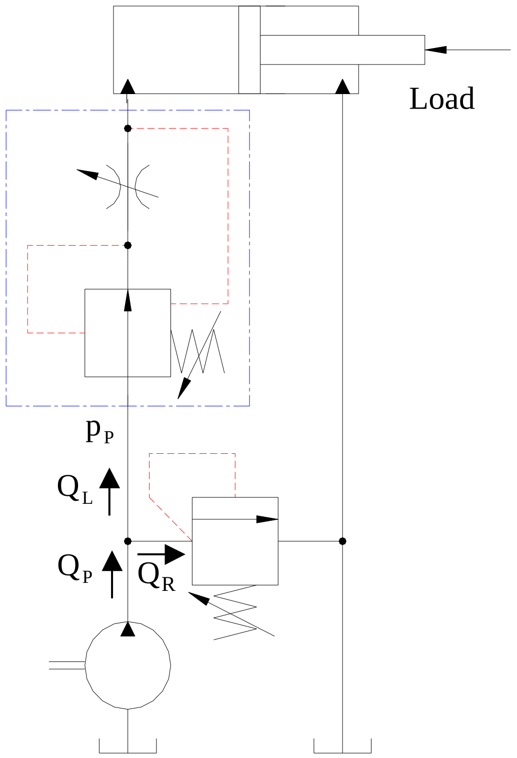


Figure 4.8 Meter in speed control of hydraulic cylinder

A constant displacement pump can be used. A pressure relief valve that continuously keeps the pump pressure at a certain level is required. It by-passes the pump flow above the set value of the 2-way flow control valve. This way the pressure at the inlet of the 2-way flow control valve is held constant as required in order for it to work properly.

The efficiency of the meter in set up may be determined from:

$$\eta = \frac{Q_L \cdot \Delta p_L}{Q_P \cdot \Delta p_P} = \left(1.0 - \frac{Q_R}{Q_P}\right) \cdot \frac{\Delta p_L}{\Delta p_R} \tag{4.3}$$

where

η	efficiency of the meter in speed control
Q_{set}	set flow value of the 2-way flow control valve, [volume/time]
Δp_L	load dependant pressure drop across the actuator, [pressure]
Q_P	pump flow, [volume/time]
Δp_P	pressure rise across the pump, [pressure]
Δp_R	crack pressure of the pressure relief valve = pump pressure, [pressure]
Q_R	flow across the pressure relief valve, [volume/time]

Obviously, the crack pressure setting of the pressure relief valve should be set as close as possible to the highest load pressure. Similarly the constant pump flow should be chosen as close as possible to the actual flow demand, controlled by the 2-way flow control valve. Hence, the overall efficiency will be low for a system with great variation in load pressures. Meter-in is well suited for more actuators, however, a 2-way flow control valve is required for each actuator.

4.3.2 Meter-out speed control

Meter out speed control is obtained by inserting a 2-way flow control valve downstream relative to the actuator, see Figure 4.9.

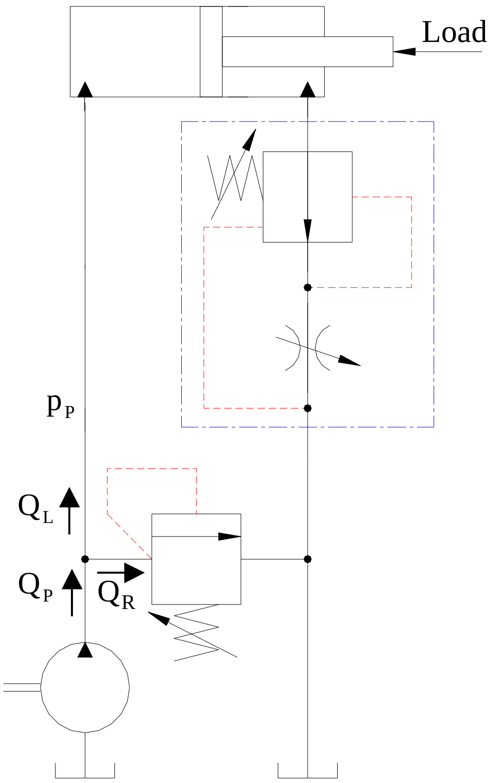


Figure 4.9 Meter out speed control of a hydraulic cylinder

A constant displacement pump can be used. A pressure relief valve that continuously keeps the pump pressure at a certain level is required. It bypasses the pump flow above the set value of the 2-way flow control valve. Considering a motor as actuator or a hydraulic cylinder with area ratio = 1.0, the efficiency of the meter out is:

$$\eta = \frac{Q_L \cdot \Delta p_L}{Q_P \cdot \Delta p_P} = \left(1.0 - \frac{Q_R}{Q_P}\right) \cdot \frac{\Delta p_L}{\Delta p_R} \tag{4.4}$$

where

η	efficiency of the meter out speed control
Q_{set}	set flow value of the 2-way flow control valve, [volume/time]
Δp_L	load dependant pressure drop across the actuator, [pressure]
Q_P	pump flow, [volume/time]
Δp_P	pressure rise across the pump, [pressure]
Δp_R	crack pressure of the pressure relief valve = pump pressure, [pressure]
Q_R	flow across the pressure relief valve, [volume/time]

The same considerations with respect to efficiency and suitability described in the previous section about meter-in hold for meter-out.
A comparison between meter-out speed control and meter-in speed control yields:

- Temperature of fluid in actuator low, because not throttled after leaving pump.
- Pressure level on actuator always maximum.
- Back pressure on hydraulic cylinders with low area ratio very high.
- Negative (runaway) loads may be controlled.

4.3.3 By-pass speed control

By-pass speed control is obtained by inserting a flow control valve parallel to the actuator. In Figure 4.10 this is shown with a 2-way flow control valve.

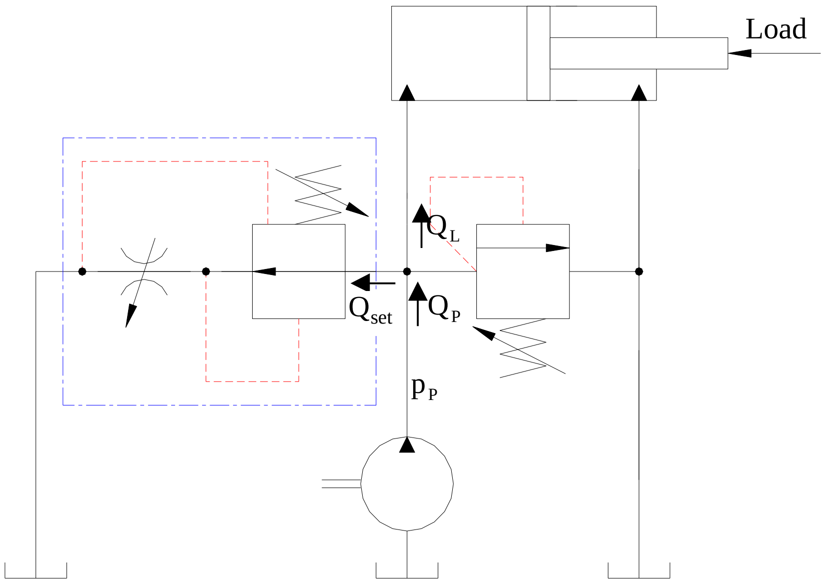


Figure 4.10 By-pass speed control by means of a 2 way flow control valve.

A constant displacement pump can be used. A pressure safety valve is used as guard against pressure overload. It does not work continuously but only when pressure peaks appear. This means that the pump pressure will never go above the actual load pressure. The excess pump flow is bypassed by means of 2-way flow control valve. The efficiency of by pass speed control is:

$$\eta = \frac{Q_L \cdot \Delta p_L}{Q_P \cdot \Delta p_P} = \frac{(Q_P - Q_{set}) \cdot \Delta p_L}{Q_P \cdot \Delta p_P} = 1.0 - \frac{Q_{set}}{Q_P} \tag{4.5}$$

where

η	efficiency of the by pass speed control
Q_L	flow into the actuator, [volume/time]
Δp_L	load dependant pressure drop across the actuator, [pressure]
Q_P	pump flow, [volume/time]
Δp_P	pressure rise across the pump = load pressure, [pressure]
Q_{set}	set flow value of the 2-way flow control valve, [volume/time]

Potentially, by pass speed control has a better efficiency than meter in and meter out, because the pump pressure never rises above the necessary pressure. Hence, the only loss associated with by pass speed control is the flow through the 2-way flow control valve. A comparison between by pass speed control and meter in speed control yields:

- Pump pressure adjusts to the actual load.
- Not suitable for systems with more than 1 actuator.
- Speed control depends on load sensitivity of primary mover.

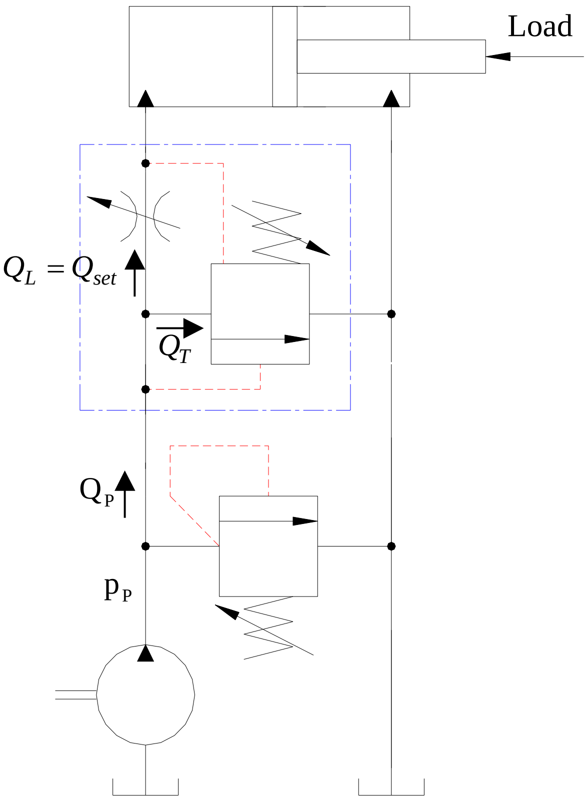


Figure 4.11 By pass speed control by means of a 3-way flow control valve.

Alternatively, by pass speed control may be obtained by directly employing a 3-way flow control valve, see Figure 4.11.
Here the excess flow is bypassed to tank directly by the valve. The only difference from the by pass speed control by means of a 2-way flow control valve is that the load sensitivity of the primary mover has been eliminated at the cost of some minor pressure drop, e.g., 5 bar, across the 3-way flow control valve. The efficiency of this type of by pass speed control becomes:

$$\eta = \frac{Q_L \cdot \Delta p_L}{Q_P \cdot \Delta p_P} = \frac{Q_{set} \cdot (\Delta p_P - \Delta p_C)}{Q_P \cdot \Delta p_P} = \frac{Q_{set}}{Q_P} \cdot \left(1.0 - \frac{\Delta p_C}{\Delta p_P}\right) \tag{4.6}$$

where

η	efficiency of the by pass speed control
Q_L	flow into the actuator, [volume/time]
Δp_L	load dependant pressure drop across the actuator, [pressure]
Q_P	pump flow, [volume/time]
Δp_P	pressure rise across the pump, [pressure]
Δp_C	pressure drop across the 3-way flow control valve, [pressure]
Q_{set}	set flow value of the 3-way flow control valve, [volume/time]

4.3.4 Negative and static load control

A negative load is defined as a load that tries to move an actuator in the same direction as the flow. Negative loads always represent a cavitation threat to the hydraulic system. They typically appear when a load has to be lowered, see Figure 4.12 to the left.

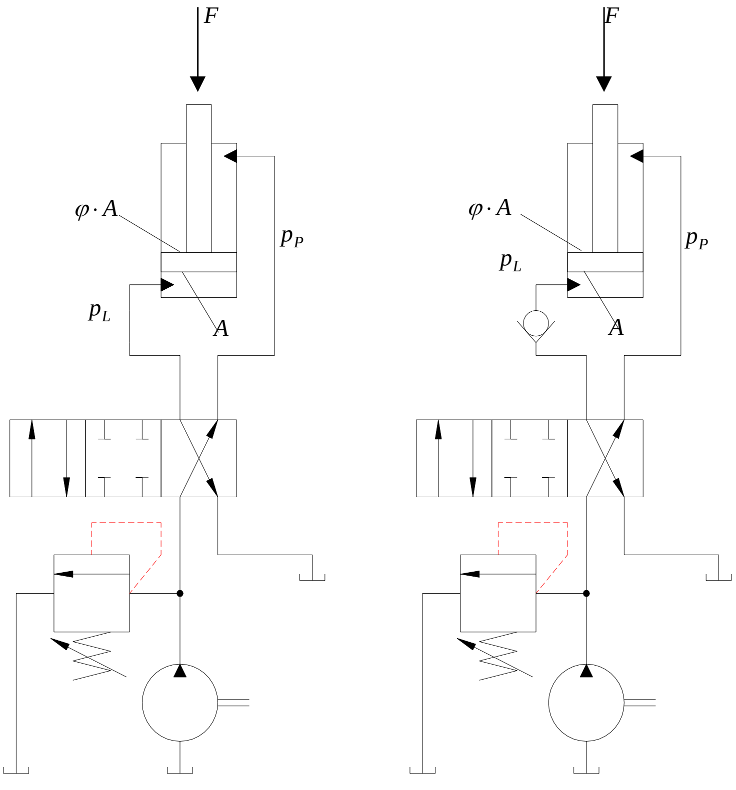


Figure 4.12 Two systems holding a load and trying to lower it.

To the left a system without any load holding capability. To the right a system with perfect load holding capability but without proper functionality.

For the simple system shown above, the only pressure build up below the cylinder piston comes from pushing flow through the tank connection of the directional control valve as well as other restrictions (piping, filter, coolers, etc.) in the flow path before it reaches the tank reservoir. A load holding pressure, p_L, that will maintain equilibrium with the load and the pump pressure, p_P, is necessary. However, if the flow required to build up this pressure is larger than what the pump is capable of delivering (including the flow gearing of the differential cylinder in Figure 4.12) then the pressure line of the pump will cavitate, i.e., in order to obtain equilibrium, p_P will try to become negative. This is impossible and the load runs away.
At the same time, any static load always represents a load drop threat. This may be caused by pipe/hose bursting and in less critical cases (load dropping slowly) by leakage from the pressurized regions to the tank reservoir. The latter is especially a problem when using spool based directional control valves (which is the typical case), as they cannot be made leakage proof.
These 2 problems: Load drop and runaway loads, may be dealt with in several ways. Basically, the load drop due to pipe/hose bursting is dealt with by mounting a seat valve directly on the actuator, see Figure 4.12 to the right. This gives a leak proof load holding capability, however, it is necessary to lift the poppet/ball from the seat when the load is supposed to be lowered. The opening pressure may be picked up in 3 different ways.

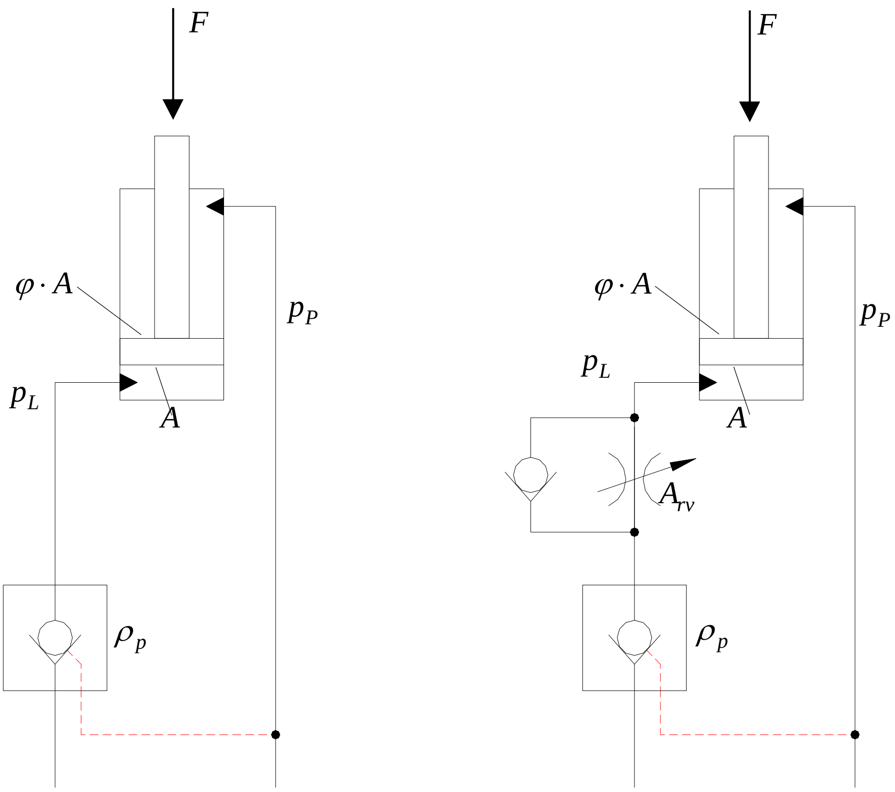


Figure 4.13 To the left is shown part of a system using a pilot operated check valve to lower a load. To the right is shown the same system but with a restrictor-check valve added.

The pilot operated check valve, described in section 3.3, uses p_P as opening pressure, see Figure 4.13 to the left. By increasing the pilot area relative to the seat area, see Figure 3.13, a relatively low pumping pressure (low power consumption) can pilot the check valve open when the load is lowered. The check valve has a relatively weak spring so that it does not offer too much resistance when the load is raised. Ignoring its contribution, the necessary pump pressure to pilot open the valve may be determined from:

$$p_P = \frac{p_L}{\rho_P} = \frac{1}{\rho_p}\left(\frac{F}{A} + p_P \cdot \varphi\right) \Rightarrow p_P = \frac{1}{\rho_p - \varphi} \cdot \frac{F}{A} \tag{4.7}$$

where

p_P	pump pressure required to pilot open the valve, [pressure]
p_L	pressure on the piston side of the cylinder, [pressure]
ρ_p	ratio of the pilot area to the seat area, $(r_x/r)^2$, see Figure 3.13
F	load, [force]
A	piston area of the cylinder, [area]
φ	cylinder area ratio

The weak spring does, however, also mean that the pilot operated check valve cannot work as a metering (pressure build up) unit to control a runaway load. It would be much too unstable as minor pressure fluctuations would cause consistently closing and opening of the valve. Hence, a pilot operated check valve only solves the load holding problem. Therefore a pilot operated check valve will normally be mounted in series with a restrictor valve, see Figure 4.13 to the right. The restrictor valve is adjusted to offer the flow restriction necessary to build up a sufficient load holding pressure, p_L. At the same time it will have a dampening effect on piston speed and pressure fluctuations in the system, and hence improve stability. If the pressure drop across the pilot operated check valve is disregarded and it is assumed that it is piloted fully open, i.e., the pilot piston is resting against a mechanical stop, then the governing equations for the system shown in Figure 4.13 are:

$$p_L = \frac{F}{A} + \varphi \cdot p_P \tag{4.8}$$

$$\frac{Q_P}{\varphi} = C_D \cdot A_{rv} \cdot \sqrt{\frac{2}{\rho} \cdot p_L} \tag{4.9}$$

where

p_L	pressure on the piston side of the cylinder, [pressure]
F	load, [force]
A	piston area of the cylinder, [area]
φ	cylinder area ratio
p_P	pump pressure required to pilot open the valve, [pressure]
Q_P	pump flow, [volume/time]
C_D	discharge coefficient of the restrictor valve
A_{rv}	discharge area of the restrictor valve, [area]
ρ	mass density of the fluid, [mass/volume]

Combining Equation (4.8) and (4.9) yields an expression for the correct setting of the discharge area of the restrictor valve for a given pump flow, a desired pump pressure and a maximum load, F_{max}:

$$A_{rv} = \frac{Q_P}{\varphi \cdot C_D \cdot \sqrt{\frac{2}{\rho} \cdot \left(\frac{F_{max}}{A} + \varphi \cdot p_P \right)}} \tag{4.10}$$

The so-called counter balance valve, see Figure 4.14 to the left, which principally corresponds to the pressure relief valve discussed in section 3.3 uses p_L as opening pressure.

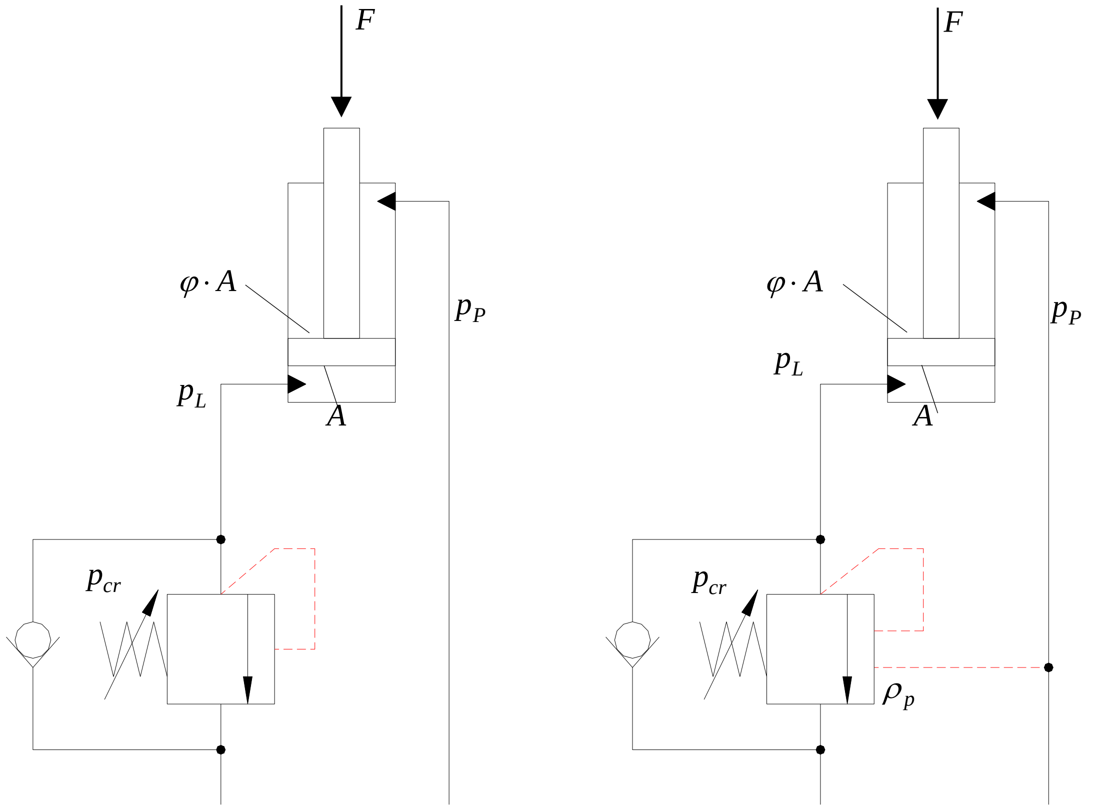


Figure 4.14 To the left is shown part of a system lowering a load by means of a counterbalance valve. To the right is shown a system lowering a load by means of an over centre valve. Both systems have a check valve in parallel with the load holding valve.

The crack pressure is set a certain percentage above what is required to maintain equilibrium with the load. This way both the load holding problem as well as the load runaway problem is taken care of. A separate check valve that allow by pass of the flow when lifting the load is necessary.

Neither the pilot operated check valve nor the counter balance valve are well suited for varying loads because they are set to handle the maximum load, thereby causing very high pumping pressures when lowering smaller loads. For systems with greatly varying loads a so-called over centre valve may be used. It is a combination of the pilot operated check valve and the counter balance valve as it uses both p_P and p_L as opening pressure, see Figure 4.14 to the right. The pump pressure will typically act on an area 3-10 times larger than the area acted upon by the load holding pressure. The ratio between the area acted upon by the pump pressure and the area acted upon by the load holding pressure is referred to as the pilot area ratio. Ignoring the pressure in the return line the governing equations for the system shown in Figure 4.14 to the right are:

$$p_L = \frac{F}{A} + \varphi \cdot p_P \tag{4.11}$$

$$p_{cr} = p_L + \rho_p \cdot p_P \tag{4.12}$$

where

p_L	pressure on the piston side of the cylinder, [pressure]
F	load, [force]
A	piston area of the cylinder, [area]
φ	cylinder area ratio
p_P	pump pressure required to pilot open the valve, [pressure]
p_{cr}	crack pressure of the over centre valve, [pressure]
ρ_p	pilot area ratio of the over centre valve

The crack pressure is defined as the load holding pressure required to open the over centre valve without the aid of any pump pressure. The crack pressure is adjusted by adjusting the initial compression of the spring. Combining Equation (4.11) and (4.12) yields an expression for the correct setting of the crack pressure for a given pump flow, a desired pump pressure and a maximum load, F_{max}:

$$p_{cr} = \frac{F_{max}}{A} + (\rho_p + \varphi) \cdot p_P \tag{4.13}$$

In general, the effect of using both p_L and p_P as opening pressures tends to stabilize the system. However, too high pilot area ratios will cause instabilities even for over centre valves. The effect of changes in the load is strongly reduced, see Figure 4.15.

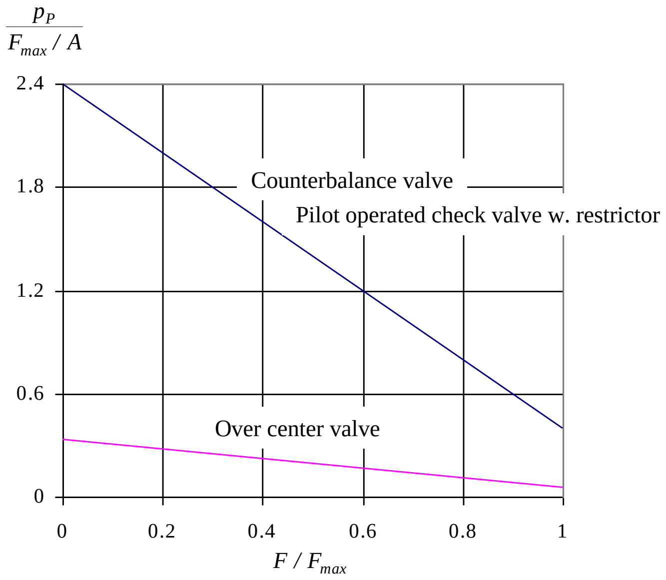


Figure 4.15 The pump pressure dependency on the load for the different types of load holding valves

4.3.5 Braking

In general, an actuator is brought to a stand still by moving a directional control valve into neutral, see Figure 4.16.

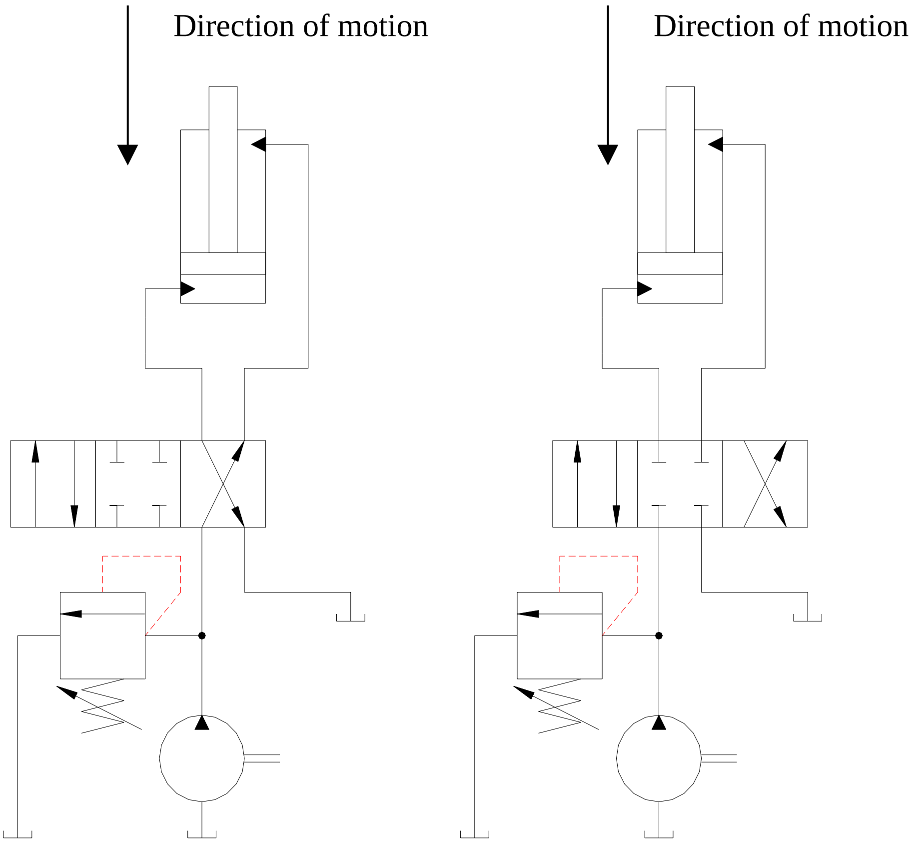


Figure 4.16 Simple hydraulic system shown lowering a load and breaking.

When the valve goes to neutral the load will, due to inertia, continue its motion. This will lead to a compression of the fluid in branch A and a decompression of the fluid in branch B. Hence p_L increases and p_P decreases. For large inertia loads and systems with insufficient damping this may easily cause severe problems, both with respect to overloading in branch A due to pressure peaks and cavitation in branch B. These problems are typically handled by inserting a shock valve and a suction valve, see Figure 4.17.

The shock is simply a pressure safety valve, see section 3.3, dimensioned to a rather small flow. It is set a certain percentage above the maximum expected static load pressure. The suction valve is a check valve, see section 3.2, with a very weak spring, so that the back pressure can crack it open and refill the branch in danger of cavitating. If the back pressure/tank reservoir pressure is not high enough, an extra check valve may be inserted in the return line with a somewhat stiffer spring, ensuring sufficient suction pressure.

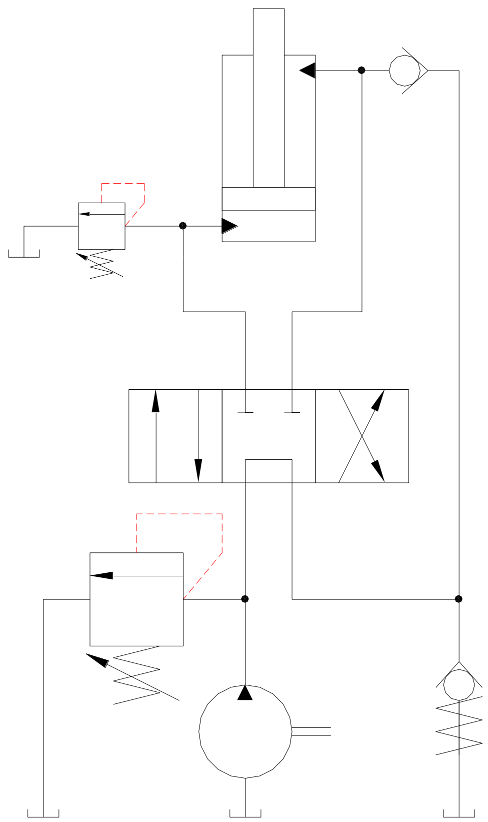


Figure 4.17 Simple system shown breaking a load. The system is equipped with a shock valve, a suction valve and a check valve in the return line to maintain a certain back pressure.

4.4 Hydrostatic Transmissions

A classical hydrostatic transmission is a hydraulic system consisting of a positive displacement pump and a positive displacement motor. There may exist more complex hydrostatic transmissions containing several pumps and/or several motors, however, the purpose remains the same, namely to transmit the rotational mechanical input power to rotational mechanical output power. The main advantages as compared to a mechanical transmission is the ease with which a variable gearing may be introduced. This may be done either via a variable displacement pump or a variable displacement motor or a combination of these. In Figure 4.18 is shown two basic types of hydrostatic transmission (open and closed) both with a variable displacement pump and a fixed displacement motor.

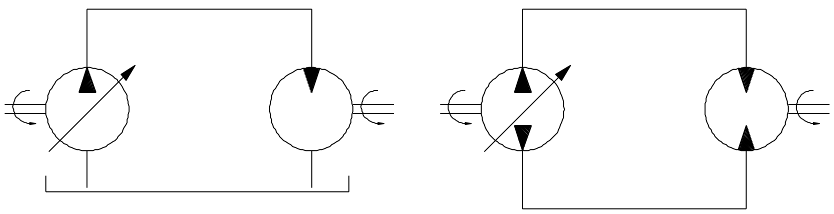


Figure 4.18 Hydrostatic transmissions; open (left) and closed (right).

The closed circuit is used for applications with frequent reversals of the motor speed. For such systems it is preferrable to change the amount as well as direction of flow smoothly via the pump avoiding an expensive and complicated valve control system. Closed circuit hydrostatic transmissions are widely used for propelling small and medium sized skid-steered vehicles employed in agriculture or on building sites where the demand on maneouvreability is so high that the relative poor efficiency of a hydrostatic transmission can be neglected. Open circuit hydrostatic transmissions requires a directional control valve in order to reverse the speed of the motor. It is primarily used in applications with a high demands on torque to volume ratio on the actuator.
In practice, any hydrostatic transmission requires a number of extra valves to ensure proper functionality during negative loads and braking, see also sections 4.3.4 and 4.3.5. For a closed hydrostatic transmission used for vehicle propelling the system shown in Figure 4.19 can be considered as a minimum.

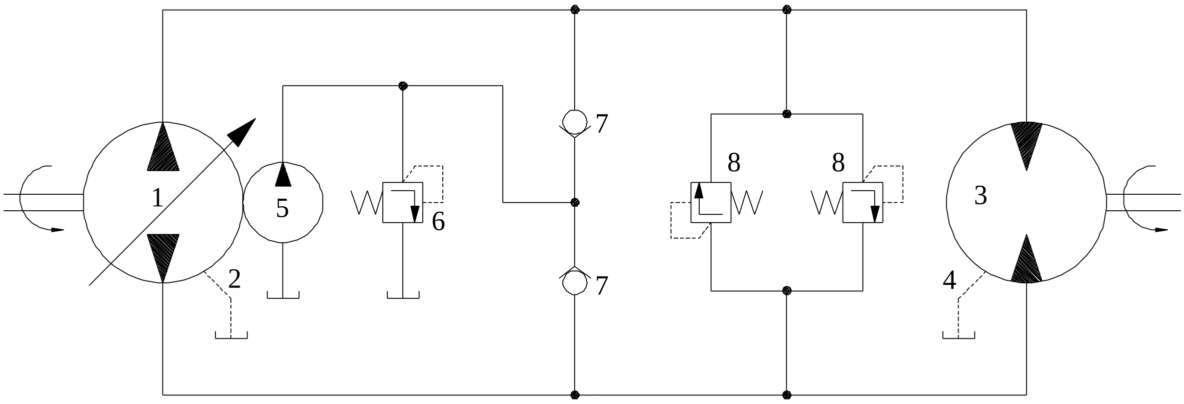


Figure 4.19 Closed hydrostatic transmissions with additional valves.

In any pump or motor there will be internal leakage from the high pressure side to the housing of the component. In order to reduce the pressure level and thereby the requirements to the sealings there must be a drain connection to a reservoir. For a closed circuit this must be added as a separate line and both the pump (1) and the motor (3) has each their drain connection to tank, (2) and (4), respectively. Depending on the physical layout these drain connections may be joined before reaching the reservoir. Because of the drainage the closed circuit needs continuous refilling. Therefore, a small circulation pump (5) is introduced that continuously pumps fluid from a reservoir into the closed system. It is mechanically connected to the pump and maintains a certain minimum pressure (for example 15 bar) by means of the pressure relief valve (6). Whenever the pressure level in either branch of the closed circuit falls below the minimum pressure the branch is refilled via the check valves (7). A major concern for closed circuits is the lack of cooling because the main pump does not pull fluid from a reservoir, i.e., the same fluid is constantly recirculated. This problem is also greatly reduced by the small circulation pump, since it continuously refills the closed circuit with fluid that has been cooled down in the reservoir. When the motor is accelerated or braked by highly dynamic external loading one branch will cavitate and the other will experience high pressure peaks. To avoid this, two shock valves (8) are added. Together with the check valves (7) they correspond to the suction-shock valve arrangement of the cylinder actuator in Figure 4.17.

As mentioned, the overall efficiency is the major drawback of a hydrostatic transmission. In Figure 4.20 a closed hydrostatic transmission where both pump and motor has variable displacement is shown.

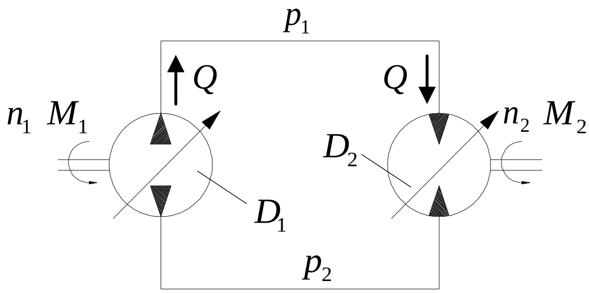


Figure 4.20 Closed hydrostatic transmissions with variable displacement pump and motor.

The displacement of the pump and motor are given as:

$$\begin{aligned} D_1 &= \alpha_1 \cdot D_{1,\max} \\ D_2 &= \alpha_2 \cdot D_{2,\max} \end{aligned} \tag{4.14}$$

Theoretically, we get the following output speed:

$$\left.\begin{aligned} Q = n_1 \cdot D_1 = n_1 \cdot \alpha_1 \cdot D_{1,\max} \\ Q = n_2 \cdot D_2 = n_2 \cdot \alpha_2 \cdot D_{2,\max} \end{aligned}\right\} \Rightarrow n_2 = \frac{\alpha_1 \cdot D_{1,\max}}{\alpha_2 \cdot D_{2,\max}} \cdot n_1 \tag{4.15}$$

Thus, introducing the gearing, i, we get:

$$i = \frac{\alpha_2 \cdot D_{2,\max}}{\alpha_1 \cdot D_{1,\max}} \tag{4.16}$$

$$n_2 = \frac{n_1}{i} \tag{4.17}$$

From Equation (4.16) it is clear that the displacement of a pump may change sign, i.e., $-1 \le \alpha_1 \le 1$ with $\alpha_1 = 0$ simply causing the motor to stand still no matter the pump speed. However, a motor with variable displacement will normally will have some safety measure that ensures: $\alpha_{2,\min} \le \alpha_2 \le 1$, where $\alpha_{2,\min}$ is some positive value, since $\alpha_2 = 0$ would give infinite motor speed for any finite flow.
The theoretical torque is:

$$\left.\begin{aligned} M_1 = \frac{D_1 \cdot \Delta p}{2 \cdot \pi} = \frac{\alpha_1 \cdot D_{1,\max} \cdot (p_1 - p_2)}{2 \cdot \pi} \\ M_2 = \frac{D_2 \cdot \Delta p}{2 \cdot \pi} = \frac{\alpha_2 \cdot D_{2,\max} \cdot (p_1 - p_2)}{2 \cdot \pi} \end{aligned}\right\} \Rightarrow M_2 = \frac{\alpha_2 \cdot D_{2,\max}}{\alpha_1 \cdot D_{1,\max}} \cdot M_1 = i \cdot M_1 \tag{4.18}$$

It is not a practical problem that $i = \infty$ for $\alpha_1 = 0$ since the output torque is determined by the external load. Hence $\alpha_1 = 0$ simply implies that theoretically no pump torque is needed to drive the motor when the pump displacement is zero.

Disregarding the losses the input power and output power balances:

$$P_2 = 2 \cdot \pi \cdot n_2 \cdot M_2 = 2 \cdot \pi \cdot \frac{n_1}{i} \cdot i \cdot M_1 = 2 \cdot \pi \cdot n_1 \cdot M_1$$
$$\Downarrow \tag{4.19}$$
$$P_2 = P_1$$

Taking into account the losses we get:

$$\left.\begin{aligned} Q &= \eta_{v1} \cdot n_1 \cdot D_1 \\ Q &= \frac{1}{\eta_{v2}} \cdot n_2 \cdot D_2 = \end{aligned}\right\} \Rightarrow n_2 = \eta_{v1} \cdot \eta_{v2} \cdot \frac{n_1}{i} \tag{4.20}$$

$$\left.\begin{aligned} M_1 &= \frac{1}{\eta_{hm1}} \cdot \frac{D_1 \cdot \Delta p}{2 \cdot \pi} \\ M_2 &= \eta_{hm2} \cdot \frac{D_2 \cdot \Delta p}{2 \cdot \pi} \end{aligned}\right\} \Rightarrow M_2 = \eta_{hm1} \cdot \eta_{hm2} \cdot i \cdot M_1 \tag{4.21}$$

$$P_2 = 2 \cdot \pi \cdot n_2 \cdot M_2 = 2 \cdot \pi \cdot \eta_{v1} \cdot \eta_{v2} \cdot n_1 \cdot \eta_{hm1} \cdot \eta_{hm2} \cdot M_1$$
$$\Downarrow \tag{4.22}$$
$$P_2 = \eta_{v1} \cdot \eta_{v2} \cdot \eta_{hm1} \cdot \eta_{hm2} \cdot P_1$$

In a real system other losses would include the pressure drops in the lines of the two branches as well as the continuous pressure supply to the circulation pump.

4.5 Accumulators

An accumulator is an energy reservoir that can be inserted into a hydraulic system. It can enhance the performance of the system in a number of ways:

- emergency power supply
- supporting power supply for short durations of high speed demand
- reduction of steady state pressure variations
- reduction of dynamic pressure peaks

4.5.1 Accumulator types

The main purpose of an accumulator is to store energy. In fluid power systems experience has shown that this is most easily down by using gas as the storing media because of its high compressibility. In practice, the gas must be separated from the fluid and this can be done in several ways. The two most popular types of accumulators are characterized by their separating mechanism:

- piston accumulator
- bladder accumulator

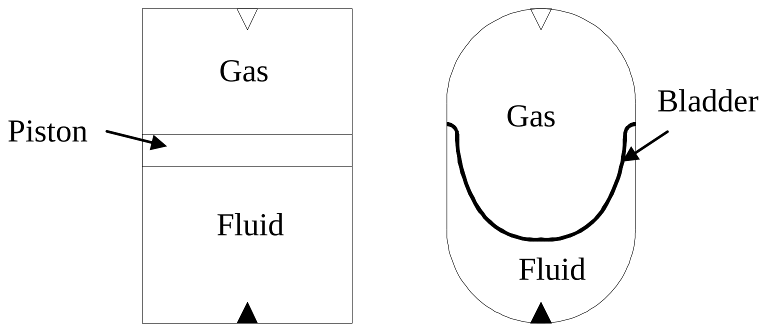


Figure 4.21 Piston accumulator (left) and bladder accumulator (right).

The two types of accumulators are illustrated in Figure 4.21. In general, the piston accumulator is used whenever high pressure and high safety are required. The drawbacks as compared to the bladder accumulator are the necessary sealing between the piston and the cylinder as well as the low response time due to the inertia of the piston.
Because the main purpose of an accumulator is to be loaded with high pressure and, simultaneously, releases its energy within short time intervals it is important to have it connected to the remaining hydraulic system in a safe way. This is typically done by means of a valve block that as a minimum has a system isolator valve, a manually actuated unloading valve and a pressure relief valve, see Figure 4.22.

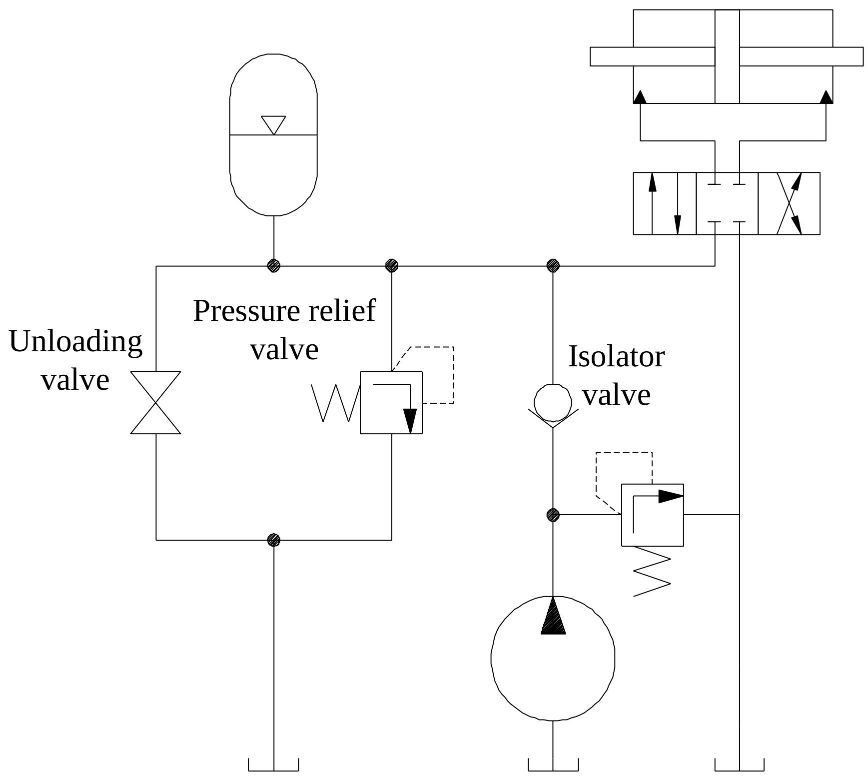


Figure 4.22 Accumulator shown built into a hydraulic system. In this system the functionality of the accumulator is to maintain the supply pressure as constant as possible.

The isolator valve protects the pump from accumulator back flow, the unloading valve allows for the accumulator to be de-energized after shutdown and the pressure relief valve protects the accumulator.

4.5.2 Governing equations

The total volume of the accumulator is fixed. It is the sum of the gas and fluid volume:

$$V_a = V_g + V_f = cst \tag{4.23}$$

When the accumulator is in its initial state (preloaded) the gas takes up all the volume, see Figure 4.23.

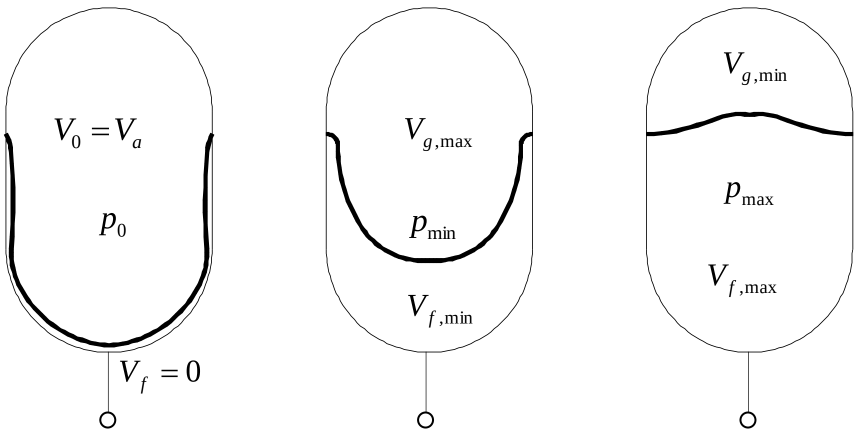


Figure 4.23 Accumulator shown in three states: initial=preloaded, loaded with minimum working pressure and loaded with maximum working pressure.

The accumulator is loaded by pressurizing the fluid whereby the gas volume is reduced. The accumulator is typically unloaded by connecting it to a part of the hydraulic system that has a lower pressure. The pressure of the gas follows that of the fluid (force equilibrium across the separator) during loading and unloading and the gas can be considered as undergoing a polytropic proces:

$$p \cdot V_g^n = cst \tag{4.24}$$

The polytropic exponent varies with the proces. If no experience or measurements are available approximation must be made regarding the proces. One way is to choose the proces that yields the most conservative design, however, it is preferrably to get more precise data from the supplier. Nitrogen gas is by far the most used gas and the polytropic exponent for Nitrogen varies strongly both with temperature, pressure and duration of proces. It is also different depending on whether the accumulator is being loaded or unloaded. If no data is available it is suggested to use the following rough approximation for the polytropic exponent corresponding to an adiabatic and an isothermal proces, respectively:

$$n = \begin{cases} 1.4 & t_p < 10\ s \\ 1.0 & t_p > 300\ s \end{cases} \tag{4.25}$$

In general, it should be avoided (especially for bladder accumulators) that the pressure falls below the preload pressure, p_0. As a rule of thumb the preload pressure should be 90% of the minimum working pressure:

$$p_0 = 0.9 \cdot p_{\min} \tag{4.26}$$

When an accumulator is used for the earlier mentioned purposes it is normally used in the following way: initially, it is loaded isothermally with the minimum pressure. Next, it is loaded to maximum pressure and at a certain time it is unloaded towards an actuator. Alternatively, it is loaded isothermally with an intermediate pressure and then passive against a prescribed inlet flow. In both cases it is normally given how much volume of fluid that the accumulator should accumulate/discharge. Also, the acceptable limits on the working pressure would typically be specified as $p_{\max}$ and $p_{\min}$. The maximum pressure, $p_{\max}$, is a design parameter that often is set by means of a pressure relief valve or simply by a maximum acceptable pressure level. The desired variation in volume, ΔV, is normally given by the requirements on the actuator motion. Knowing these values it is possible to compute the maximum and minimum gas volume, see Figure 4.23:

$$p_{\max} \cdot V_{g,\min}^{n_L} = p_{\min} \cdot V_{g,\max}^{n_L} \Rightarrow V_{g,\max} = \left\{ \frac{p_{\max}}{p_{\min}} \right\}^{\left(\frac{1}{n_L}\right)} \cdot V_{g,\min} \tag{4.27}$$

In Equation (4.27) n_L is the polytropic exponent corresponding to the loading and/or unloading proces. Introducing the known volume change yields:

$$\begin{gathered} \Delta V = V_{g,\max} - V_{g,\min} = \left[\left\{ \frac{p_{\max}}{p_{\min}} \right\}^{\left(\frac{1}{n_L}\right)} - 1 \right] \cdot V_{g,\min} \\ \Downarrow \\ V_{g,\min} = \frac{1}{\left[\left\{ \frac{p_{\max}}{p_{\min}} \right\}^{\left(\frac{1}{n_L}\right)} - 1 \right]} \cdot \Delta V \qquad V_{g,\max} = \frac{1}{\left[1 - \left\{ \frac{p_{\min}}{p_{\max}} \right\}^{\left(\frac{1}{n_L}\right)} \right]} \cdot \Delta V \end{gathered} \tag{4.28}$$

Since, the initialization should be considered isothermal the total volume of the accumulator may be determined as:

$$p_0 \cdot V_0 = p_{\min} \cdot V_{g,\max} \Rightarrow V_0 = V_a = \frac{p_{\min}}{p_0} \cdot V_{g,\max} \tag{4.29}$$

Hence, the data necessary to specify an accumulator, namely the total volume and corresponding preload pressure may be determined from Equations (4.26) and (4.29), respectively.

---- oo 0 oo ----

5
Synthesis of Hydraulic Systems

Hydraulic System Design

5.1 Introduction

The designer rarely needs to design the individual components. His task is to conceive a strategy for solving the problem – of cause appropriate to the particular application, then to represent this strategy on a circuit diagram, and at last to select the components for his system from a wide range of commercial stock.
This chapter will outline a basic approach for arriving at a hydraulic system capable of performing a specific task – giving a stepwise approach to synthesis of hydraulic systems. Hence, the chapter will address how to combine and size some of the basic components of a hydraulic system: Pumps, valves, actuators, lines, filters, coolers and tank reservoir.

5.2 Steady state approach

When designing a hydraulic system the following steps should be addressed, see also: "Design and Steady-state Analysis of Hydraulic Control Systems" by Jacek S. Stecki et al., ISBN 83-86219-94-7:

1. Define required operating cycle for entire system.
2. Define the required operating cycle for each sub function.
3. Decide on system concept:
4. Setup a hydraulic diagram with the necessary components.
5. Decide on pressure level.

6. Select and size the actuators.
7. Select and size the primary mover and pump(s).
8. Select the type of hydraulic fluid.
9. Select size of hydraulic lines.
10. Select control elements.
11. Determine overall efficiency of system.
12. Select and size tank reservoir and/or cooling elements.
13. Select suitable filtering.
14. Do a system analysis and determine reliability.
15. Prescribe procedures for system assembly, monitoring, operation and component replacing.
16. Estimate costs.

“The procedure outlined requires use of static or equilibrium relationships and component manufactures’ data. There can be no certainty as to the actual dynamic response and performance of the proposed system.”

In the following steps 1 - 13 will be described in different levels of detail.

5.2.1 Define operating cycles

Fundamental to any kind of model based dimensioning of both hydraulic and mechanical systems is the definition of the relevant situations in which the system is expected to operate. In this text a time sequence of working situations are referred to as an operating cycle. All operating cycles that influence the requirements for the system should be defined in details. The number and complexity of operating cycles for a system may vary significantly from simple (e.g. a motor driven fan) to advanced (e.g. a passenger car) but in any case a number of basic advantages are obtained when specifying the operating cycles adequately. Some of the pronounced advantages are:

1. The necessary background for a model based approach to the sizing of both the mechanical and the hydraulic system becomes available.
2. Together with proper documentation traceability in the development of any given system is obtained.
3. Optimization and/or improvement of existing designs is simplified.
4. The amount of experimental work and field-tests is reduced.

A given operating cycle may be defined at several detail levels depending on how precisely the mechanical system topology has been defined. If all the degrees of freedom of the mechanical system are well defined then the operating cycle should address the active or passive actuation of each independent degree of freedom. Further, the exterior loading and disturbance (e.g. payload, press force, impact loading, driving conditions, inertial loads etc.) should be described as time series. Also, such parameters as temperature, humidity and level of contamination should be considered together with any parameter that might influence the operation of the system and its components.

It is important to accept that the defined/chosen operating cycles not necessarily describe all possible working situations. The important thing is that they represent an appropriate set of operational demands to the system. Especially within mobile hydraulics it is impossible to foresee all kind of behavior by the operator-in-the-loop and it is a futile excercise to try and do it.

If the principle layout of the mechanical design is determined then the operating cycles should be formulated at a detail level that makes it possible to derive the corresponding power requirements of each actuator. Hence, for a hydraulically actuated system it should be possible to set up the required force and speed for each cylinder and the required torque and angular velocity for each motor, see Figure 5.1..2.

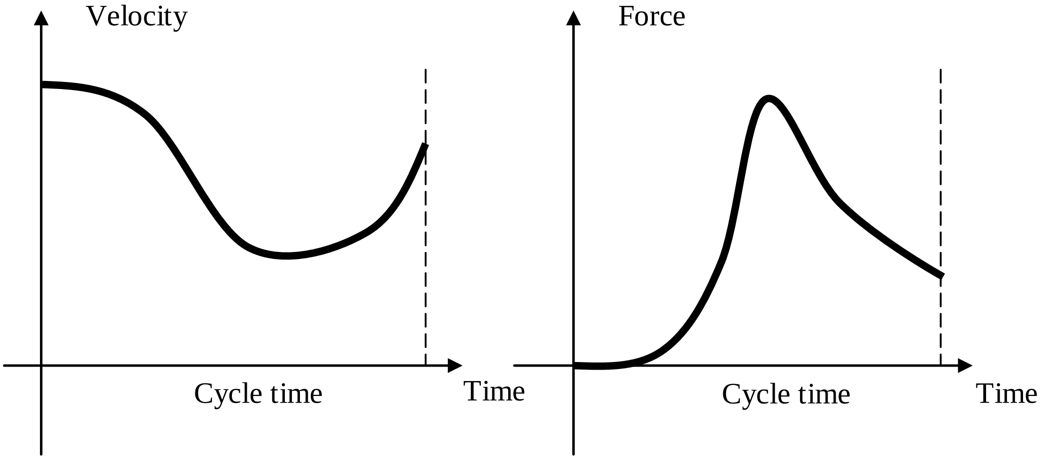


Figure 5.1 Requirements to a hydraulic cylinder across an operating cycle.

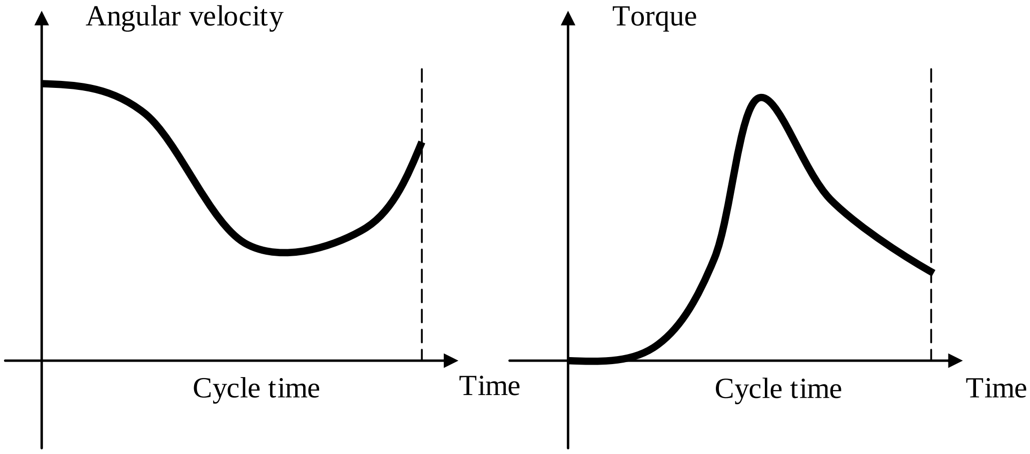


Figure 5.2 Requirements to a hydraulic motor across an operating cycle.

Steps 1 and 2 should return curves as the ones shown in the above figures.

5.2.2 Hydraulic system concept

Steps 3 and 4 correspond to finding a concept for the hydraulic actuation. Basically, this means determining how the fluid power should be generated and directed to the different actuators. At this stage it should be determined:

- Number of pumps
- For each pump how should it be driven and controlled
- For each actuator how should it be controlled

In hydraulics it is customary to use fewer pumps than actuators (often only one pump). This is due to costs because the pump is one of the most expensive components in any hydraulic system. More pumps can be justified if for instance two simple inexpensive pumps can replace one complicated and expensive one. Otherwise, adding pumps are in general done with a view to improve the efficiency.

When there are more actuators than pumps it is, in general, necessary to insert valves to direct and control the fluid power to each actuator. The basic conceptual building blocks are the types of pump and actuator control described in Chapter 4 together with the

basic valves described in Chapter 3. As an example, a pressure controlled pump may be used in conjunction with flow controlled actuators if the speed of the mechanical system should be controlled. Alternatively, if the force supplied by the actuator is the main concern then pressure controlling valves should be utilized.
The proper combination of pumps, valves and control methods cannot be chosen without doing some sizing, i.e., the correct topology of the hydraulic system must be determined iteratively and should include computation of the dimensions of the main components. An important tool during these iterative steps is the standardized approach to depicting hydraulic systems as so-called CETOP-diagrams. These diagrams are of a conceptual nature and are well suited both during the design phase but also as final documentation. However, a CETOP-diagram alone is not enough to properly describe the operation of a hydraulic system and it should always be accompanied by some kind of written description of the intended behavior of the system during operation.

5.2.3 Pressure level

The pressure level is typically chosen to lie within 150-250 bar. If pressure levels within this interval results in very small flow demands, e.g., less than 3-5 l/min or very large flow demands, e.g., > 1000 l/min then pressure level should probably be reduced or increased, respectively. Alternatively, the entire system concept might need to be reevaluated.

5.2.4 Actuator sizing

If the actuator is a motor, the displacement may be determined from:

$$D > \frac{2 \cdot \pi \cdot M_{max}}{p} \tag{5.1}$$

where

D	displacement of the motor, [volume/revolution]
M_{max}	maximum load torque on the motor, [moment]
p	chosen pressure level, [pressure]

The greater than sign allows for an estimated hydro-mechanical efficiency of the motor and pressure drops within the system to be taken into account at this stage.
If the actuator is a cylinder, the piston area may be determined from:

$$A > \frac{F_{max}}{p} \tag{5.2}$$

where

A	piston area of the cylinder, [area]
F_{max}	maximum load on the cylinder, [force]
p	chosen pressure level, [pressure]

The greater than sign allows for an estimated hydro-mechanical efficiency of the cylinder, back pressure on the annulus area as well as pressure drops within the system to be taken into account at this stage.

5.2.5 Primary mover and pump sizing

The necessary pump flow is determined based on the situation during the system operating cycle, where the total flow demand is at its maximum:

$$Q_{max} = \left(\sum_{i}^{motors} D_i \cdot n_i + \sum_{i}^{cylinders} A_i \cdot v_i \right)_{max} \qquad \textbf{(5.3)}$$

where

Q_{max}	maximum required theoretical flow, [volume/time]
D_i	displacement of the i'th motor, [volume/revolution]
n_i	required rotational speed of the i'th motor, [revolution/time]
A_i	piston area (or by reversed flow the annulus area) of the i'th cylinder, [area]
v_i	required speed of the i'th cylinder, [speed]

The power demand to the primary mover may, relatively simple, be determined from the maximum required theoretical flow and the pressure level:

$$P_{max} > Q_{max} \cdot p \qquad \textbf{(5.4)}$$

where

P_{max}	maximum required power from the primary mover, [power]
Q_{max}	maximum required theoretical flow, [volume/time]
p	pressure level, [pressure]

The greater than sign allows for an estiamte of the total efficiency of the pump and the volumetric efficiency of the actuators to be taken into account at this stage.
It should bet noted that in mobile hydraulics the primary mover is not a free choice, since it is the engine of the vehicle. Otherwise, the primary mover is chosen among stationary combustion engines or electric (AC and DC) motors.
Knowing the rotational speed of the primary mover, the pump displacement may be determined:

$$D > \frac{Q_{max}}{n} \qquad \textbf{(5.5)}$$

where

D	pump displacement, [volume/revolution]
Q_{max}	theoretical maximum flow demand, [volume/time]
n	rotational speed of the primary mover, [revolutions/time]

Here the greatet than sign accounts for an estimated volumetric efficiencies of pump and actuators. The rotational speed of the primary mover, *n*, will depend on the torque required by the pump. However, this torque is a function of the pump displacement.

$$M > \frac{p \cdot D}{2 \cdot \pi} \qquad \textbf{(5.6)}$$

where

M	output torque from the primary mover, [moment]

p	chosen pressure level, [pressure]
D	pump displacement, [volume/revolution]

The greater than sign covers an estimated hydro-mechanical efficiency of the pump at this stage. Clearly Equation (5.5) and (5.6) depend on each other. Hence, if the dependency between torque and rotational speed is relatively complex for the primary mover, then the pump displacement needs to be determined iteratively.

5.2.6 Choose hydraulic fluid

Hydraulic fluids are gone through in detail elsewhere. Its main purpose is to transfer the hydraulic power, lubricate the moving parts and protect against corrosion. The dominant type of hydraulic fluid is mineral oil. They are replaced by water-based fluids in applications with fire hazards, and by environmental friendly fluids (biologically degradable) when special attention to the surroundings must be addressed.
The mineral oils are categorized according to DIN 51524 into: H, HL, HLP, HV and HLPD. Except for type H that is rarely used the others are supplied with additives that improve

- corrosion protection, HL, HLP, HV and HLPD
- oxidation protection, HL, HLP, HV and HLPD
- wear reduction, HLP, HV and HLPD
- viscosity independency on temperature, HV and HLPD
- ability to self-clean and resistance to dissolve water, HLPD

The main parameter of a hydraulic fluid is the viscosity and its dependency on temperature. It is necessary that the viscosity of the fluid at start and during operation stay within the acceptable limits of the components of the system. Hence, it is necessary to know the temperature of the surroundings as well as the overall efficiency of the system to get an exact estimate of the possible viscosity range of the fluid. This should be compared with the acceptable viscosity range for each component of the system. However, these computations can only be performed when the entire system sizing has been carried out. As an initial approximation it may be expected that the operating temperature may lie in the range 40-60 °C.

5.2.7 Select line dimensions

The hydraulics lines consist of pipes and hoses. First of all, the pipe and hose dimensions must be chosen so that their allowable pressure is above the pressure level of the system. Also, the dimensions should be chosen so a suitable fluid velocity is obtained. The following values are suggested:

- Suction lines (from tank reservoir to pump) : *0.5 – 2.0 m/s*
- Delivery line (from pump to actuators): *3.0 – 10 m/s*
- Return line (from actuators to tank reservoir): *1.0 – 3.0 m/s*

Especially, in the suction line is it essential to avoid any pressure drops so that the suction pressure of the pump may be adequate for it to work optimally. Both in the delivery as well as the return line there is a trade off between strength (smaller diameters needs less thickness to withstand same pressure) and low velocity (larger diameters).

The pressure drop in any hydraulic line depends on several parameters. Firstly, the type of flow, laminar or turbulent, has to be established. This is done based on the dimensionless Reynolds number:

$$Re = \frac{4 \cdot v \cdot A}{\upsilon \cdot O} \tag{5.7}$$

where

v	mean fluid velocity, [length/time]
A	cross sectional area of the flow path, [area]
υ	kinematic viscosity, [area/time]
O	circumference of the flow area, [length]

Hence, for a circular line (pipes and hoses), the Reynolds number is:

$$Re = \frac{4 \cdot Q}{\upsilon \cdot \pi \cdot d} \tag{5.8}$$

where

Q	flow in the pipe, [volume/time]
υ	kinematic viscosity, [area/time]
d	inner diameter of the line, [length]

At low Reynolds numbers the flow is laminar, whereas it becomes turbulent rather abruptly around a critical value of approximately 2300.
The pressure loss due to flow in a line is given by:

$$\Delta p = \frac{\lambda \cdot L \cdot \rho \cdot v^2}{2 \cdot d} \tag{5.9}$$

where

λ is a dimensionless resistance number
L is the length of the line, [length]
ρ is the mass density, [mass/volume]
v is the mean fluid velocity, [length/time]
d is the inner diameter of the line, [length]

The resistance number can be found when the Reynolds number and the type of flow have been established. For laminar flow it is:

$$\lambda = \frac{64}{Re} \tag{5.10}$$

For turbulent flow it is:

$$\lambda = \frac{0.3164}{Re^{0.25}} \tag{5.11}$$

Equation (5.8) $\rightarrow$ (5.11) is the necessary tools to determine the pressure losses in the hydraulic lines of the system. Knowing the approximate mechanical system as well as the position of the actuators, it is possible to reasonably estimate the different line lengths.

Another pressure loss source in the hydraulic lines is associated with any bends or branching of the flow path. As a rough estimate on the pressure drop around a bend the following formula may be employed:

$$\Delta p = \frac{\xi \cdot \rho \cdot v^2}{2} \cdot \frac{\beta}{90} \tag{5.12}$$

where

ξ	dimensionless pressure drop coefficient number
ρ	mass density, [mass/volume]
v	mean fluid velocity, [length/time]
d	inner diameter of the line, [length]
β	angle of the bend, see Figure 5.1, [degrees]

The pressure drop coefficient depends on the ratio between the bend radius and the inner diameter, see Figure 5.3, and may roughly be determined

$$\xi = \frac{0.3}{\left(\frac{r_b}{d}\right)^{0.6}} \tag{5.13}$$

where

ξ	dimensionless pressure drop coefficient number
r_b	radius of the bend, [length]
d	inner diameter of the line, [length]

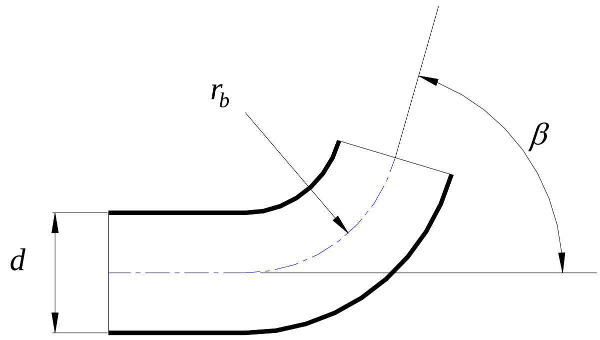


Figure 5.3 Illustration of the variables associated with the pressure drop across a bend in a hydraulic line.

5.2.8 Select control elements

The necessary control elements are determined from the design concept. The valves may either be mounted on a plate or screwed into a housing as cartridge valves. In general, it is a good idea to keep the valves close together and to use ports with the same dimensions. I.e., if the delivery line is, e.g. ½", then the valves should, preferably have ½" connections. It is important to know the demands from the valves to contamination (filtering) and viscosity (fluid and fluid temperature).

5.2.9 Determine overall efficiency of system

The overall efficiency of the system is determined from:

$$\eta = \frac{P_{out}}{P_{in}} = \frac{\sum_{i}^{motors} 2 \cdot \pi \cdot M_i \cdot n_i + \sum_{i}^{cylinders} F_i \cdot v_i}{M_{PM} \cdot n_{PM}} \tag{5.14}$$

where

η	efficiency of the system at a certain time
P_{out}	power delivered by the system, [power]
P_{in}	power put into the system, [power]
M_i	output torque delivered by the i'th motor, [moment]
n_i	rotational speed of the i'th motor, [revolutions/time]
F_i	output force delivered by the i'th cylinder, [force]
v_i	speed of the i'th cylinder, [length/time]
M_{PM}	output torque from the primary mover, [moment]
n_{PM}	rotational speed of the primary mover, [revolutions/time]

Normally, the efficiency will vary during the operating cycle of the entire system. Hence the overall efficiency of the system should be determined as:

$$\eta_{tot} = \int_{t=0}^{t_c} \frac{P_{out}}{P_{in}} \cdot dt \tag{5.15}$$

where

η_{tot}	overall efficiency of the system
P_{out}	power delivered by the system, [power]
P_{in}	power put into the system, [power]
t_c	duration of the operating cycle for the entire system, [time]

5.2.10 Tank and cooler sizing

The losses associated with the efficiency of the system will always lead to heating of the fluid. In general, all the power losses in the system are used to generate heat in the fluid. Assuming that the temperature of the fluid is uniform within the entire system and that the only heat exchange with the environment is via the surface area of the tank reservoir then a heat balance may be set up for a steady state situation:

$$\Phi_s = \Phi_e \tag{5.16}$$

where

Φ_s	heat flux generated by the system, [power]
Φ_e	heat flux transmitted to the surroundings/environment, [power]

The average heat flux generated by the system may be determined as:

$$\Phi_s = P_{in} - P_{out} = (1 - \eta_{tot}) \cdot 2 \cdot \pi \cdot M_{PM} \cdot n_{PM} \tag{5.17}$$

The heat flux transmitted to the environment from the tank reservoir may be determined as:

$$\Phi_e = k_c \cdot A_r \cdot \Delta T \quad \textbf{(5.18)}$$

where

Φ_e	heat flux transmitted to the surroundings/environment, [power]
k_c	heat transmission coefficient for the tank, [power/(area·degree)]
A_r	area of the cooling surface of the tank reservoir, [area]
ΔT	temperature difference between the hydraulic fluid and the surroundings, [degree]

Typical values for the heat transmission coefficient are: 10..15 W/(m^2·K) for steel reservoirs and 6..9 W/(m^2·K) for cast iron reservoirs. Combining Equation (5.17) and (5.18) leads to an expression for the necessary surface area of the tank reservoir based on an acceptable fluid temperature:

$$A_r = \frac{\Phi_s}{k_c \cdot \Delta T} \quad \textbf{(5.19)}$$

As a rule of thumb the tank reservoir should be approximately 3..5 times the pump flow. In cases where the requirements to the reservoir surface area becomes too large, heat exchangers are widely used. They are either water or air driven, and their power consumption capability is easily determined once a tank reservoir has been chosen:

$$\Phi_{he} = \Phi_s - k_c \cdot A_r \cdot \Delta T \quad \textbf{(5.20)}$$

where

Φ_{he}	required cooling capability of the heat exchanger, [power]

If steady state is not reached for the temperature during an operating cycle then Equation (5.19) may be considered somewhat conservative and the more general equation for the development of the fluid temperature as a function of time may be employed:

$$T(t) = \left[\frac{\Phi_s}{k_c \cdot A_r} + T_S\right] + \left[T_0 - \frac{\Phi_s}{k_c \cdot A_r} - T_S\right] \cdot e^{-\frac{k_c \cdot A_r}{M_{fl} \cdot c_{fl}} \cdot t} \quad \textbf{(5.21)}$$

where

t	is the time, [time]
M_{fl}	is the mass of the fluid, [mass]
c_{fl}	is the specific heat of the fluid, [energy/(mass·degree)]
T_s	is the temperature of the surroundings, [degree]
T_0	is the initial temperature of the fluid, [degree]

In (5.21) it is assumed that the power loss is constant in time.

5.2.11 Filtering

In order for the hydraulic system to function properly over a longer period proper filtering is crucial. Filters may, basically, be inserted in suction lines (protect pump),

delivery lines (protect valves and actuators) and return lines (remove picked up contamination). Return line filtering is most common as the demands to and hence cost of a return line filter is smaller than that of a delivery line (high pressure) filter. Filters in the suction line protect the expensive pump but might cause cavitation if the pressure drop across it becomes too large. Filters typically come with a by pass check valve that allow free flow if the filter is blocked.
The efficiency of the filter is expressed by means of a β value. It is defined as follows:

$$\beta_x = \frac{n_{in}}{n_{out}} \tag{5.22}$$

Where

β_x	efficiency w.r.t. to particles of the size x µm
n_{in}	number of particles of the size x µm pr. *100 ml* entering the filter
n_{out}	number of particles of the size x µm pr *100 ml* leaving the filter

Hence a filter with $\beta_{10} = 75$ will pick up *98.7%* of *10 µm* particles trying to pass through it. In general, the component information will include requirements to fluid cleanliness and recommendations to the necessary filtering.
Fluid cleanliness is classified according to ISO 4406 and consists of 2 numbers that define the allowable number of *5 µm* and *15 µm* particles, respectively, in a *100 ml* sample.

5.3 Dynamic design considerations

The design approach described in Section 5.2 is well suited for systems with a predominantly static behavior. However, many hydraulically actuated systems have a strongly dynamic nature and this needs to be taken into account. The hydraulic system influences the dynamic performance of the entire system and, simultaneously, the dynamic performance of the entire system influences the choice of hydraulic components. The most important type of hydraulically actuated systems where dynamic design considerations have to be made are, in general, referred to as servo systems.

5.3.1 Servo systems

A servo system is a tracking control system that receives a reference signal, measures its own output and controls the output to be in accordance with the reference. For a hydraulic servo system the output to be controlled is typically a position, speed or force and, accordingly, the servo system is labelled either a position servo, speed servo or a force servo. In Figure 5.4 a typical valve controlled hydraulic servo system is shown with its basic components. A typical hydraulic servo consists of:

- Constant pressure source
- Directional control valve
- Hydraulic lines between valve and actuator
- Actuator
- Mechanical system - payload

- Output sensor
- Servo amplifier

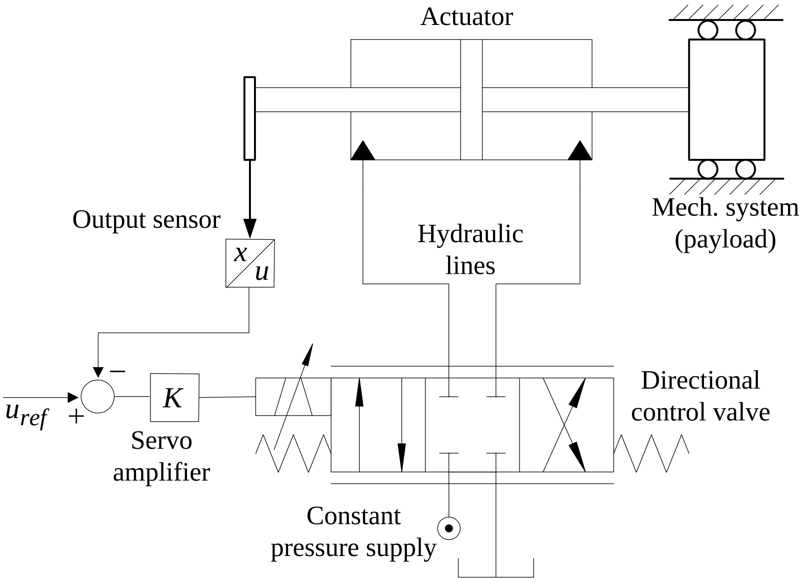


Figure 5.4 Hydraulic servo system with linear actuator.

In order to reduce disturbances and non-linearities in the system the supply pressure is normally held at a constant value by means of accumulators together with either variable displacement pumps or pressure control valves. This is done at the expense of efficiency which can become quite small depending on the characteristics of the working cycles.

The valve is a high-end directional control valve and it is characterized by a high bandwidth (low response time), internal control loop that positions the spool according to an electrical input signal and with finely machined tolerances that ensures smooth transition in performance when the valve is operating around the neutral position. Directional control valves for servo applications are referred to as either servo valves (servo applications) or proportional valves (spool position is propotional to input signal). The distinction between the two types is not well defined but there is a tendency to refer to valves depending on the type of actuation used to move the spool:

- Single-stage - the spool is directly actuated by means of an electrical linear force motor. The inner control loop is realised via electrical position feedback.
- Two-stage - the spool is hydraulically actuated by means of an electrically actuated pilot stage. The inner control loop is realised via mechanical position feedback.
- Three-stage - the main spool is hydraulically actuated by means of a two stage valve. The inner control loop of the main spool is realised via electrical position feedback.

The two- and three-stage control valves have higher bandwidth but also requires extra filtering (built in), requires a pilot flow, costs more and their response time depends on the level of the supply pressure. The three-stage control valve is the most expensive and is only used for high power applications where flow and pressure demands cannot be met by a two-stage valve. The valves are also, classically, referred to according to the layout of the spool around neutral position, see Figure 5.5.

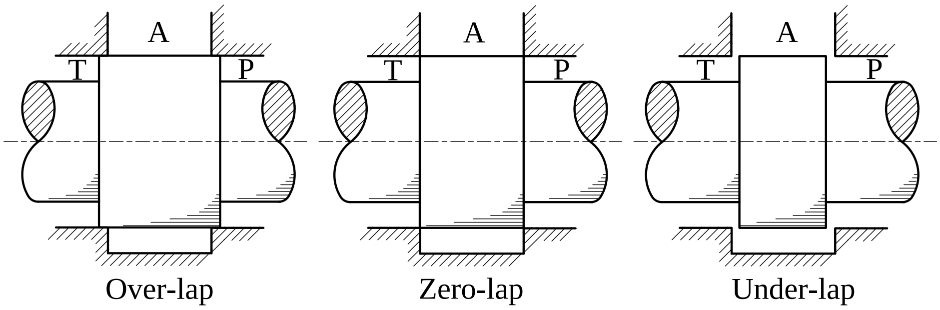


Figure 5.5 Physical layout of spool with over-lap, zero-lap and under-lap.

This greatly influences the performance of the valve around its neutral position and for such applications this must be taken into account. Normally, a number of steady state characteristics are given for a commercial valve that describe how the valve is lapped.
The hydraulic lines between the valve and the actuator are normally reduced in size as much as possible in order to maintain as high an effective stiffness as possible of the fluid. This is important in order to have as low a response time as possible for the hydraulic part of the servo system. Appropriate peak fluid veclocity for such lines may be as high as *30 m/s*.
The actuators are the usual rotary=motor and linear actuators=cylinder gone through in chapter 2. Normally, the inertia of the actuators are negligible in hydraulically actuated systems when compared with that of the payload. The use of cylinders pose a number of challenges that is not encountered when using motors. For high speed applications the cylinder friction may pose a problem and more expensive cylinders where extra measures have been taken to reduce both stiction and coulomb friction are used. Also, the effective stiffness of hydraulic cylinders change significantly with stroke, see Section 6.3, and this must be considered in the design phase. Also, cylinders with differential areas should be avoided for applications where the valve operates around its neutral position because the dynamic characteristics of the system will be changing abruptly all the time. Because of this, cylinders with identical pressure areas, i.e. $\varphi = 1$, are used frequently, see Figure 5.4. When using differential area cylinders different layouts may be utilized, see Figure 5.6.

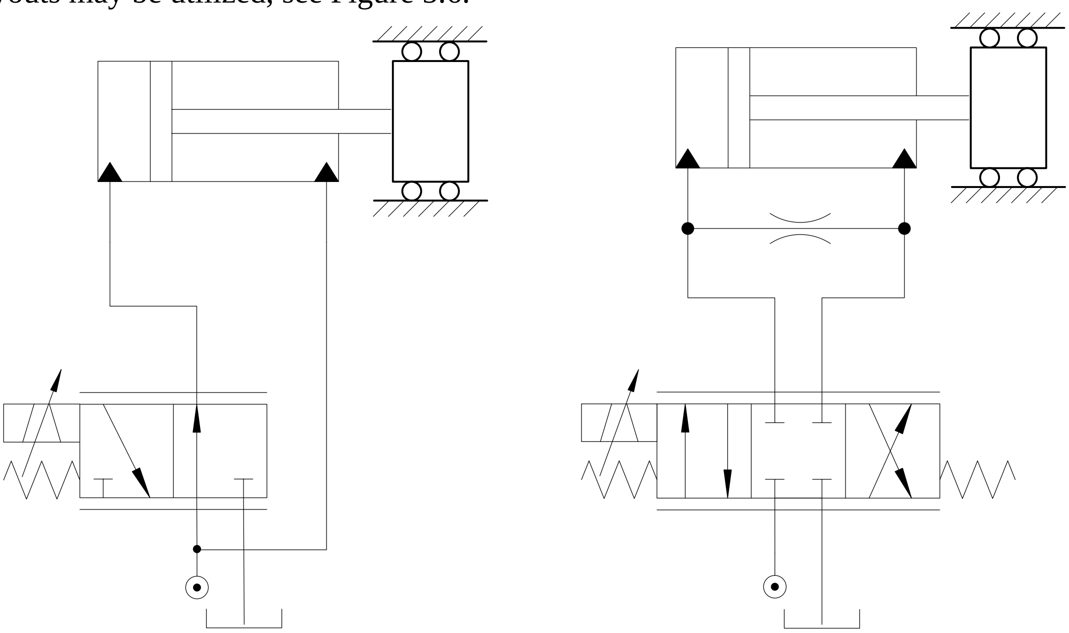


Figure 5.6 Hydraulic servo system with differential area cylinder.

When a hydraulic motor is used as actuator the controlled variables of the different types of servo systems are: rotation, rotational speed and torque. For such systems the effective stiffness is almost constant, see Section 6.3.
The mechanical system should be well understood, especially its effective inertia and its effective stiffness, see Section 6.3. Often the mechanical system to be actuated is quite stiff as compared to the hydraulic system, however, this is not always the case, and in such situations the extra flexibility introduced by the mechanical structure must be included when computing the eigenfrequency of the system. Special care must be paid to any type of sliding connections where friction may appear since this can greatly reduce the controllability of the mechanical system. Ideally, the possible implications of friction should be investigated either via model based sensitivity analysis or, if this is not sufficient for a reliable design, via experimental work.
The sensor obviously depends on the type of servo and can be in the shape of a potentiometer, an encoder, a tachometers or an accelerometer for a position or speed servo. For a force servo it may be sufficient to measure the pressure drop across the actuator, however, if friction is high it may be necessary to measure the actual force produced at the piston rod or at the payload.
The servoamplifier receives the measured signal and a command reference signal. The difference is fed into a controller that generates a command signal to the valve. In some cases the electronics is integrated in the valve. It is important to note that depending on the type of servo the valve is operating around neutral (position servo) or around a metered out position (speed and force servo), see Figure 5.7. Hence, a differential area cylinder can be used for a force servo because the jumps in dynamic characteristics around the neutral valve position are avoided.

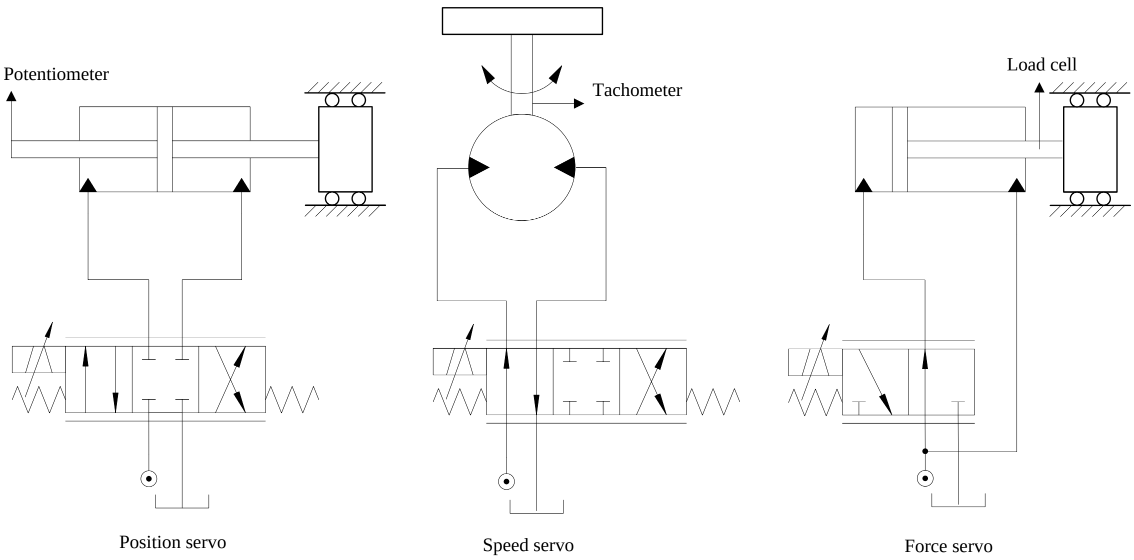


Figure 5.7 Position (equal area cylinder), speed (motor) and force servo (differential area cylinder).

5.3.2 Motion reference

When generating motion references for servo systems it is necessary to take into account the natural eigenfrequency of the hydraulic mechanical system. In Section 6.3 it is gone through in detail how to compute the eigenfrequency of hydraulic mechanical

systems. This requires the computation of the spring stiffness of the hydraulic system and the effective inertia of the mechanical system. It should be noted that if the mechanical system has a flexibility that cannot be ignored it should be treated as a spring in series with the spring representing the hydraulic system.
The natural eigenfrequency gives an indication of the limit that the mechanical system imposes on the desired motion. Consider a 2nd order under-damped system with a natural eigenfrequency, ω_n, a damping, ζ and a mass, *m*. The mass is traveling at a speed, $\dot{y} = v_0$, and the motion of the mass should be ramped down via the reference input x, see Figure 5.8. In principle this corresponds to halting a hydraulically controlled payload by ramping down the input flow.

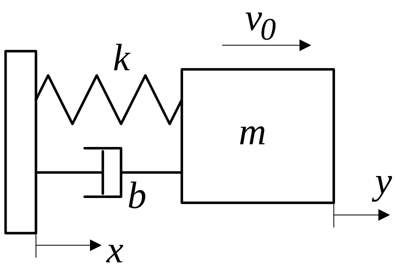


Figure 5.8 A second order system with a mass traveling at a speed $\dot{y} = v_0$.

Initially, $\dot{x} = \dot{y} = v_0$ and then the reference velocity is ramped down according to:

$$\dot{x} = v_0 \cdot \left[1 - \frac{t}{t_R} \right] \tag{5.23}$$

In (5.23) *t* is the time and t_R is the ramp time during which the mass is decelerated. The analytical solution to the motion of the mass is:

$$\begin{gathered} y = C_0 + C_1 \cdot t + C_2 \cdot t^2 - C_0 \cdot e^{-\alpha \cdot t} \cdot \left[cos(\beta \cdot t) + \frac{\alpha}{\beta} \cdot sin(\beta \cdot t) \right] \\ C_0 = \frac{v_0}{\omega_n^2 \cdot t_R} \qquad C_1 = v_0 \qquad C_2 = -\frac{v_0}{2 \cdot t_R} \\ \alpha = \zeta \cdot \omega_n \qquad \beta = \omega_n \cdot \sqrt{1 - \zeta^2} \end{gathered} \tag{5.24}$$

In Figure 5.9 the variations in the reference motion and the motion of the mass are shown. The reference velocity becomes zero at $t = t_R$. At that instant the position error, referred to as the overshoot, is:

$$e = \left| x_{t=t_R} - y_{t=t_R} \right| = \frac{v_0}{\omega_n^2 \cdot t_R} \cdot \left\{ 1 - e^{-\alpha \cdot t_R} \cdot \left[cos(\beta \cdot t_R) + \frac{\alpha}{\beta} \cdot sin(\beta \cdot t_R) \right] \right\} \tag{5.25}$$

The relative overshoot is the absolute position error relative to the nominal travel:

$$\varepsilon = \frac{e}{x_{t=t_R}} = \frac{2}{\omega_n^2 \cdot t_R^2} \cdot \left\{ 1 - e^{-\alpha \cdot t_R} \cdot \left[\cos(\beta \cdot t_R) + \frac{\alpha}{\beta} \cdot \sin(\beta \cdot t_R) \right] \right\} \tag{5.26}$$

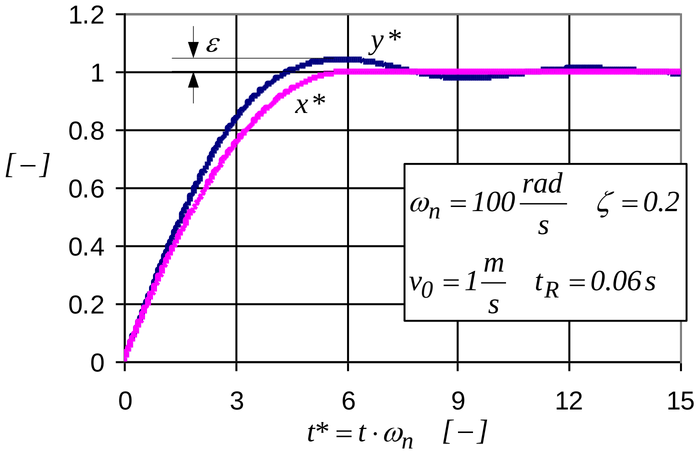


Figure 5.9 Reference motion and actual motion of a 2nd order system subjected to a ramp down of the velocity. The relative overshoot is shown.

In Figure 5.10 the relative overshoot is plotted as a function of the ramp time and the damping. The eigenfreguency has no influence on the curves.

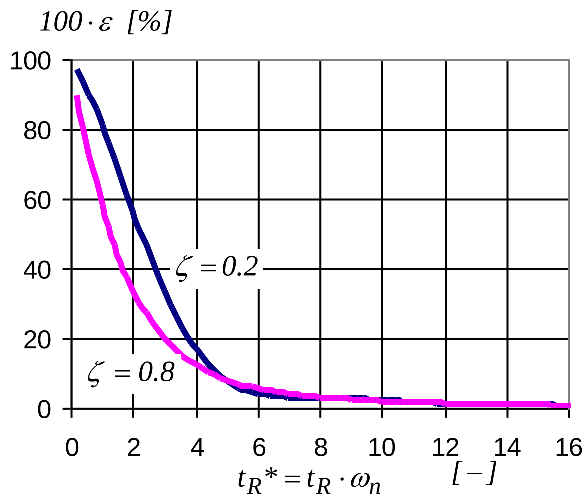


Figure 5.10 Relative overshoot as a function of ramp time and damping.

Since most hydraulic systems have dynamic characteristics that are not far from those of the system in Figure 5.8 the above results may be utilized for a hydraulic servo system. From Figure 5.10 it is clear that for a typical hydraulic mechanical system ramp times should be considered whenever motion is prescribed. Also, it is clear that a useful ramp time depends on the acceptable overshoot. Hence, if high overshoot is acceptable small ramp times may be prescribed and vice versa. As a rule of thumb, the ramp time of a prescribed motion should obey the inequality:

$$t_R \geq \frac{6}{\omega_n} \tag{5.27}$$

It is very important to keep in mind that the eigenfrequency of a real system may be substantially smaller than the one computed for a model of the system. The discrepancy is mainly because the modeled stiffness typically is higher than the actual stiffness. This can be taken into account in many ways, for example by using a smaller stiffness in the computation or by means of experimental work that can reveal the actual eigenfrequency(ies) of the system to be controlled.
For stiff systems with high natural frequency the limitation on the ramp time might come from other sources such as:

- pressure level required to generate the ramp acceleration
- response time of directional control valve
- response time of variable displacement pump if present

These lower bounds on the ramp time must be considered from application to application, however, in general, ramp times should be kept as small as possible because this reduces the maximum speed and thereby the energy usage and the pump flow. This can be illustrated by considering a simple task of translating a payload of mass, $m = 1kg$, a distance, $\Delta s = 1m$, within a given time $\Delta t = 1s$. In that case the maximum velocity and the maximum kinetic energy depend on the ramp time as shown in Figure 5.11.

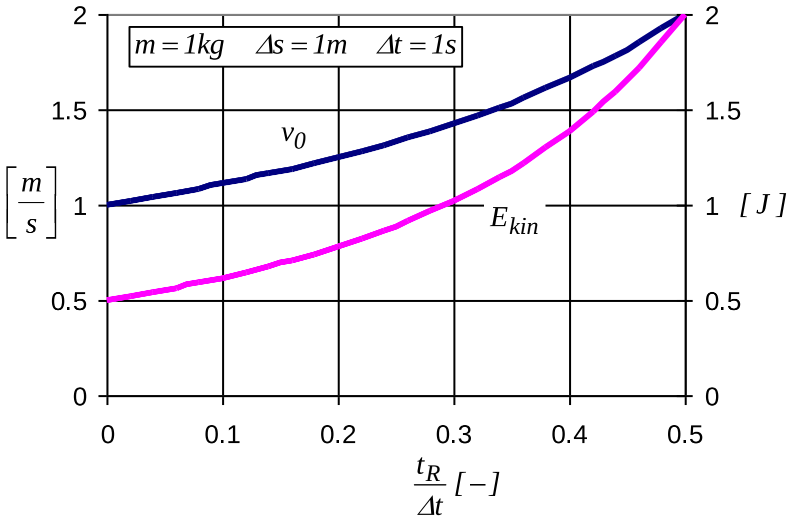


Figure 5.11 Maximum velocity and maximum kinetic energy as function of ramp time.

Considering losses it should be noted that most of the flow losses in valves and lines also increase with the maximum velocity.
Let us consider the basic operation; to move a payload a certain distance within a certain time. Let Δs denote the travel associated with the operation. let Δt denote the time available for the operation and let σ be a dimensionless number, $\sigma < 0.5$, that denotes the ratio between the ramp time and total time. Initially, some considerations should be made concerning the shape of the velocity profile. In Figure 5.12 the velocity profile associated with a linear ramp, i.e., constant acceleration, and a cosine-shaped ramp, i.e., a sinus-shaped acceleration, are shown. Clearly, the sinusoid has the advantage that it increases the degree of continuity at the ramp boundaries. This will, especially for very stiff systems, reduce impact loads, vibrations and noise. The disadvantage is the increased demands on the valve control and the higher maximum acceleration (half way through the ramp).

The maximum velocity is the same for both profiles:

$$v_0 = \frac{1}{1-\sigma} \cdot \frac{\Delta s}{\Delta t} \tag{5.28}$$

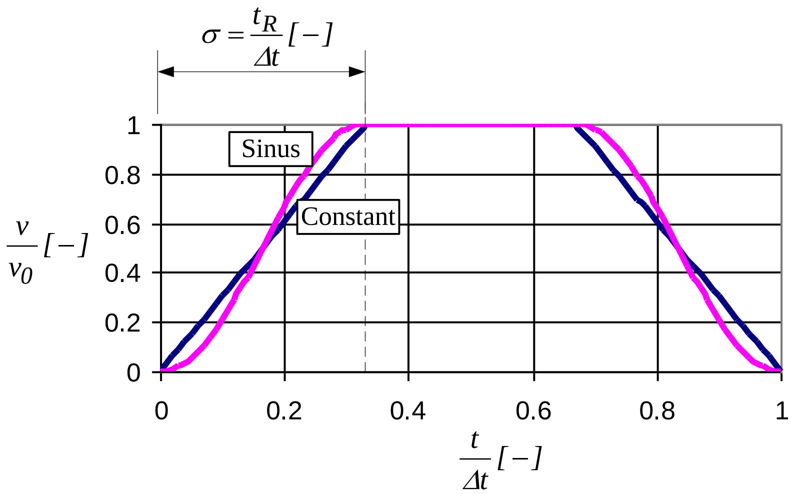


Figure 5.12 Two different velocity profiles; constant acceleration and sinusoid acceleration. Both time and velocity are normalized.

The velocity as a function of time may be computed as:

$$v^{(Linear)} = \begin{cases} v_0 \cdot \dfrac{t}{t_R} & 0 \le t \le t_R \\ v_0 & t_R \le t \le \Delta t - t_R \\ v_0 \cdot \left[1 - \dfrac{t^*}{t_R}\right] & \Delta t - t_R \le t \le \Delta t \end{cases}$$

$$v^{(Sinusoid)} = \begin{cases} \dfrac{v_0}{2} \cdot \left[1 - cos\left(\pi \cdot \dfrac{t}{t_R}\right)\right] & 0 \le t \le t_R \\ v_0 & t_R \le t \le \Delta t - t_R \\ \dfrac{v_0}{2} \cdot \left[1 + cos\left(\pi \cdot \dfrac{t^*}{t_R}\right)\right] & \Delta t - t_R \le t \le \Delta t \end{cases} \tag{5.30}$$

$$t^* = t - \Delta t + t_R$$

5.3.3 Servo valve specifications

Servo valves are highly specialized components and as such, they are normally chosen from the catalogue data of experienced manufacturers. In order to choose a servo valve from a catalogue two values need to be computed:

1. The minimum rated flow, $Q_{r,min}$, of the valve.
2. The minimum bandwidth, $\omega_{v,min}$, of the valve

To compute the minimum rated flow it is necessary to perform steps 1..5 in the systematic design approach, see Section 5.2, but only for the degree of freedom controlled by the servo valve. As an example, the servo system of figure 5.4 is considered, see Figure 5.13.

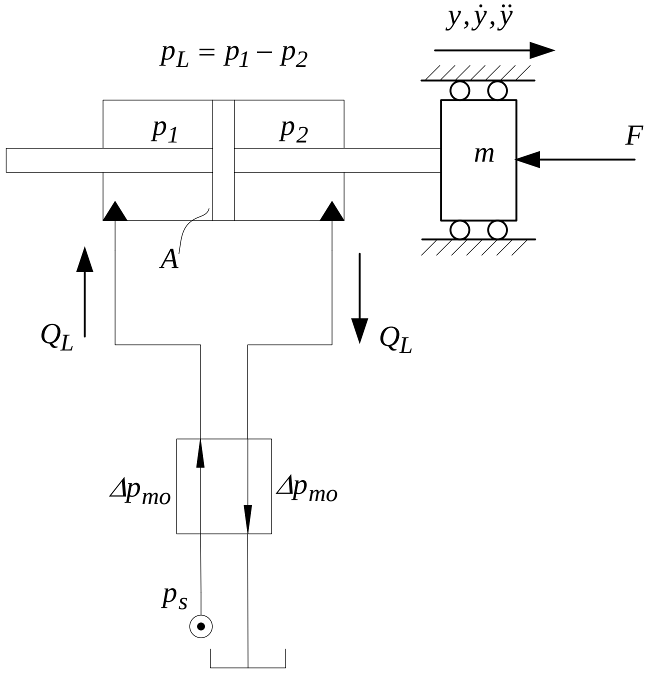


Figure 5.13 Servo system with 4/3-way servo valve supplying fluid power to an equal area actuator that translates a payload.

In this case the prescribed motion of the piston, y, $\dot{y}$ and $\ddot{y}$, and the applied force on the piston, F, must be determined over the entire operating cycle. Next, the pressure level, i.e., the supply pressure, p_s , should be chosen. The value of the supply pressure depends on the chosen valve type. In general, as high a pressure level as possible should be chosen with a view to reduce the size and cost of the pump and other flow transmitting components. Because the servo valves use throttling across their metering orifices for functionality it is necessary to take this into account when sizing the actuator. The pressure drop across the equal area cylinder is introduced, see Figure :

$$p_L = \Delta p_{cyl} = p_1 - p_2 \tag{5.31}$$

Using a symmetrical valve we have the following correlation between the supply pressure, the load pressure and the load flow:

$$Q_L = C_d \cdot w \cdot x \cdot \sqrt{\frac{2}{\rho} \cdot \Delta p_{mo}} = C_d \cdot w \cdot x \cdot \sqrt{\frac{2}{\rho} \cdot \frac{p_s - p_L}{2}} = C_d \cdot w \cdot x \cdot \sqrt{\frac{1}{\rho} \cdot (p_s - p_L)} \tag{5.32}$$

In Equation (5.32) the area gradient of the metering orifices of the valve is w and the spool travel is x. The pressure drop across each of the metering orifices is introduced as $\Delta p_{mo} = 0.5 \cdot (p_s - p_L)$. The fluid power delivered to the cylinder piston is:

$$P_{F \to C} = p_L \cdot Q_L = p_L \cdot C_d \cdot w \cdot x \cdot \sqrt{\frac{1}{\rho} \cdot (p_s - p_L)} \tag{5.33}$$

In order to maximize the available power for a given valve opening the following can be set up:

$$\begin{aligned}
&\frac{\partial P_{F\to C}}{\partial p_L}=0\Rightarrow\frac{\partial\left\{p_L\cdot\sqrt{p_s-p_L}\right\}}{\partial p_L}=0\\
&\Downarrow\\
&\sqrt{p_s-p_L}-\frac{p_L}{2\cdot\sqrt{p_s-p_L}}=0\Rightarrow p_L=\frac{2}{3}\cdot p_s
\end{aligned}\qquad\textbf{(5.34)}$$

Hence, the load pressure should be set to 67% of the supply pressure in order to maximize output power.
Knowing the load pressure the size of the actuator can be computed from:

$$p_L\cdot A=F+m\cdot\ddot{y}\Rightarrow A\geq\frac{\left(F+m\cdot\ddot{y}\right)_{max}}{p_L}\qquad\textbf{(5.35)}$$

In catalogues the inequality of Equation (5.35) is turned into an equation by adding a safety factor of *1.3*:

$$A=1.3\cdot\frac{\left(F+m\cdot\ddot{y}\right)_{max}}{p_L}\qquad\textbf{(5.36)}$$

Based on Equation (5.36) it is possible to find a commercially available cylinder and now the so-called no-load flow demand can be computed for the entire operating cycle. Clearly, the load pressure as it is computed in Equation (5.34) is a maximum value. During an operating cycle the load on the piston may vary significantly. Therefore it is necessary to compute the actual flow demand, the actual load pressure and the corresponding no-load flow demand for the entire operating cycle:

$$\begin{aligned}
&Q(t)=A\cdot\dot{y}(t)\\
&p_L(t)=\frac{F(t)+m\cdot\ddot{y}(t)}{A}\\
&Q_{NL}(t)=Q(t)\cdot\sqrt{\frac{p_s}{p_s-p_L(t)}}
\end{aligned}\qquad\textbf{(5.37)}$$

The maximum value of the no-load flow represents the basic flow requirement to the valve: in a situation where the valve is fully open and there is no load on the piston the valve flow should be larger than or equal to the maximum no-load flow:

$$Q_{v,NL@p_S}\geq max\left\{Q_{NL}(t)\right\}=Q_{NL,max}\qquad\textbf{(5.38)}$$

In catalogues the ability of the valve to transmit flow is given as a rated flow. The rated flow corresponds to a no-load flow for a supply pressure equal to a certain rated pressure, p_r. For multiple stage servovalves this rated pressure is, classically, equal to *70 bar* which corresponds to *35 bar* pressure drop across each metering orifices in the no load situation. So the flow demand may finally be transformed to a value that is comparable with the rated flow:

$$Q_{r,min} = 1.1 \cdot Q_{NL,max} \cdot \sqrt{\frac{p_r}{p_s}} \qquad Q_r \geq Q_{r,min} \tag{5.39}$$

A safety factor of *1.1* is normally introduced at this point. This concludes the computation of the minimum required value of the rated flow. It is recommended to choose a valve that has a rated flow that is as close as possible to the minimum required value. A large valve will be costly and compromise the accuracy of the total system.
Secondly, the minimum bandwidth of the valve must be determined. Here, experience dictates that the valve must be faster than the hydraulic-mechanical system, i.e., the valve should be able to operate at frequencies higher than the lowest eigenfrequency of the hydraulic-mechanical system, ω_n. A rule of thumb is that the operating frequency that corresponds to a $90°$ phase lag for the valve should be three times larger than ω_n:

$$\omega_{v,min} = 3 \cdot \omega_n \tag{5.40}$$

Most servo valves has a frequency characteristic that roughly ressembles that of a critically damped 2nd order system. In Figure 5.14 this has been illustrated with three idealised Bode plots of the same valve. It is important to note that the valve has a much higher bandwidth if only a certain percentage of the total spool travel is activated.

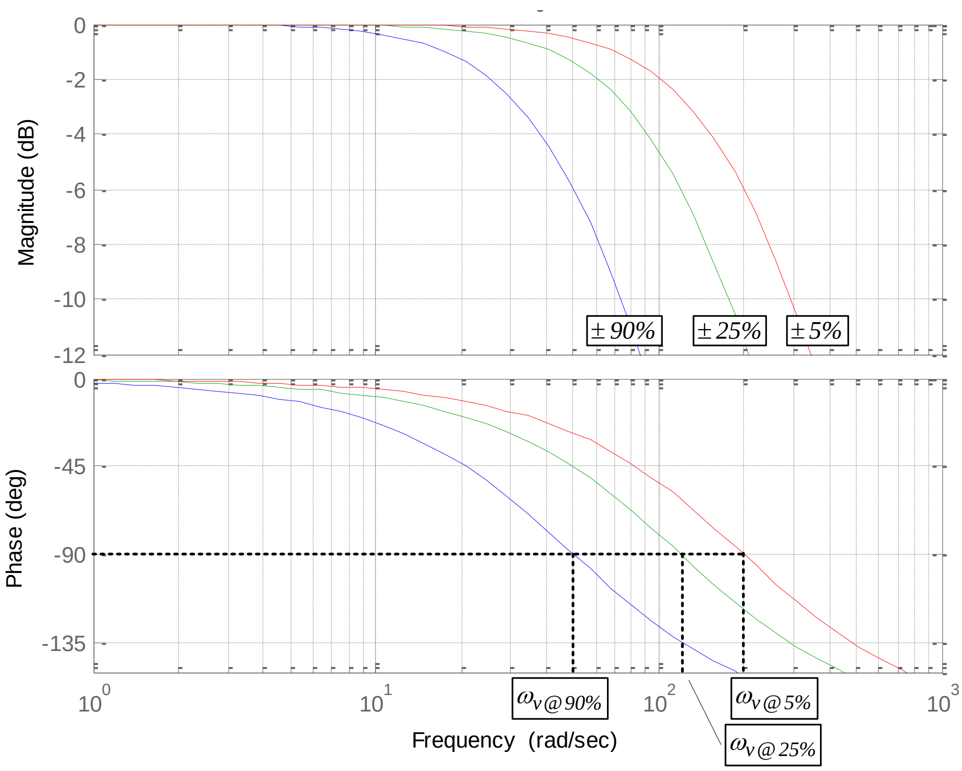


Figure 5.14 Idealised Bode plots of servo valve for three different levels of actuation.

In Figure 5.14 the effective bandwidth of the valve ($90°$ phase lag) is referred to as ω_V with reference to the actuation level. The valve actuation that corresponds to computation of the minimum rated flow should off course be used to determine whether the valve is suitable:

$$\omega_V \geq \omega_{V,min} \qquad \textbf{(5.41)}$$

Equations (5.39) and (5.41) represents the criteria used in practice when choosing servo valves.

5.4 Load sensing systems

Fluid power systems for systems that have several actuators with substantial variations in their power demand may easily encounter very poor efficiencies. In many cases this problem is handled by introducing a so-called load sensing (LS) system. An LS-system is characterized by one or more components that are capable of

- sensing the current load situation as pressure signals, and
- adjusting the power demand to the pump(s) of the system accordingly.

These type of systems are especially popular within mobile hydraulics where large variations in actuator loading and several actuators in activity simultaneously are common practice. In the following different LS system configurations will be introduced.

5.4.1 Several actuators

In Figure 5.15 a simplified hydraulic diagram is shown for a system with two actuators and a fixed displacement pump. The directional control valves used to distribute the fluid power to the two actuators are electro-hydraulically actuated proportional 4/3-way valves.

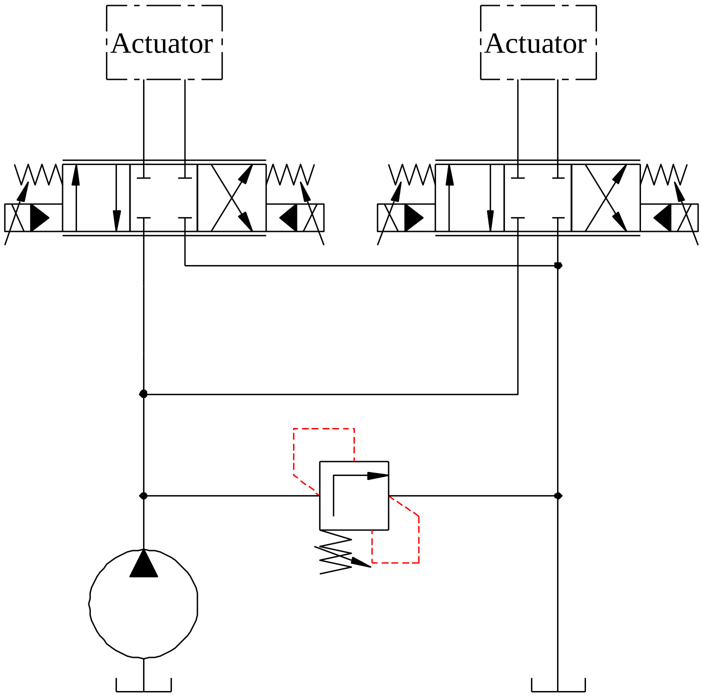


Figure 5.15 Simplified hydraulic diagram with two actuators controlled by electro-hydraulic proportional valves.

The electrical input to the proportional valves can be generated in several ways. If there is an operator-in-the-loop then the input is normally generated by means of a manually operated joy stick. If there is no operator, then some kind of closed loop control is normally employed that requires position or velocity feedback from the actuator, see Fig. 5.16.

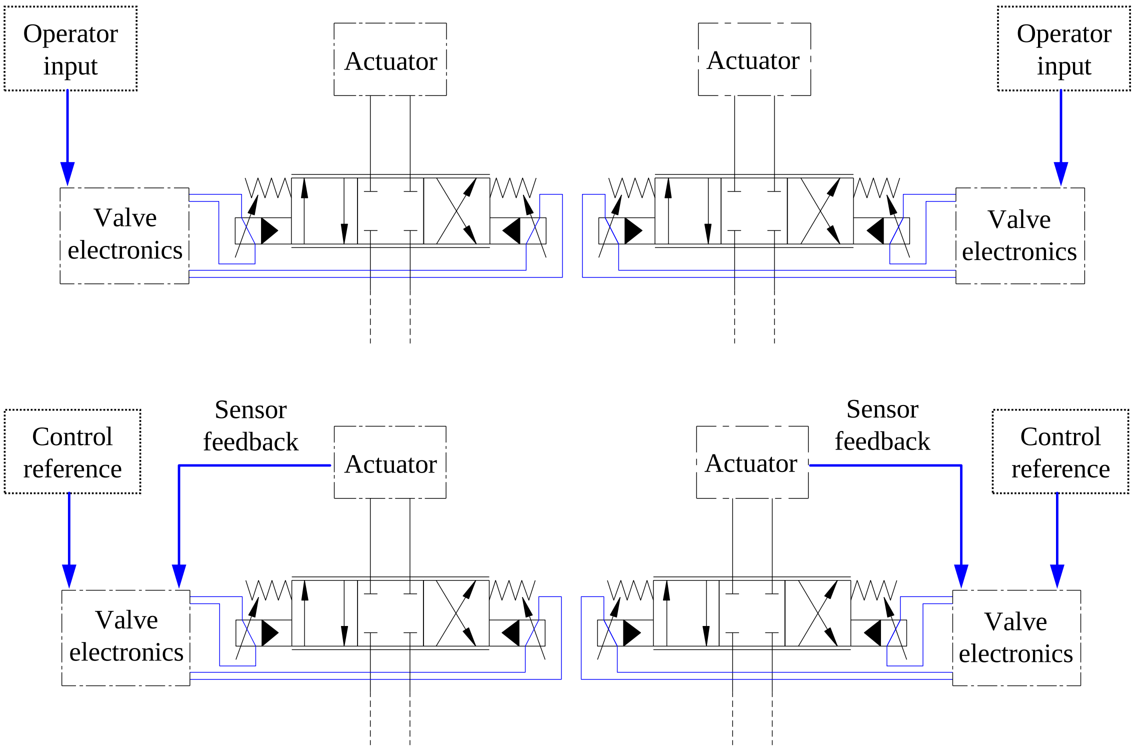


Figure 5.16 Typical actuation of electro-hydraulic proportional valve with and without an operator in the loop, respectively.

In any case, some kind of valve electronics (either integrated into the valve or a stand alone component) is required to manipulate the input signal into a spool position. This is typically obtained via an integrated feedback control system with a positional sensor attached to the spool and a power unit capable of driving a servo system that controls the distribution of the valve actuation hydraulic pressure.
The system shown in Fig. 5.15 will always be difficult to control by an operator in the loop. The main problem is caused by the multiple actuators that may require fluid power at different levels and simultaneously. This means that the operator must adjust the spool position continuously to ensure that heavily loaded actuators receive their designated share of the total available pump flow. In most cases this is either very time consuming or actually impossible. Because of this, the pressure compensated valve has been developed. In Fig. 5.17 the design shown in Fig. 5.15 has been modified so as to include pressure compensators for each actuator circuit.
The compensator that has been added upstream to each valve will try to maintain a constant pressure drop across the main spool P-to-A or P-to-B orifices depending on the current spool position. This means that the only parameter in the orifice equation that may still vary is the discharge area of the spool. The discharge area always depends on the spool position, i.e., the valve flow is proportional to the discharge area and thereby to the spool position and thereby to the input signal. Hence, the name proportional now refers to the valve flow and not only the spool position.

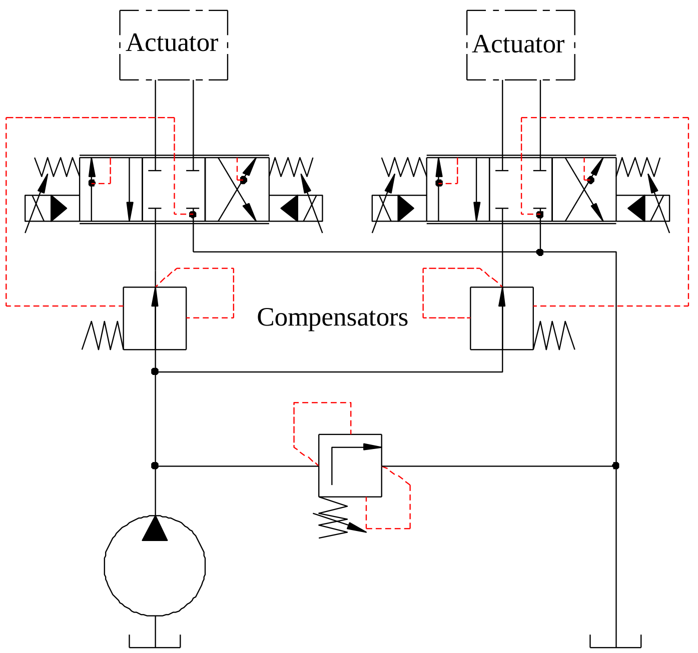


Figure 5.17 Pressure compensated electro-hydraulic proportional valves.

In the following section different types of systems using pressure compensators are introduced.

5.4.2 LS design principles

In Fig. 5.18 the circuit of Fig. 5.17 is repeated. The system has a fixed displacement pump and a pressure relief valve set to a constant crack pressure. The flow of the fixed displacement pump should be set to a value slightly higher than the maximum required actuator flow, so that the difference, ΔQ, may be used to keep the pressure relief valve open.

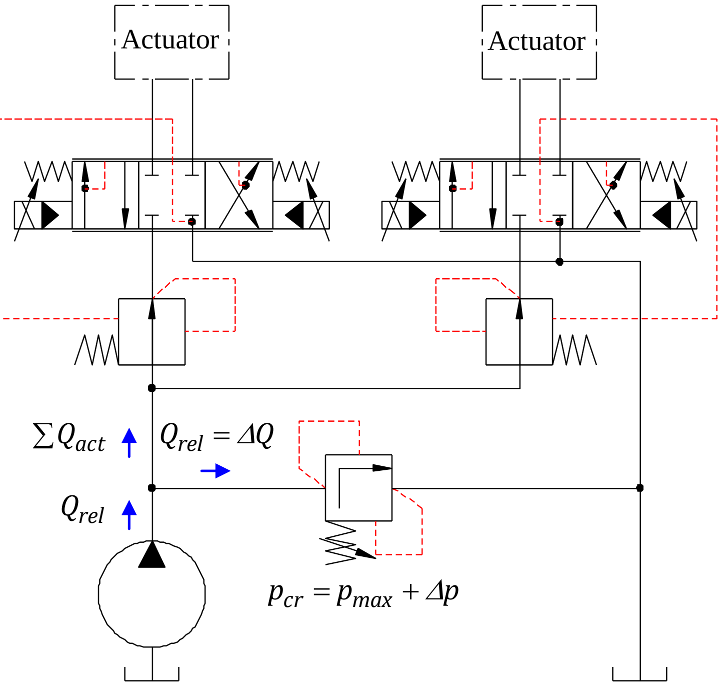


Figure 5.18 Hydraulic circuit labelled: Design A.

Also, the crack pressure of the pressure relief valve should be set to a value slightly higher, Δp, than the maximum operational actuator pressure. This design will be referred to as Design A.

Next, the fixed displacement system is turned into an LS-system, see Fig. 5.19. The LS pressure is obtained by means of check valves that compare relevant pressure levels with the largest value reported back as the LS pressure of the entire circuit. This LS pressure is compared to that of the other circuit(s) and the highest value qualifies as the overall LS pressure of the valve.

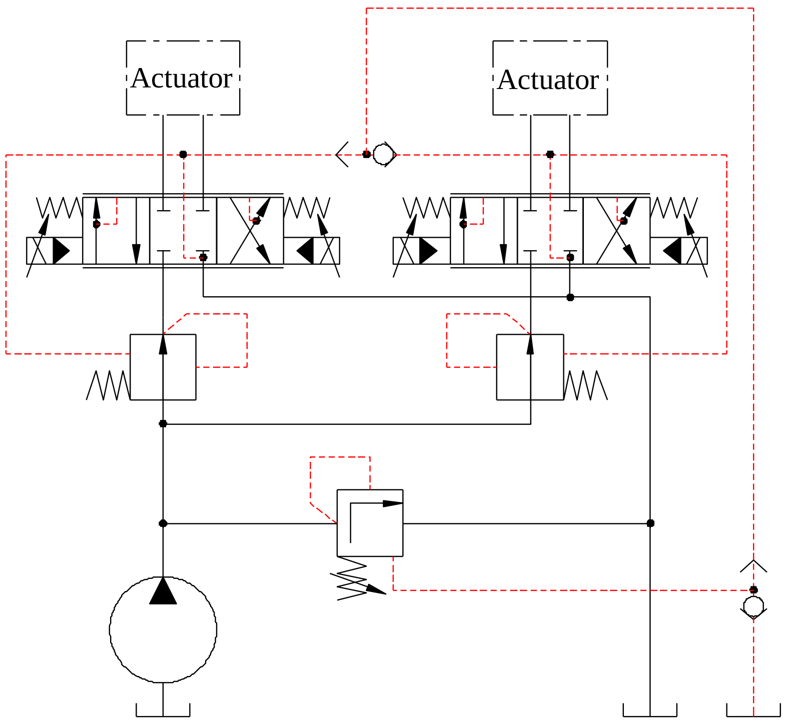


Figure 5.19 Hydraulic circuit labelled: Design B.

Rather than employing a fixed displacement pump it is also possible to introduce variable displacement pumps with pressure control as described in section 4.2.1 of these notes. For that purpose we introduce a simplified notation for variable displacement pumps with pilot operated pressure control and the LS pressure control, se Figure 5.20.

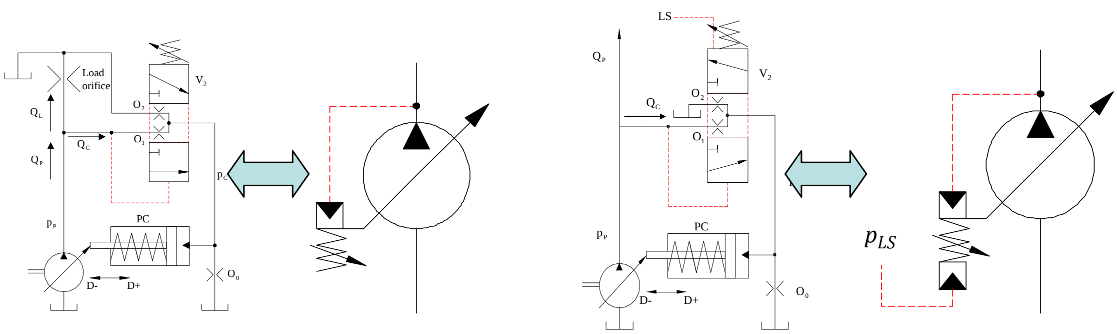


Figure 5.20 Simplified diagram notation for variable displacement pumps with pilot operated pressure control and LS pressure control.

These pumps may be used in a non LS system and an LS system as shown in Fig. 5.21 and Fig. 5.22.

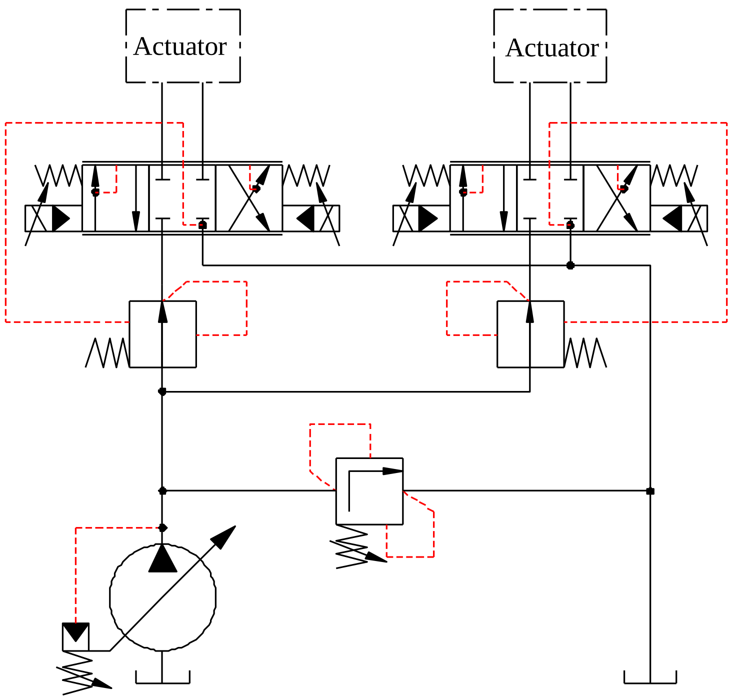


Figure 5.21 Hydraulic circuit labelled: Design C.

The pressure relief valve now remains closed at all time and the pump only supplies the required flow.

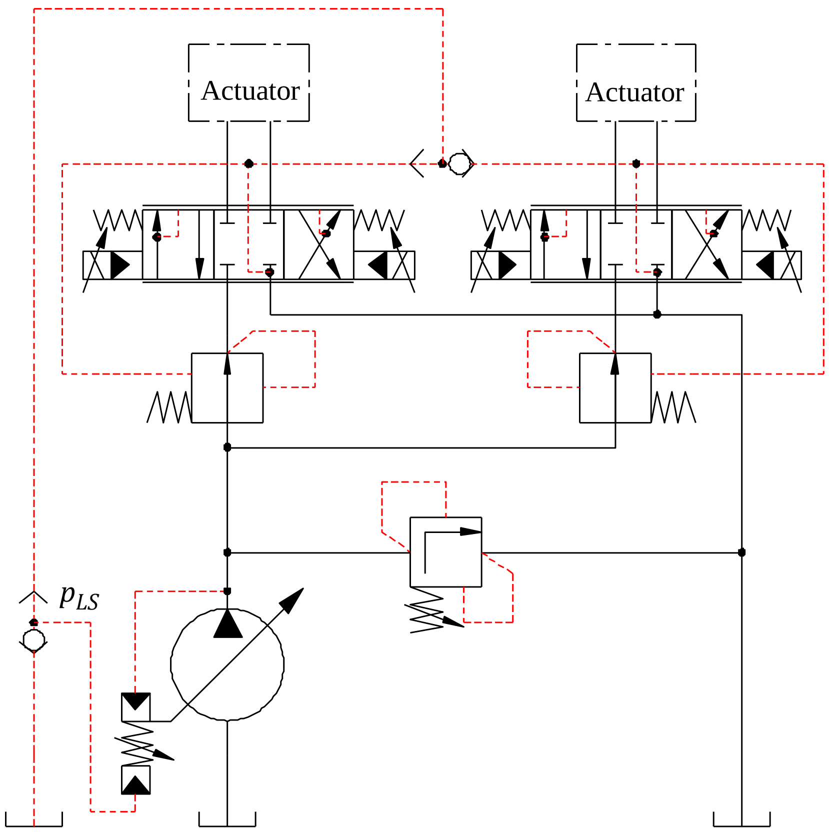


Figure 5.22 Hydraulic circuit labelled: Design D.

5.5 System efficiencies

In the previous section four different hydraulic circuits were presented that all are capable of handling systems with several actuators that are manually controlled by an operator in the loop. A simplified load case is considered. The total duration of the load case is 60 seconds and in that period both actuators are subjected to high pressure load (typically positive load), low pressure load (typically negative load) and idle (not actuated). If the actuators are chosen carefully, then it is reasonable to set the same pressure demands for both circuits at high and low pressure. The power difference is normally seen in the pump flow, hence, the volume flow demand of actuator 2 is set to the double of the volume flow demand of actuator 1. Finally, the different periods of high pressure, low pressure and idle for the two actuators are offset relative to each other as would be the case in many applications.
In Table 5.1 the volume flow, the pressure and the corresponding power level used to investigate the design put forward in the previous sections are displayed for both actuators in the 60 second window of the load case.

Actuator #1			
Interval [s]	Volume flow [l/min]	Pressure [bar]	Power [kW]
0-10	50	180	15
10-20	50	180	15
20-30	50	30	2.5

30-40	50	30	2.5
40-50	0	0	0
50-60	0	0	0
Actuator #2			
Interval [s]	Volume flow [l/min]	Pressure [bar]	Power [kW]
0-10	0	0	0
10-20	100	30	5
20-30	100	30	5
30-40	100	180	30
40-50	100	180	30
50-60	0	0	0

For comparison the accumulated energy consumption at the pump is displayed for each design together with the accumulated energy demand at the actuators. Please note, that for Design A and B a flow safety margin of 10 l/min is used, i.e., the pump flow is always 160 l/min. Also, for all design a pressure safety margin of 20 bar is used, i.e., the pump pressure must be 20 bar higher than the maximum/current pressure demand. These number yields the accumulated energy curves shown in Fig. 5.23.

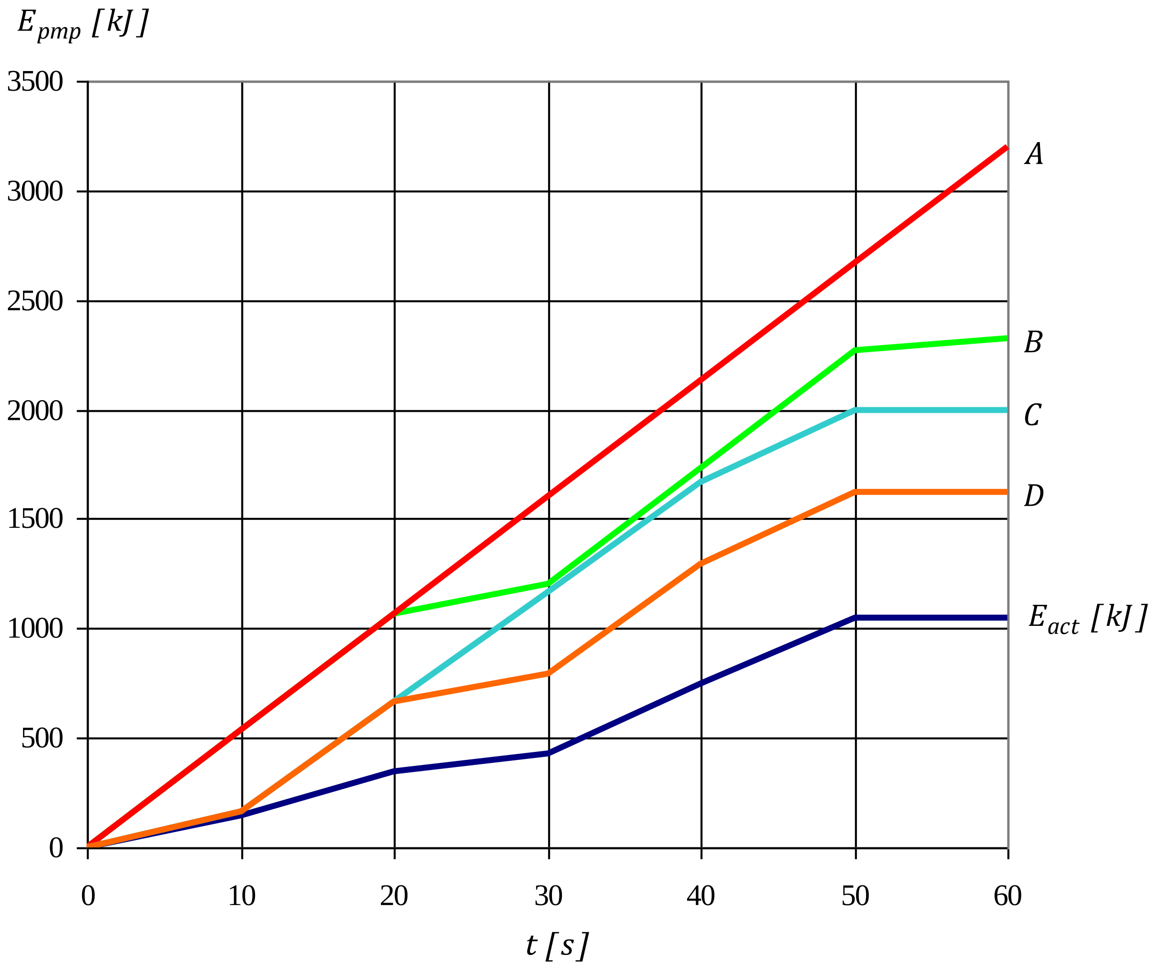


Figure 5.23 Accumulated energy consumption at the pump for Design A..D as compared to the accumulated energy demand at the actuators.

As can be seen, Design D has the best efficiency for this load case and, indeed, for almost any application. However, it is also clear that as long as both actuators are working, but with different pressure levels, then Design D will also experience energy losses. For this load case this is relevant for the intervals 10-20 seconds and 30-40 seconds, respectively. The only way to remove these types of losses are by adding more pumps (ideally, one pump for each actuator), however, for a practical system this will increase costs substantially.

----- oo 0 oo -----

6
Analysis of Hydraulic Systems

Hydraulic System Design

6.1 Introduction

In the previous chapter on synthesis of hydraulic systems a step-wise approach to the sizing of a given concept was introduced. This approach and indeed any type of model based approach to the design of hydraulic systems requires that the system performance can be simulated. For a hydraulic system the important questions that need to be answered via the simulation are both functionality and performance. Basically, it should be investigated whether the desired control of the actuators is possible and, simultaneously, at what power, pressure and flow levels this can be achieved.
The ongoing competition between manufacturers of hydraulically actuated systems continuously set new references for main competition parameters such as price, efficiency, controllability, weight and safety. Since development time and costs must be kept at a minimum this leaves the design engineer in a challenged position typically facing a complex task of a strongly dynamic and multidisciplinary nature. The use of simulation offers a number of potential advantages if applied with common sense. They include reduction of development time, reduction of experimental costs, improved documentation and facilitation of optimization of already existing systems. The difficulties lie not so much in the complexity of the governing equations but propably more often in the ability to estimate important equation parameters with a sufficient precision. Also, the type of analysis needs consideration. Hydraulically actuated systems are often highly dynamical, however, in the initial design stages when the hydraulic system is at a conceptual level it is typically easier and more rewarding to do steady state simulation since this will give a good indication of both functionality, price and efficiency. The steady state simulation does, however, not give any indication on dynamic performance, i.e., instability, vibrations, controllability, fluid compression, pressure peaks etc. In order to investigate these phenomena a dynamic simulation is r

equired. In the following subsections methods for steady state and dynamic modeling and simulation are presented.

6.2 Steady State Modeling and Simulation

Physically, steady state simulation of a hydraulic system is characterized by:

- constant actuator speed
- constant pump speed
- incompressible fluid
- all mechanical parts in valves are stationary

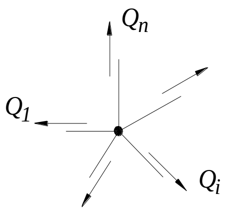
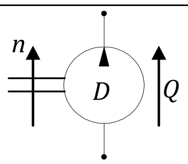
The steady state equations for the basic components of a hydraulic system are gone through in the following tables. For each component a short descriptive name is given together with a symbol. Also, the governing equations are listed both in SI-units and typical fluid power units. The typical fluid power units (FLP-units) as defined in this note are listed in Table 6.1:

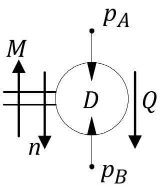
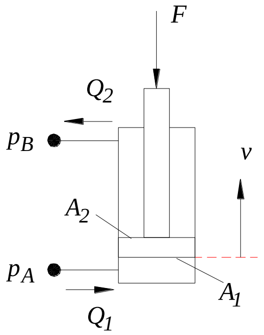
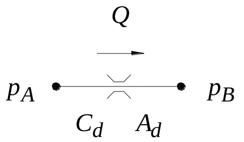
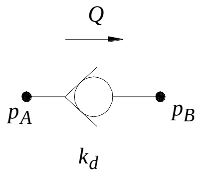
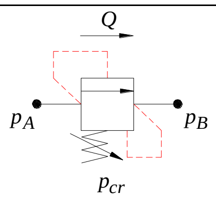
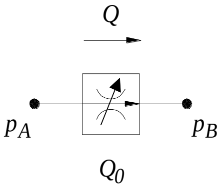
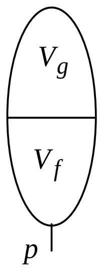
Table 6.1 *SI-units and FLP-units*

Symbol	Description	SI-units	FLP-units
Q	Flow	m^3 / s	l / min
n	Rotational speed	o / s	o / min
D	Displacement	m^3 / o	cm^3 / o
M	Torque	Nm	Nm
p	Pressure	N / m^2	bar
F	Force	N	N
v	Speed	m / s	m / s
A	Area	m^2	mm^2

This yields the following basic equations for some of the most common components in hydraulic systems. The pressure node is simply a volume of fluid without significant pressure variations.

Table 6.2 *Steady state equations for basic components*

Description	Symbol	Equations SI-units	Equations Hyd-units
Pressure node		$\sum_{i=1}^{n} Q_i = 0$	$\sum_{i=1}^{n} Q_i = 0$
Pump		$Q = n \cdot D$	$Q = \frac{n \cdot D}{1000}$

Motor		$Q = n \cdot D$ $M = \dfrac{D \cdot (p_A - p_B)}{2 \cdot \pi}$	$Q = \dfrac{n \cdot D}{1000}$ $M = 0.0159 \cdot D \cdot (p_A - p_B)$
Cylinder		$Q_1 = v \cdot A_1$ $Q_2 = v \cdot A_2$ $F = p_A \cdot A_1 - p_B \cdot A_2$	$Q_1 = 0.06 \cdot v \cdot A_1$ $Q_2 = 0.06 \cdot v \cdot A_2$ $F = \dfrac{p_A \cdot A_1 - p_B \cdot A_2}{10}$
Orifice		$Q = C_d \cdot A_d \cdot \sqrt{\dfrac{2}{\rho} \cdot (p_A - p_B)}$	$Q = 0.89 \cdot C_d \cdot A_d \cdot \sqrt{p_A - p_B}$
Check valve		$Q = 0 \quad p_A \leq p_B$ $Q > 0 \quad p_A = p_B$	$Q = 0 \quad p_A \leq p_B$ $Q > 0 \quad p_A = p_B$
Pressure relief valve		$Q = 0 \quad p_A - p_B \leq p_{cr}$ $Q > 0 \quad p_A - p_B = p_{cr}$	$Q = 0 \quad p_A - p_B \leq p_{cr}$ $Q > 0 \quad p_A - p_B = p_{cr}$
2-way flow control valve		$Q = \dfrac{Q_0}{\sqrt{\Delta p_0}} \cdot \sqrt{p_A - p_B} \quad p_A - p_B \leq \Delta p_0$ $Q = Q_0 \quad p_A - p_B > \Delta p_0$	$Q = \dfrac{Q_0}{\sqrt{\Delta p_0}} \cdot \sqrt{p_A - p_B} \quad p_A - p_B \leq \Delta p_0$ $Q = Q_0 \quad p_A - p_B > \Delta p_0$
Accumulator		$p \cdot V_g^n = p_0 \cdot V_0^n$ $V_a = V_f + V_g = cst$	$p \cdot V_g^n = p_0 \cdot V_0^n$ $V_a = V_f + V_g = cst$

In the above a number of simplifications have been made. As an example efficiencies are not included in the pump and actuator equations. Also, the spring of the check valve is ignored, the slope of the pressure relief valve characteristic is not included. The 2-way flow control valve is simplified to an orifice if the pressure drop is smaller than the closing pressure of the spring of the main spool and to a perfect flow controller if the pressure drop is above this value. All of these simplified models can, however, be adjusted/ if so desired. As an example, consider the pressure relief valve and imagine that the slope of the p-Q characteristics should be included. In that case the governing equations simply change into:

$$\begin{aligned} Q &= 0 & p_A - p_B &\le p_{cr} \\ Q &= \frac{p_A - p_B - p_{cr}}{\alpha} & p_A - p_B &> p_{cr} \end{aligned} \tag{6.1}$$

where $\alpha \left[\frac{pressure}{flow} \right]$ is the slope of the p-Q characteristic of the valve. If the slope for some reason is not constant but varies significantly this may also be introduced. It is simply a question of how detailed information is required at the current stage of the design evaluation. In Table 6.2 only a few valves are shown, however, similar equations may be set up for any type of valve.

The most important aspect of the governing equations for the valves in Table 6.2 are the fact that there are at least/typically **two modes of operation**. Hence, a governing equation exists for both modes and for each mode there is an inequality that must be fulfilled in order for the mode to be active. In the following the choice of mode of operation is referred to as the **configuration parameter** of the valve. For a pure steady state analysis, the configuration parameter of each valve must be chosen beforehand, i.e., it is necessary to make a number of qualified guesses. Only af choosing/guessing the configuration parameter of each valve can the governing equations be formulated and solved. After solving the equations each choice of configuration parameter must be validated by checking whether the corresponding inequality is fulfilled. If not, the configuration parameter must be changed, and the analysis redone. Potentially, this leads to 2^n possible different system configurations, where n is the number of valves with two modes of operation. In practice, however, the mode of operation of most valves are easily recognized for a given situation.

A step-wise approach for steady-state analysis of any hydraulic system can now be set up:

1. Identify pressure nodes.
2. Identify the components that demarcate each pressure node.
3. Choose/guess configuration parameters for all components with more than one mode of operation.
4. Set up equations:
   a) Flow continuity for each pressure node.
   b) Flow continuity for each pump and actuator.
   c) Static equilibrium for each actuator.
   d) Flow through restrictions (orifices, filters etc.).
   e) Equations associated with the choice of configuration parameters.
   f) Static equilibrium for spool and poppet valves.
5. Solve equations numerically.
6. Are the computed variables physically meaningful?
   - Yes: Analysis completed.
   - No: Have all combinations of configuration parameters been examined?
     - Yes: Analysis cannot be carried out.
     - No: Go to 3) and choose/guess configuration parameters differently.

As an example the hydraulic system shown in Figure 6.1 has been subjected to this type of analysis. From the figure it is seen that a total number of 9 equations has been established. In this case, there is a single component with two operation modes, namely the pressure relief valve, and the guess is that the valve is closed. For a given system with a known pump speed and a known output torque the 9 equations may be solved to yield the 9 unknowns: $p_{1..3}$ $Q_{1..5}$ n_m. Hence, pressure, flow and actuator speed are the classical output of steady state analysis, however, it is also possible to prescribe the motor speed and introduce the motor displacement as a variable thereby changing the equation solving from pure analysis to component sizing. After solving the equations it is necessary to examine that all pressures are non-negative and that the choice of configuration parameter is correct. In this case this is simply done by ensuring that p_1 is smaller than p_{cr}.

Flow continuity, nodes

$$Q_1 - Q_2 - Q_3 = 0$$
$$Q_3 - Q_4 = 0$$
$$Q_4 - Q_5 = 0$$

Flow continuity, pumps and actuators

$$Q_1 - n_p \cdot D_p = 0$$
$$Q_4 - n_m \cdot D_m = 0$$

Actuator equilibrium

$$M_m - \frac{D_m \cdot (p_2 - p_3)}{2 \cdot \pi} = 0$$

Restriction flow

$$Q_3 - C_D \cdot A_D \cdot \sqrt{\frac{2}{\rho} \cdot (p_1 - p_2)} = 0$$
$$Q_5 - C_D \cdot A_D \cdot \sqrt{\frac{2}{\rho} \cdot p_3} = 0$$

Operation mode / Configuration parameter

$$Q_2 = 0$$

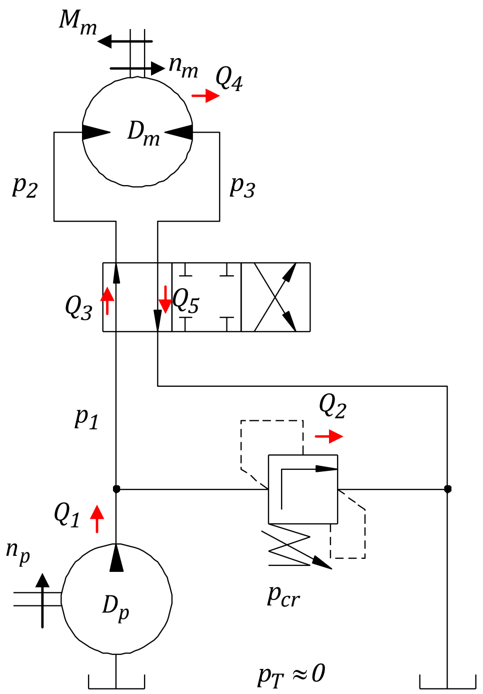


Figure 6.1. *Hydraulic circuit and corresponding steady state equations.*

Clearly, the number of equations might seem excessive, and the system of equations could easily be reduced substantially by substituting the different expressions into each other.

The set of equations are non-linear because of the orifice equations and therefore have to be solved numerically, typically by means of Newton-Raphson iteration. Often the numerical solver will have difficulties solving the equations simply because they are formulated in SI-units. If this is the case then one should simply reformulate the problem using FLP-units. Also, the numerical solver may encounter problems with the orifice equation because the sign of the pressure drop may become negative during iteration. This may be avoided by using the following formulation:

$$Q = C_D \cdot A_D \cdot SIGN(p_A - p_B) \cdot \sqrt{\frac{2}{\rho} \cdot |p_A - p_B|}$$
$$SIGN(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} \tag{6.2}$$

Also, numerical problems will be encountered if for some reason the correct solution includes zero flow through an orifice. In that case the orifice equation must be replaced with a restriction flow equation that reflects the laminar regime, i.e., $Q \propto \Delta p$.

6.3 Dynamic Modeling and Simulation

Physically, dynamic simulation of a hydraulic system is mainly characterized by:
- acceleration of all mechanical parts
- compressible fluid

Dynamic simulation allows for a much more detailed investigation of the performance of the hydraulic system; however, it does also require that further estimations of especially system damping and oil stiffness in order to predict behavior correctly.
Steady state simulation corresponds to solving a set of algebraic equations whereas dynamic simulation corresponds to solving a mixed set of differentials and algebraic equations. The differential equations are time dependent, i.e., the problem is an initial value problem, where the pressure in the pressure nodes, the volumes of the accumulators and the position and velocity of all mechanical degrees of freedom must be known at the start of simulation.
Most of the basic equations shown in Table 6.2 remain the same. Most notable is the difference when considering flow continuity of a pressure node. If the fluid is compressible then conservation of mass yields the following differential equation for a volume of fluid = pressure node:

$$\dot{p} = \frac{\beta \cdot (Q - \dot{V})}{V} \tag{6.3}$$

This equation gives the pressure gradient in a given volume of fluid, V. The net-flow into the volume is Q (positive if flow enters the volume) and the time derivative of the expansion (displacement flow) is $\dot{V}$ (positive if the volume is expanding). If this value is positie at a certain time the pressure is going up and vice versa. The effective stiffness of the fluid, which greatly depends on temperature, dissolved air, hosing and tubing, is β. In this case we have a 1st-order differential equation and only one initial condition is required, namely the initial pressure in the volume: p.
As an example consider the volume shown in Figure 6.2 which is bounded by 2 cylinders, 2 motors, 2 pumps and 2 orifices.

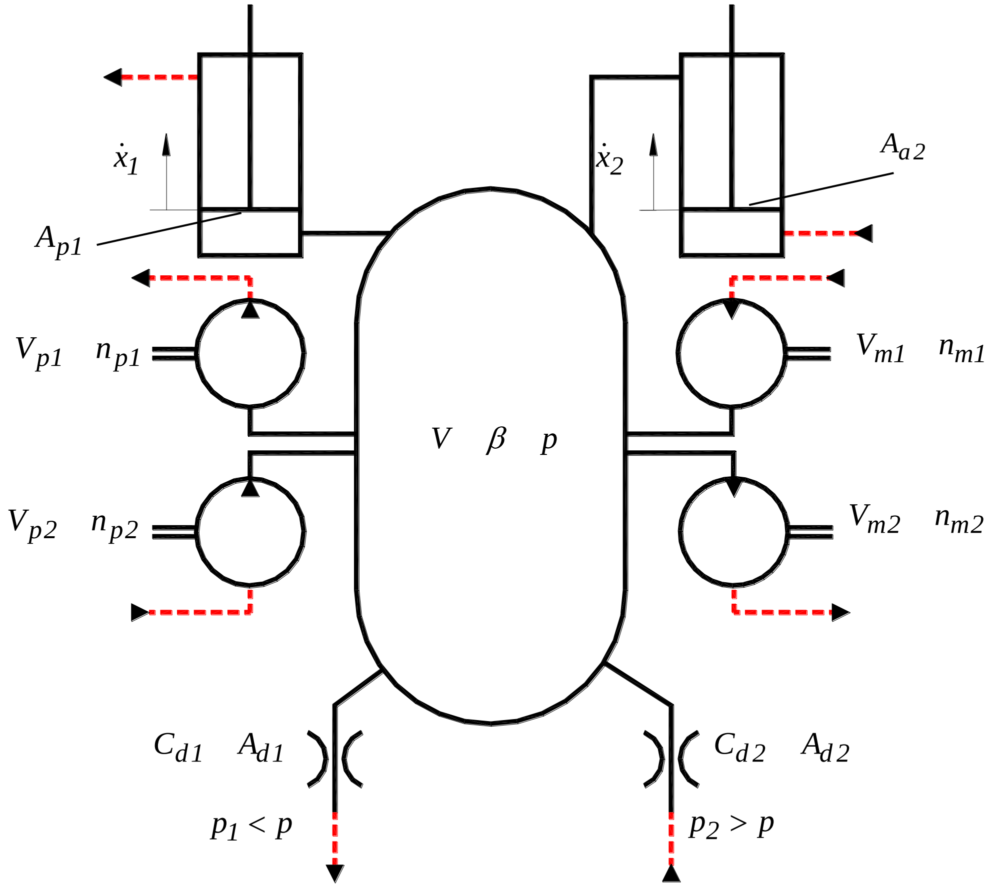


Figure 6.2. *A volume bounded by cylinders, motors, pumps and orifices.*

The differential equations for the pressure gradient of this volume may be written using Equation 6.3 with the following values for Q and $\dot{V}$:

$$
\begin{aligned}
&Q = -D_{p1} \cdot n_{p1} + D_{p2} \cdot n_{p2} + D_{m1} \cdot n_{m1} - D_{m2} \cdot n_{m2} - \\
&C_{d1} \cdot A_{d1} \cdot \sqrt{\frac{2}{\rho} \cdot (p - p_1)} + C_{d2} \cdot A_{d2} \cdot \sqrt{\frac{2}{\rho} \cdot (p_2 - p)} \\
&\dot{V} = A_{p1} \cdot \dot{x}_1 - A_{a2} \cdot \dot{x}_2
\end{aligned}
\tag{6.4}
$$

Just like a mechanical system typically contains more than one body a hydraulic system will typically need to be modelled with several volumes. In a hydraulic system the different volumes are typically separated by either: Pumps, motors, cylinders or orifices. Displacement flows in volumes are typically caused by hydraulic cylinders or accumulators. If we consider the steady state equations for an accumulator then the time derivative yields a linear equation containing both the pressure and volume gradient:

$$
\begin{aligned}
&\dot{p} \cdot V_g^n + n \cdot p \cdot V_g^{n-1} \cdot \dot{V}_g = 0 \quad \& \quad \dot{V}_g + \dot{V}_f = 0 \\
&\Downarrow \\
&\dot{V}_f = \frac{\dot{p}}{n \cdot p} \cdot V_g
\end{aligned}
\tag{6.5}
$$

In the above equation the polytropic exponent *n* depends on whether the compression/decompression of the accumulator is predominantly adiabatic, see also section 4.5. In general Equation (6.5) is a simplification because *n* may vary significantly, hence, care should be taken in the modeling if the performance of the hydraulic system is very sensitive to the value of *n*.
Equation (6.5) introduces another state variable, namely the fluid volume of the accumulator. The initial fluid volume is readily derived from the total accumulator volume (which is constant), the preload conditions and the current pressure:

$$V_f = V_a - \left(\frac{p_0}{p}\right)^{\frac{1}{n}} \cdot V_0 \qquad V_g = \left(\frac{p_0}{p}\right)^{\frac{1}{n}} \cdot V_0 \tag{6.6}$$

Hence, if a volume is connected to an accumulator it has two states: the pressure and the fluid volume of the accumulator and their gradients must be solved simultaneously using Equations (6.3) and (6.5).
In general, a purely hydraulic system may be solved numerically in the following steps:

1. Identify all volumes in the system and set up the pressure build up equation for each and identify all accumulators and set up the (Circuit diagrams useful here).
2. Identify all orifices and their dependency on the motion of mechanical parts in valves.
3. Determine initial pressure for each volume and initial fluid volume for each accumulator.
4. Calculate pressure gradients for each volume and volume gradient for each accumulator.
5. Calculate acceleration of all movable mechanical parts in valves.
6. Update pressure in each volume and volume of each accumulator.
7. Update position and velocity of all movable mechanical parts in valves.
8. If the analysis is not yet concluded then go back to 4.

Typically some mechanical bodies, cylinders or valves will be part of the system. This means that their position and velocity must be updated simultaneously in order to update volumes, orifice flows and net-flows into volumes for the next step.
As may readily be observed from Equation (6.7) the position and velocity of the mechanical system is needed in order to do a dynamic simulation of the hydraulic system. In fact, any type of dynamic simulation of a hydraulic system requires a simultaneous dynamic simulation of the actuated mechanical system and, depending on the level of detail, also of the movable mechanical parts within the hydraulic valves. Hence, dynamic simulation of hydraulics is closely connected to dynamic simulation of mechanics.
The mechanical system may, in general, be divided into a number of bodies. In the planar case, the governing dynamic equations for a body are:

$$\begin{aligned} m \cdot \ddot{\underline{r}} &= \Sigma \underline{F} \\ J \cdot \ddot{\theta} &= \Sigma M \end{aligned} \tag{6.7}$$

In Equation (6.7) m is the mass and J is the mass moment of inertia with respect to the mass center of the body. Furthermore, $\underline{r}$ is the coordinates of the mass center of the body and θ is the rotation of the body, both measured relative to some reference coordinate system. $\sum \underline{F}$ is the sum of all the forces acting on the body, normally divided into applied forces (gravity, springs, dampers, actuators, friction, wind resistance, rolling resistance etc.) and reactive forces (connections with other bodies and ground). $\sum M$ is the sum of the force moment of $\sum \underline{F}$ with respect to the mass center and all the moments (force couples) acting on the body.
The dynamic equilibrium equations contain 3 scalar equations corresponding to the 3 degrees of freedom of a body. All of them are differential equations, and in general needs to be solved numerically. Being 2nd order differential equations 2 initial conditions is required for each coordinate, namely the initial position and initial velocity:

$$\underline{r} \quad \dot{\underline{r}} \quad \theta \quad \dot{\theta}$$

Knowing the positions and velocities of the bodies will typically also be necessary in order to determine the resulting forces and moments, i.e., the right hand side of Equation (6.7).
Alternatively, the steady state equations of the actuators, see Table 6.2, may be generalized to take into account the dynamics of the mechanical system. For the motor a simplified dynamic equation can be set up:

$$J_{eff} \cdot \ddot{\theta} = \frac{D \cdot \Delta p_M}{2 \cdot \pi} - M = M_{tM} - M \qquad (6.8)$$

In Equation (6.8) the sign conventions are as follows: rotation of the shaft, θ (and its time derivatives; $\dot{\theta}$ and $\ddot{\theta}$), is defined as positive in the same direction as the hydraulically generated torque on the shaft produced by a positive pressure drop across the motor. The applied moment on the output shaft, M, is positive in the opposite direction. The introduction of the hydro-mechanical efficiency, η_{hmM}, should be done with care. Normally, the hydromechanical efficiency is measured in a situation where the motor is motoring, i.e., the direction of the applied moment is opposite to that of the angular speed. In that case:

$$J_{eff} \cdot \ddot{\theta} = \eta_{hmM} \cdot \frac{D \cdot \Delta p_M}{2 \cdot \pi} - M \qquad (6.9)$$

However, if the motor is working as a pump, i.e., the direction of the applied moment is in the same direction as that of the angular speed (negative load $M<0$) we get:

$$J_{eff} \cdot \ddot{\theta} = \frac{1}{\eta_{hmM}} \cdot \frac{D \cdot \Delta p_M}{2 \cdot \pi} - M \qquad (6.10)$$

Note: in Equation (6.10) both Δp_M and M have negative values.
The effective mass moment of inertia, J_{eff}, and the applied moment, M, must be related to the output shaft of the motor. In general, they are both functions of θ and $\dot{\theta}$ and

may be determined from energy considerations. The applied moment may be computed as follows:

$$M = \frac{-dW_{ext}}{d\theta} \tag{6.11}$$

In Equation (6.11) W_{ext} is the work done by the external forces/moments (gravity, friction, rolling resistance etc.) on the mechanical system actuated by the motor.
The effective mass moment of inertia may be computed according to:

$$J_{eff} = \frac{2 \cdot E_{kin}}{\dot{\theta}^2} \tag{6.12}$$

In Equation (6.12) E_{kin} is the kinetic energy of the mechanical system.
Consider the system shown in Figure 6.3.

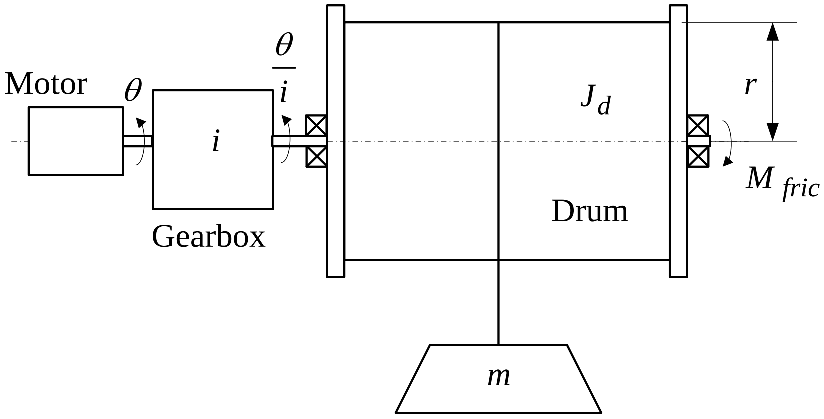


Figure 6.3. *A hydraulic motor is driving a winch drum via a gearbox. The drum is connected to a payload and is also subjected to a certain rotational friction.*

For an infinitesimal rotation of the motor shaft the work done on the system by the external loading can be computed and, subsequently, the moment applied to the output shaft of the motor:

$$\begin{aligned} &W_{ext} = -m \cdot g \cdot dy - M_{fric} \cdot d\theta_d = -m \cdot g \cdot r \cdot \frac{d\theta}{i} - M_{fric} \cdot \frac{d\theta}{i} \\ &\Downarrow \\ &M = \frac{m \cdot g \cdot r + M_{fric}}{i} \end{aligned} \tag{6.13}$$

The effecitve mass moment of inertia may be computed as:

$$E_{kin} = \frac{1}{2}\cdot J_d \cdot \dot{\theta}_d^2 + \frac{1}{2}\cdot m \cdot v^2 = \frac{1}{2}\cdot J_d \cdot \left(\frac{\dot{\theta}}{i}\right)^2 + \frac{1}{2}\cdot m \cdot \left(r \cdot \frac{\dot{\theta}}{i}\right)^2$$
$$\Downarrow \qquad (6.14)$$
$$J_{eff} = \frac{J_d}{i^2} + \frac{m \cdot r^2}{i^2}$$

Notice, that the mass moment of inertia of the motor rotor and the gearbox have been neglected. Because of the high torque pr. volume ratio of hydraulic motors neglecting the inertia of the motor rotor and the gearbox is normally a reasonable assumption.
Next, consider the system shown in Figure 6.4. It is a four-wheel drive vehicle subjected to a total rolling resistance of F_R, and propelled by a hydraulic motor via a geared chain drive $i = \frac{z_2}{z_1}$. Due to symmetry the vehicle is modeled as a planar system.

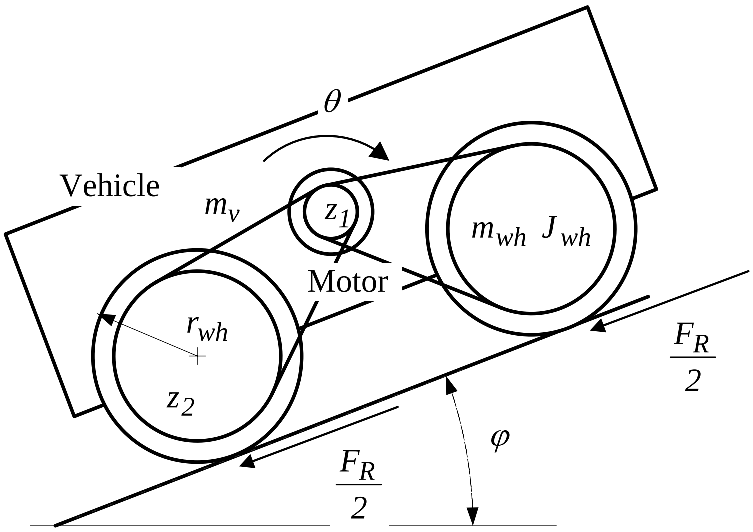


Figure 6.4. *A four-wheel drive vehicle propelled up an incline by a hydraulic motor is shown. Power is transmitted from the motor to the wheels via chain drives.*

As in the previous example a infinitesimal rotation of the motor is used to determine the moment applied to the output shaft of the motor:

$$W_{ext} = -(m_v + 2\cdot m_{wh})\cdot g \cdot dy - F_R \cdot ds = -(m_v + 2\cdot m_{wh})\cdot g \cdot r_{wh} \cdot \frac{d\theta}{i}\cdot \sin\varphi - F_R \cdot r_{wh} \cdot \frac{d\theta}{i}$$
$$\Downarrow \qquad (6.15)$$
$$M = \frac{r_{wh}\cdot \{F_R + (m_v + 2\cdot m_{wh})\cdot g \cdot \sin\varphi\}}{i}$$

The effecitve mass moment of inertia may be computed as:

$$E_{kin} = 2 \cdot \frac{1}{2} \cdot J_{wh} \cdot \dot{\theta}_{wh}^2 + \frac{1}{2} \cdot (2 \cdot m_{wh} + m_v) \cdot v^2 = J_{wh} \cdot \left(\frac{\dot{\theta}}{i}\right)^2 + \frac{1}{2} \cdot (2 \cdot m_{wh} + m_v) \cdot \left(r_{wh} \cdot \frac{\dot{\theta}}{i}\right)^2$$
$$\Downarrow \tag{6.16}$$
$$J_{eff} = \frac{2 \cdot J_{wh}}{i^2} + \frac{(2 \cdot m_{wh} + m_v) \cdot r_{wh}^2}{i^2}$$

Having determined the effective mass moment of inertia it is possible to compute the eigenfrequency of the hydraulic-mechanical system composed of the lines to and from the motor, the motor and the mechanical system represented by the effective mass moment of inertia:

$$\omega_n = \sqrt{\frac{k_\theta}{J_{eff}}}$$
$$k_\theta = \frac{\beta \cdot D^2}{\pi^2 \cdot (D + V_L)} \tag{6.17}$$

In Equation (6.17) V_L is simply the total volume of the fluid lines leading up to the motor and away from it.

Similar to that of the motor a simplified dynamic equation can be set up for the cylinder:

$$m_{eff} \cdot \ddot{x} = A \cdot (p_1 - \varphi \cdot p_2) - F = F_{tC} - F \tag{6.18}$$

In Equation (6.18) the sign conventions are as follows: piston travel, x (and its time derivatives; $\dot{x}$ and $\ddot{x}$), is defined as positive when the cylinder is extracting. The applied force on the piston, F, is positive in the opposite direction. As in the case of the motor the introduction of the hydro-mechanical efficiency, η_{hmC}, should be done with care. Equation (2.47) is used in its general form to get an expression for the friction force as a function of the hydromechanical efficiency:

$$\eta_{hmC} = \frac{F_C}{|F_{tC}|} = \frac{|F_{tC}| - F_{mC}}{|F_{tC}|} = 1.0 - \frac{F_{mC}}{|F_{tC}|} \Rightarrow F_{mC} = (1 - \eta_{hmC}) \cdot |F_{tC}| \tag{6.19}$$

If the cylinder is extracting the theoretical cylinder force is defined as $F_{tC} = A \cdot (p_1 - \varphi \cdot p_2)$, see Equation (2.43). If the piston is pushing against a load we have:

$$|F_{tC}| = F_{tC} \Rightarrow F_{mC} = (1 - \eta_{hmC}) \cdot F_{tC} = (1 - \eta_{hmC}) \cdot A \cdot (p_1 - \varphi \cdot p_2)$$
$$\Downarrow \tag{6.20}$$
$$m_{eff} \cdot \ddot{x} = F_{tC} - F_{mC} - F = \eta_{hmC} \cdot A \cdot (p_1 - \varphi \cdot p_2) - F$$

If the cylinder is extracting and is pulled by the load we have:

$$\begin{aligned}
&\left|F_{tC}\right| = -F_{tC} \Rightarrow F_{mC} = -(1-\eta_{hmC}) \cdot F_{tC} = -(1-\eta_{hmC}) \cdot A \cdot \left(p_1 - \varphi \cdot p_2\right) \\
&\Downarrow \\
&m_{eff} \cdot \ddot{x} = F_{tC} - F_{mC} - F = \left(2-\eta_{hmC}\right) \cdot A \cdot \left(p_1 - \varphi \cdot p_2\right) - F
\end{aligned} \tag{6.21}$$

Note that in Equation (6.21) both $F_{tC} = A \cdot \left(p_1 - \varphi \cdot p_2\right)$ and F have negative values.
If the cylinder is retracting the theoretical cylinder force is defined as $F_{tC} = A \cdot \left(\varphi \cdot p_2 - p_1\right)$, see Equation (2.44). If the cylinder is pulling a load we have:

$$\begin{aligned}
&\left|F_{tC}\right| = F_{tC} \Rightarrow F_{mC} = (1-\eta_{hmC}) \cdot F_{tC} = (1-\eta_{hmC}) \cdot A \cdot \left(\varphi \cdot p_2 - p_1\right) \\
&\Downarrow \\
&m_{eff} \cdot \ddot{x} = -F_{tC} + F_{mC} - F = -\eta_{hmC} \cdot A \cdot \left(\varphi \cdot p_2 - p_1\right) - F
\end{aligned} \tag{6.22}$$

Note that in Equation (6.22) $F_{tC} = A \cdot \left(\varphi \cdot p_2 - p_1\right)$ has a positive value whereas *F* has a negative value.
If the cylinder is retracting and is pushed by the load we have:

$$\begin{aligned}
&\left|F_{tC}\right| = -F_{tC} \Rightarrow F_{mC} = -(1-\eta_{hmC}) \cdot F_{tC} = -(1-\eta_{hmC}) \cdot A \cdot \left(\varphi \cdot p_2 - p_1\right) \\
&\Downarrow \\
&m_{eff} \cdot \ddot{x} = -F_{tC} + F_{mC} - F = -(2-\eta_{hmC}) \cdot A \cdot \left(\varphi \cdot p_2 - p_1\right) - F
\end{aligned} \tag{6.23}$$

Note that in Equation (6.23) $F_{tC} = A \cdot \left(\varphi \cdot p_2 - p_1\right)$ has a negative value and *F* has a positive value.
The effective mass, m_{eff}, and the applied force, *F*, must be related to the piston of the cylinder. In general, they are both functions of x and $\dot{x}$ and may, similar to the values associated with the motor, be determined from energy considerations. The applied force may be computed as follows:

$$F = \frac{-dW_{ext}}{dx} \tag{6.24}$$

The effective mass may be computed according to:

$$m_{eff} = \frac{2 \cdot E_{kin}}{\dot{x}^2} \tag{6.25}$$

As an example consider the system shown in Figure 6.5.

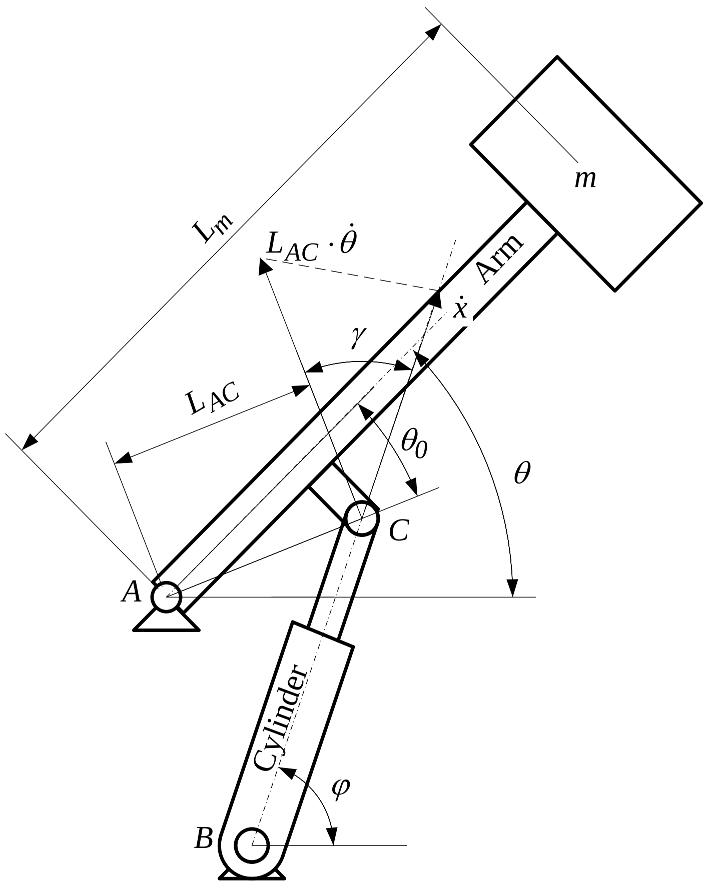


Figure 6.5. *A hydraulic cylinder is rotating an arm with a payload at the end.*

For a infinitesimal extraction of the cylinder the work done on the system by the external loading can be computed and, subsequently, the force applied to the cylinder piston:

$$W_{ext} = -m \cdot g \cdot dy_m = -m \cdot g \cdot L_m \cdot \cos\theta \cdot d\theta$$
$$\Downarrow$$
$$F = m \cdot g \cdot L_m \cdot \cos\theta \cdot \frac{\dot{\theta}}{\dot{x}} = m \cdot g \cdot L_m \cdot \cos\theta \cdot \frac{\dot{\theta}}{L_{AC} \cdot \dot{\theta} \cdot \cos\gamma} = m \cdot g \cdot \frac{L_m \cdot \cos\theta}{L_{AC} \cdot \cos\gamma} \qquad (6.26)$$
$$\gamma = \theta - \theta_0 + \frac{\pi}{2} - \varphi \qquad \varphi = tg^{-1}\left(\frac{y_C - y_B}{x_C - x_B}\right)$$

In Equation (6.26) it is utilized that the arm and the piston must have the same absolute velocity in point C.
The effective mass may be computed as:

$$E_{kin} = \frac{1}{2} \cdot m \cdot v^2 = \frac{1}{2} \cdot m \cdot \left(L_m \cdot \dot{\theta}\right)^2 = \frac{1}{2} \cdot m \cdot L_m^2 \cdot \left(\frac{\dot{x}}{L_{AC} \cdot \cos\gamma}\right)^2$$
$$\Downarrow \qquad (6.27)$$
$$m_{eff} = m \cdot \left(\frac{L_m}{L_{AC} \cdot \cos\gamma}\right)^2$$

Having determined the effective mass it is possible to compute the eigenfrequency of the hydraulic-mechanical system composed of the lines to and from the cylinder, the cylinder and the mechanical system represented by the effective mass:

$$\omega_n = \sqrt{\frac{k}{m_{eff}}} \qquad k = \frac{\beta \cdot A^2}{V_1} + \frac{\beta \cdot (\varphi \cdot A)^2}{V_2} \tag{6.28}$$

In Equation (6.28) V_1 and V_2 are simply the total volume of the fluid leading up to the cylinder piston side and to the cylinder rod side, respectively. This include the volume in the lines as well as the volume in the cylinder. Hence, in general the volumes are functions of the piston position.

6.4 Numerical solution

The actual solving of the coupled set of differential and algebraic equations can be performed in several ways. Today, a several software packages exist that allow for relatively fast and easy modeling and simulation of physical systems. The packages may vary with respect to modeling concepts and solver algorithms, however, the fundamentals remain the same and in the following the basic architecture of the numerical simulation of hydraulic-mechanical systems is presented.
As described in the previous chapter the state variables must be initialized. Normally, that include:

$$\underline{X} = \begin{bmatrix} \underline{p} & \underline{V}_f & \underline{x} & \underline{\dot{x}} & \underline{\theta} & \underline{\dot{\theta}} \end{bmatrix} \tag{6.29}$$

Here, $\underline{X}$ is an algebraic vector of state variables consisting of all volume pressures, all accumulator fluid volumes, all cylinder positions and velocities and all motor angular positions and angular velocities.
Based on the state variables Equations (6.3), (6.5), (6.8) and (6.18) may be set up to yield a linear set of equations:

$$\underline{\underline{M}}(\underline{X}) \cdot \underline{\dot{X}} = \underline{Y}(\underline{X}) \Rightarrow \underline{\dot{X}} = \underline{\underline{M}}^{-1}\underline{Y} \tag{6.30}$$

Setting up the coefficient matrix $\underline{\underline{M}}(\underline{X})$ and the right hand side $\underline{Y}(\underline{X})$ is done by solving the algebraic equations, i.e., the steady state equations of components where the dynamic properties may be ignored.
Often, the equations are not coupled very closely meaning that inverting the coefficient matrix $\underline{\underline{M}}(\underline{X})$ can be avoided or divided into the inversion of smaller sub matrices.

Having computed the time derivative of the state variables the next step is the time integration. The simplest way of doing this is by means of a so-called forward-Euler:

$$t^{(new)} = t + dt$$
$$\underline{X}^{(new)} = \underline{X} + \underline{\dot{X}} \cdot dt \tag{6.31}$$

As an example, let us consider the hydraulic system shown in Figure 6.6 (see also
ate analysis and Figure 6.1) and set up the combined set of differential and algebraic equations.

Pressure build up equations

$$\dot{p}_1 = \frac{\beta}{V_1} \cdot \left(Q_1 - Q_3 - Q_2 - \dot{V}_f\right)$$

$$\dot{p}_2 = \frac{\beta}{V_2} \cdot \left(Q_3 - Q_4\right) \quad \dot{p}_3 = \frac{\beta}{V_3} \cdot \left(Q_4 - Q_5\right)$$

Flow continuity pumps and actuators

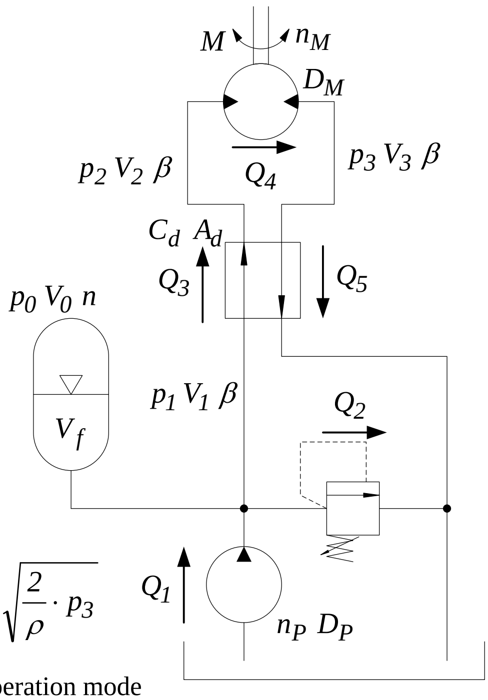
$$Q_1 = n_P \cdot D_P \qquad Q_4 = n_M \cdot D_M$$

Actuator dynamic equilibrium

$$J_{eff} \cdot \ddot{\theta} = \frac{D_M \cdot \left(p_2 - p_3\right)}{2 \cdot \pi} - M$$

Restriction flows

$$Q_3 = C_d \cdot A_d \cdot \sqrt{\frac{2}{\rho} \cdot \left(p_1 - p_2\right)} \qquad Q_5 = C_d \cdot A_d \cdot \sqrt{\frac{2}{\rho} \cdot p_3}$$



Accumulator equations

$$\dot{V}_f = \frac{V_g}{n \cdot p_1} \cdot \dot{p}_1 \qquad V_g = \left(\frac{p_0}{p_1}\right)^{\frac{1}{n}} \cdot V_0$$

Operation mode

$$Q_2 = 0$$

Figure 6.6. *Hydraulic circuit and corresponding set of differential and algebraic equations.*

One important result from doing dynamic time domain simulation is that the operation mode is no longer a guess, now it can be evaluated directly from the state variables. In this case since we have access to p_1 we simply compare it with the crack pressure of the pressure relief valve.

In order to run a simulation obviously some input must be prescribed in time. Typically, this can be the load on the motor output shaft, the motion of the directional control valve and the angular speed of the pump.

6.5 Valve modeling

Valves come in a very large different variations and design. They do, however, almost all have one important similarity: they facilitate one or more hydraulic connections. The connections can be characterized as:

1. variable generated by moving bodies or constant
2. type of operation (manual, control signal, hydraulic pilot signal)
3. on/off or proportional
4. negligeable dynamics of moving bodies

In fact, almost any type of valve can be modeled as one or more of these connections. Each connection is a hydraulic restriction can be formulated as:

$$\Delta p = f(u,Q) \tag{6.32}$$

In (6.32) the function f normally reflects both the geometry and the flow regime (laminar or turbulent).

The dimensionless parameter should reflect whether the connections if fully closed, $u=0$, fully opened $u=1$, or somewhere in between, $0<u<1$. This value can be obtained in different ways depending on the type of connection. It the connections if fixed, so is u (typically to $u=1$ but not necessarily). If the connection is variable then u is also a variable and should be computed based on the type of operation and the importance of the dynamics. If the operation is manual og by means of an electrical control signal then u is an input parameter in the modeling that either is given as part of the load case to be investigated or from measurements. If, however, it stems from hydraulic pilot signals then it must be computed from the state values of the different pilot signals together with the other forces that may act on a moving body within a valve such as: stiction, Coulomb friction, viscous friction, flow forces and spring forces. It is vital for the success of the modeling that saturation is considered for this case, i.e., taking into account that $0 \le u \le 1$. In practice this corresponds to including the reactive forces from seats etc. whenever the spool is pushed into an extreme position. Some connections can be modeled as on/off, i.e., $u=0$ or $u=1$. Since no physical connection is totally on/off this should be used when the intermediate positions are either uninteresting or happens very fast as compared to the remaining model. If the dynamics of the moving bodies should be included then it is often a good first approximation to simply model the dimensionless parameters as a second order system with a damping that is close or equal to critical damping and a natural frequency that reflects the bandwidth of the connection.

The volume flow will typically be computed from an orifice equation that takes into account whether the flow in the connection is laminar, turbulent or a mix of these. The main advantage of using (6.32) is that even for complex hydraulic circuits it is possible to address each valve in a systematic way and modeling it as one or more connections characterized by the four points above.

----- oo 0 oo -----

----- oo 0 oo -----

7
Control of Hydraulic Servo Systems

Control of Hydraulic Servo Systems

7.1 Introduction

Hydraulic servo systems are always controlled by means of some kind of closed loop control utilizing some feedback signal to control a servo valve or a variable displacement pump. In the following focus will be on valve controlled circuits where one or more high response valve, typically referred to as a servo valves, are the control elements. There exist a wide variety of control methods that are closely related to the type of task that the hydraulic servo system should undertake and also to the type of actuator and its connection to the valve and payload. Again, it has been necessary to narrow the focus to position and velocity control of symmetrical actuators.

7.2 Dynamics of Hydraulic Servo System

Let us consider the hydraulic servo system shown in Figure 7.1. The set of parameters includes the normalized position of the spool of the servo valve, $-1 \le u \le 1$. For this system it is possible to set up the ideal governing equations:

$$Q_L = A \cdot \dot{y} \tag{7.1}$$

$$m \cdot \ddot{y} = p_L \cdot A - F \tag{7.2}$$

$$Q_v = C_d \cdot A_{d0} \cdot u \cdot \sqrt{\frac{1}{\rho} \cdot (p_s - \frac{u}{|u|} \cdot p_L)} \tag{7.3}$$

In (7.3) it is assumed that the servo valve is symmetric and that the maximum discharge area is A_{d0}. Also note that both the load pressure, p_L, the load flow, Q_L, and the spool travel, u, may take on negative as well as positive values.

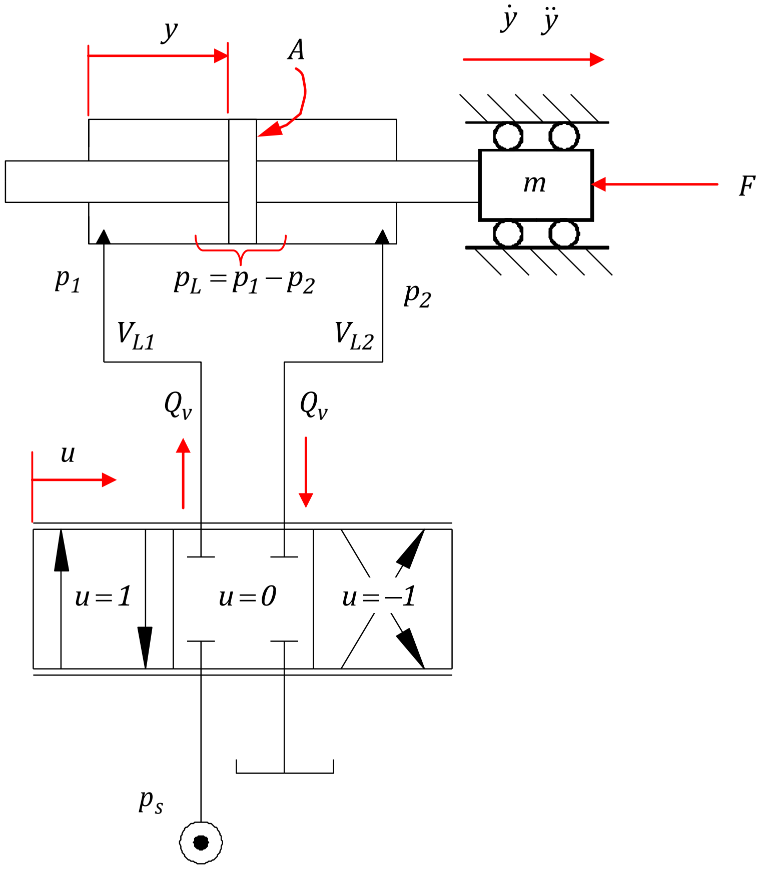


Figure 7.1 Hydraulic servo system using a symmetrical cylinder to translate the payload.

Taking into account the flexibility of the fluid we get the following differential equation for the chamber pressures:

$$\dot{p}_1 = \frac{\beta}{V_{L1} + y \cdot A} \cdot (Q_v - A \cdot \dot{y}) \tag{7.4}$$

$$\dot{p}_2 = \frac{\beta}{V_{L2} + (h - y) \cdot A} \cdot (A \cdot \dot{y} - Q_v) \tag{7.5}$$

In (7.5) the stroke of the cylinder, h, is introduced. Assuming that the cylinder is in midstroke, $y = 0.5 \cdot h$, and that the volume of the hoses are identical, $V_{L1} = V_{L2} = V_L$, we can derive the following expressions for the load pressure gradient:

$$\dot{p}_L = \dot{p}_1 - \dot{p}_2 = \frac{\beta}{V_{L1} + y \cdot A} \cdot (Q_v - A \cdot \dot{y}) - \frac{\beta}{V_{L2} + (h - y) \cdot A} \cdot (A \cdot \dot{y} - Q_v) \Rightarrow$$

$$\dot{p}_L = \frac{\beta}{V_L + \frac{h}{2} \cdot A} \cdot (2 \cdot Q_v - 2 \cdot A \cdot \dot{y}) = \frac{4 \cdot \beta}{V_0} \cdot (Q_v - A \cdot \dot{y}) \tag{7.6}$$

In (7.6) the total fluid volume, $V_0 = A \cdot h + 2 \cdot V_L$, between the valve and the actuator has been introduced.
In order to further examine the dynamic characteristics of the hydro-mechanical system in the frequency domain it is necessary to linearize the governing equations and do a laplace transformation. The linearized governing equations are:

$$\Delta Q_L = A \cdot \Delta \dot{y} \quad \textbf{(7.7)}$$

$$m \cdot \Delta \ddot{y} = \Delta p_L \cdot A - \Delta F \quad \textbf{(7.8)}$$

$$\Delta Q_v = K_{qu} \cdot \Delta u - K_{qp} \cdot \Delta p_L \quad \textbf{(7.9)}$$

$$\Delta \dot{p}_L = \frac{4 \cdot \beta}{V_0} \cdot (\Delta Q_v - A \cdot \Delta \dot{y}) \quad \textbf{(7.10)}$$

The linearization is carried out for a steady state situation, denoted by superscript ss, hence, all the variables must be considered as deviations from this situation.

$$\Delta Q_L = Q_L - Q_L^{(ss)} \quad \textbf{(7.11)}$$

$$\Delta Q_v = Q_v - Q_v^{(ss)} \quad \textbf{(7.12)}$$

$$\Delta p_L = p_L - p_L^{(ss)} \quad \textbf{(7.13)}$$

$$\Delta \dot{p}_L = \dot{p}_L - \dot{p}_L^{(ss)} = \dot{p}_L \quad \textbf{(7.14)}$$

$$\Delta \dot{y} = \dot{y} - \dot{y}^{(ss)} \quad \textbf{(7.15)}$$

$$\Delta \ddot{y} = \ddot{y} - \ddot{y}^{(ss)} = \ddot{y} \quad \textbf{(7.16)}$$

The governing equations for the steady state situation are:

$$Q_L^{(ss)} = A \cdot \dot{y}^{(ss)} \quad \textbf{(7.17)}$$

$$p_L^{(ss)} \cdot A = F^{(ss)} \quad \textbf{(7.18)}$$

$$Q_v^{(ss)} = C_d \cdot A_{d0} \cdot u^{(ss)} \cdot \sqrt{\frac{1}{\rho} \cdot (p_s - \frac{u^{(ss)}}{\left| u^{(ss)} \right|} \cdot p_L^{(ss)})} \quad \textbf{(7.19)}$$

$$Q_v^{(ss)} = A \cdot \dot{y}^{(ss)} = Q_L^{(ss)} \quad \textbf{(7.20)}$$

Special attention must be given to the linearization of the only non-linear equation (7.3) leading to the linear equation (7.9). The theoretical definition of the two coefficients of (7.9) are readily derived from the Taylor expansion:

$$K_{qu} = \left. \frac{\partial Q_v}{\partial u} \right|_{ss} = C_d \cdot A_{d0} \cdot \sqrt{\frac{1}{\rho} \cdot (p_s - \frac{u^{(ss)}}{\left| u^{(ss)} \right|} \cdot p_L^{(ss)})} \quad \textbf{(7.21)}$$

The coefficient, K_{qu}, will be referred to as the flow gain. It is the sensitivity of the load flow relative to the spool travel and with the load pressure held constant. It appears directly in the total gain of the control of the hydraulic servo systems and is of major importance in control of hydraulic servo systems in general.

$$K_{qp} = -\frac{\partial Q_v}{\partial p_L}\bigg|_{ss} = \frac{C_d \cdot A_{d0} \cdot \left|u^{(ss)}\right|}{2 \cdot \sqrt{\rho \cdot \left(p_s - \frac{u^{(ss)}}{\left|u^{(ss)}\right|} \cdot p_L^{(ss)}\right)}} \tag{7.22}$$

The coefficient, K_{qp}, will be referred to as the flow-pressure gain. It is the sensitivity of the load flow relative to the load pressure with the spool travel held constant. The flow-pressure gain represents damping in any hydraulic servo system. Finally, a third coefficient can be computed from the first two:

$$K_{pu} = \frac{K_{qu}}{K_{qp}} = \frac{2 \cdot \left(p_s - \frac{u^{(ss)}}{\left|u^{(ss)}\right|} \cdot p_L^{(ss)}\right)}{\left|u^{(ss)}\right|} \tag{7.23}$$

This coefficient is the pressure gain, and it reflects the sensitivity of the load pressure relative to the spool travel with the load flow held constant (locked piston). Although it does not enter directly into the control equations it is of major importance when evaluating the ability of the control loop to handle disturbances.
Clearly, the steady state situation strongly influences the value of the three coefficients. Especially, the so-called 0-position is often chosen as steady state reference because it yields a conservative controller design. The 0-position is defined as:

$$\begin{aligned} p_L^{(0)} &= 0 \\ u^{(0)} &= 0 \\ Q_v^{(0)} &= Q_L^{(0)} = 0 \end{aligned} \tag{7.24}$$

Theoretically, that gives the following coefficients:

$$\begin{aligned} K_{qu,t}^{(0)} &= C_d \cdot A_{d0} \cdot \sqrt{\frac{1}{\rho} \cdot p_s} \\ K_{qp,t}^{(0)} &= 0 \\ K_{pu,t}^{(0)} &= \infty \end{aligned} \tag{7.25}$$

Note that an extra index, *t*, has been added to indicate that these are theoretical values. In fact, the theoretical flow coefficient is in good accordance with values observed in practice for any steady state situation. On the contrary, the actual values of the flow-pressure coefficient and the pressure coefficient differ strongly from the theoretical values when the spool is in neutral or close by, $u^{(ss)} \approx 0$. In the 0-position this is most evident. An acceptable way of handling the discrepancy, is simply to introduce a minimum steady state spool travel, $u_\varepsilon = u_{min}^{(ss)}$, that replaces the actual steady state spool position, $u^{(ss)}$, if its numerical value becomes to small:

$$u_{lim}^{(ss)} = \begin{cases} \dfrac{u^{(ss)}}{\left|u^{(ss)}\right|} \cdot u_{\varepsilon} & \left|u^{(ss)}\right| \leq u_{\varepsilon} \\ u^{(ss)} & \left|u^{(ss)}\right| \geq u_{\varepsilon} \end{cases} \tag{7.26}$$

Typical values that reflects the leakage would be $u_{\varepsilon} = 0.002 \cdots 0.05$ depending on the wear conditions of the leakage paths between spool and housing. Using (7.26) it is possible to set up more realistic expressions for the coefficients in the 0-situation:

$$\begin{aligned} K_{qu}^{(0)} &= K_{qu,t}^{(0)} = C_d \cdot A_{d0} \cdot \sqrt{\frac{1}{\rho} \cdot p_s} \\ K_{qp}^{(0)} &= \frac{C_d \cdot A_{d0} \cdot u_{lim}^{(ss)}}{2 \cdot \sqrt{\rho \cdot p_s}} \\ K_{pu}^{(0)} &= \frac{K_{qu}^{(0)}}{K_{qp}^{(0)}} = \frac{2 \cdot p_s}{u_{lim}^{(ss)}} \end{aligned} \tag{7.27}$$

Next, the governing equations (7.7 ... 7.10) are subjected to Laplace transformation:

$$Q_L(s) = A \cdot s \cdot y(s) \tag{7.28}$$

$$m \cdot s^2 \cdot y(s) = p_L(s) \cdot A - F(s) \tag{7.29}$$

$$Q_v(s) = K_{qu} \cdot u(s) - K_{qp} \cdot p_L(s) \tag{7.30}$$

$$s \cdot p_L(s) = \frac{1}{C} \cdot (Q_v(s) - A \cdot s \cdot y(s)) \tag{7.31}$$

In (7.31) the total capacitance of the hydraulic fluid, C, has been introduced. It is defined as:

$$C = \frac{V_0}{4 \cdot \beta} \tag{7.32}$$

In Figure 7.2 the equations (7.28 ... 7.31) are shown as block diagrams with the spool position as input and the piston speed as output. The disturbance, F, is neglected in the simplifications.

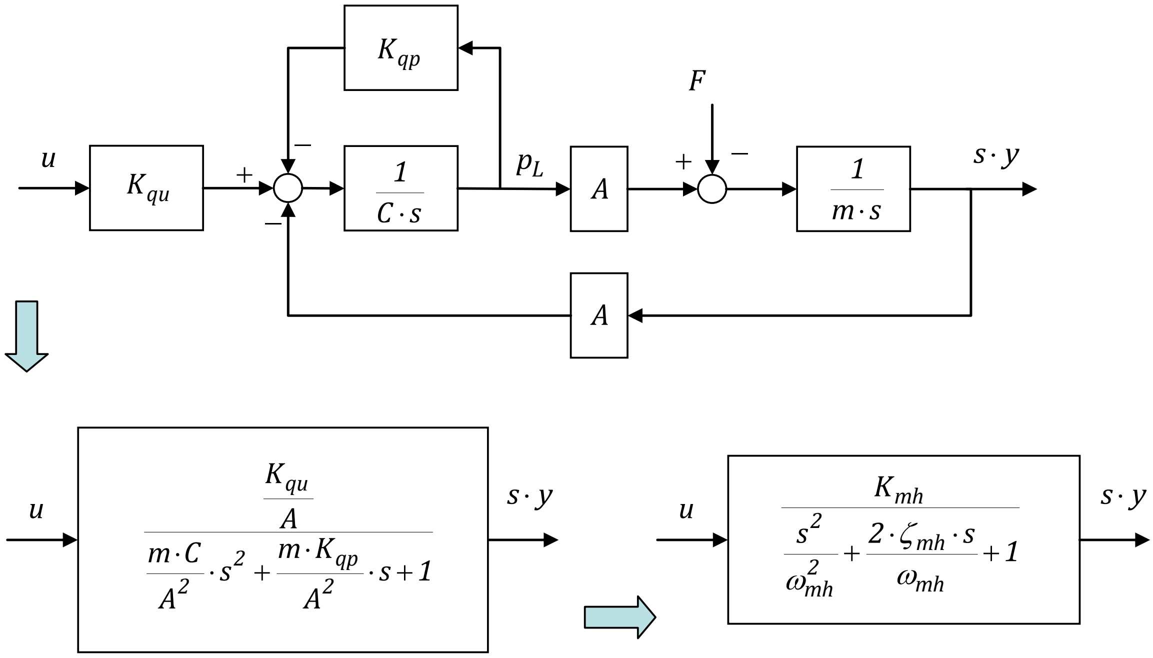


Figure 7.2 Block diagram of hydraulic servo system.

The transfer function between piston speed and spool position describes a 2nd order system with the gain K_{mh}, the eigenfrequency ω_{mh}, and the damping ζ_{mh}. The index refers to mechanical-hydraulic system. The transfer function and the associated parameters are given as:

$$G_{mh}(s) = \frac{s \cdot y}{u} = \frac{K_{mh}}{\frac{s^2}{\omega_{mh}^2} + \frac{2 \cdot \zeta_{mh} \cdot s}{\omega_{mh}} + 1} \tag{7.33}$$

$$K_{mh} = \frac{K_{qu}}{A} \tag{7.34}$$

$$\omega_{mh} = \frac{A}{\sqrt{m \cdot C}} \tag{7.35}$$

$$\zeta_{mh} = \frac{K_{qp}}{2 \cdot A} \cdot \sqrt{\frac{m}{C}} \tag{7.36}$$

In the time domain the piston speed can be written as a function of time when subjected to a the step input, u_{in}, to the dimensionless spool travel. The function is:

$$\frac{\dot{y}(t)}{u_{in}} = K_{mh} - \frac{K_{mh}}{\sqrt{1-\zeta_{mh}^2}} \cdot e^{-\zeta_{mh} \cdot \omega_{mh} \cdot t} \cdot sin\left(\omega_{mh} \cdot \sqrt{1-\zeta_{mh}^2} \cdot t + tg^{-1}\left(\frac{\sqrt{1-\zeta_{mh}^2}}{\zeta_{mh}}\right)\right) \tag{7.37}$$

The size of the step input must lie in the interval $-1 \le u_{in} \le 1$. The time it takes for the piston speed to reach the steady state value the first time is called the crossing time, t_{cr}, and the subsequent relative overshoot is referred to as Ω. They can be computed as:

$$t_{cr} = \frac{\pi - tg^{-1}\left(\frac{\sqrt{1-\zeta_{mh}^2}}{\zeta_{mh}}\right)}{\omega_{mh} \cdot \sqrt{1-\zeta_{mh}^2}} \tag{7.38}$$

$$\Omega = \frac{\dot{y}_{max} - u_{in} \cdot K_{mh}}{u_{in} \cdot K_{mh}} = e^{-\left(\frac{\zeta_{mh}}{\sqrt{1-\zeta_{mh}^2}} \cdot \pi\right)} \tag{7.39}$$

The soft parameters, K_{qu}, K_{pq} and C may be determined experimentally if the crossing time and the relative overshoot has been measured. In that case the damping and the eigenfrequency of the hydraulic-mechanical system are first computed as:

$$\zeta_{mh} = \frac{k_\Omega}{\sqrt{1+k_\Omega^2}} \qquad k_\Omega = -\frac{ln(\Omega)}{\pi} \tag{7.40}$$

$$\omega_{mh} = \frac{\pi - tg^{-1}\left(\frac{\sqrt{1-\zeta_{mh}^2}}{\zeta_{mh}}\right)}{t_{cr} \cdot \sqrt{1-\zeta_{mh}^2}} \tag{7.41}$$

The gain, K_{mh}, can be taken from the steady state value of the piston speed as:

$$K_{mh} = \frac{\dot{y}(t \to \infty)}{u_{in}} \tag{7.42}$$

Finally, the soft parameters are computed:

$$K_{qu} = A \cdot K_{mh} \tag{7.43}$$

$$C = \frac{A^2}{m \cdot \omega_{mh}^2} \tag{7.44}$$

$$K_{qp} = 2 \cdot A \cdot \zeta_{mh} \cdot \sqrt{\frac{C}{m}} \tag{7.45}$$

The governing equations have very much the same structure for a hydraulic servo system with a motor rather than a symmetrical cylinder as actuator. If the motor has a displacement of D and drives an inertia of J then we get the following transfer function from the valve spool position, u, to the angular speed of the motor, $s \cdot \theta$

$$G_{mh}(s) = \frac{s \cdot \theta}{u} = \frac{K_{mh}}{\frac{s^2}{\omega_{mh}^2} + \frac{2 \cdot \xi_{mh} \cdot s}{\omega_{mh}} + 1} \tag{7.46}$$

$$K_{mh} = \frac{K_{qu}}{D_\omega} \quad \textbf{(7.47)}$$

$$\omega_{mh} = \frac{D_\omega}{\sqrt{J \cdot C}} \quad \textbf{(7.48)}$$

$$\zeta_{mh} = \frac{K_{qp}}{2 \cdot D_\omega} \cdot \sqrt{\frac{J}{C}} \quad \textbf{(7.49)}$$

$$D_\omega = \frac{D}{2 \cdot \pi} \quad \textbf{(7.50)}$$

The total volume used to compute the capacitance, C, is also different from the cylinder actuator, $V_0 = D + 2 \cdot V_L$

7.3 Closed Loop Control

In position control the task of the hydraulic servo system is to follow either a reference position of the cylinder piston or a reference rotation of the motor output shaft. In that case, some position feedback from the actuator is required. A typical setup is shown in Figure 7.3.

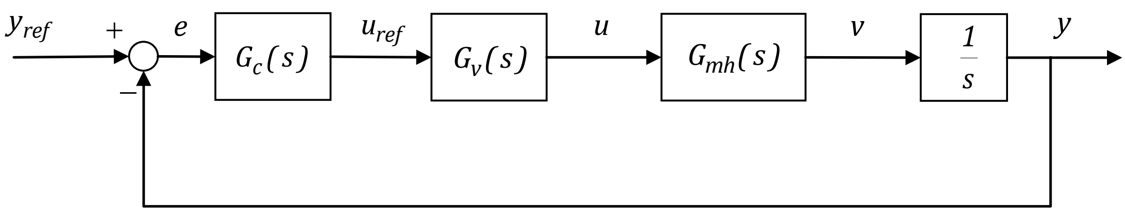


Figure 7.3 Block diagram of position servo.

The G_v block represents the closed loop spool position control inside the valve and the dynamics of the electrical-mechanical system of the valve. For simplified analysis this block is normally modeled as a second order system:

$$G_v(s) = \frac{1}{\frac{s^2}{\omega_v^2} + \frac{2 \cdot \zeta_v \cdot s}{\omega_v} + 1} \quad \textbf{(7.51)}$$

Similarly, the G_{mh} block represents the second order system derived in the previous section, see (7.33) and (7.46):

$$G_{mh}(s) = \frac{K_{mh}}{\frac{s^2}{\omega_{mh}^2} + \frac{2 \cdot \zeta_{mh} \cdot s}{\omega_{mh}} + 1} \quad \textbf{(7.52)}$$

The performance of any hydraulic servo system depends on the bandwidth of the product of these two transfer functions. As explained in Chapter 5, the typical approach is to select a valve with a deadband three times larger, or, $\omega_v \geq 3 \cdot \omega_{mh}$. If this design rule is observed, the overall dynamics of the valve and hydraulic-mechanical system will approximately be that of the hydraulic-mechanical system. If the valve has a smaller deadband then the total deadband of the system is markedly reduced. In Figure 7.4 the bode plot of G_{mh} is shown together with the product $G_v \cdot G_{mh}$ for $\omega_v = \omega_{mh}$ and $\omega_v = 3 \cdot \omega_{mh}$.

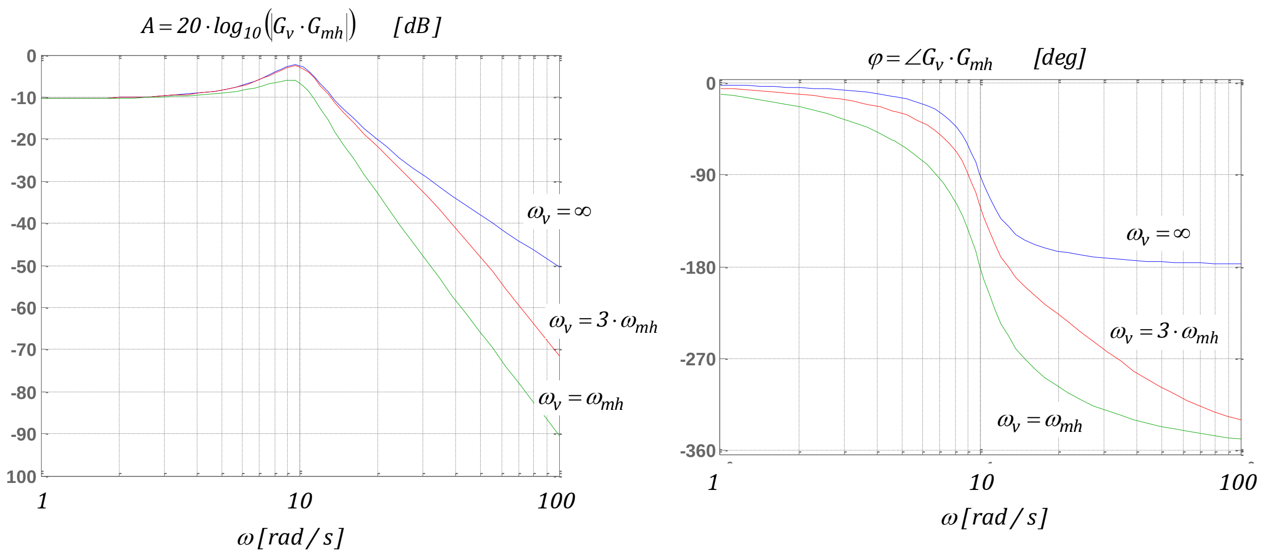


Figure 7.4 Bode plot of transfer functions of different combinations of servo valve dynamics and hydraulic-mechanical system dynamics. The parameters of the transfer function are: $K_{mh} = 0.3$, $\omega_{mh} = 10 \frac{rad}{s}$, $\zeta_{mh} = 0.2$ and $\zeta_v = 0.8$.

Clearly, choosing $\omega_v = \omega_{mh}$ yields a system with distinctly poorer dynamic performance whereas choosing $\omega_v \geq 3 \cdot \omega_{mh}$ justifies the simplification:

$$G_{vmh} = \frac{K_{mh}}{\frac{s^2}{\omega_{vmh}^2} + \frac{2 \cdot \zeta_{mh} \cdot s}{\omega_{vmh}} + 1} \approx G_v \cdot G_{mh} \qquad \textbf{(7.53)}$$

$$\omega_{vmh} = 0.9 \cdot \omega_{mh}$$

Equation (7.53) is valid for frequencies up to and around ω_{mh}. Frequencies beyond this value is rarely of interest in hydraulic servo systems. So we simply disregard the valve dynamics and continue working with an effective eigenfrequency of the valve-mechanical-hydraulic system, ω_{vmh}, that is 90% of ω_{mh}. It must be emphasized again, that this approach is only valid for $\omega_v \geq 3 \cdot \omega_{mh}$.

In Figure 7.3 the control block contains the control law. In this section we will investigate pure proportional control as well as proportional control compensated with a lead-lag network. If the control law is simply proportional control, $G_c = K_p$, we get the following open loop transfer function for the position servo:

$$G_{po} = K_p \cdot G_{vmh} \cdot \frac{1}{s} = \frac{K_p \cdot K_{mh}}{s \cdot \left(\frac{s^2}{\omega_{vmh}^2} + \frac{2 \cdot \zeta_{mh} \cdot s}{\omega_{vmh}} + 1 \right)} \tag{7.54}$$

We want to choose the gain of the controller, K_p, as high as possible in order to obtain a good performance with respect to precision and reaction time. The upper limit is introduced via the stability criterion of Nyquist that simply states:
In order for the closed loop transfer function to be stable then the gain of the open loop transfer function must be less than unity when the phase lag is $180°$.
In Figure 7.5 the bode plot of a typical open loop transfer function is shown. The damping is set to $\zeta_{mh} = 0.2$ which is a typical value for the, in general, poorly damped hydraulic-mechanical systems.

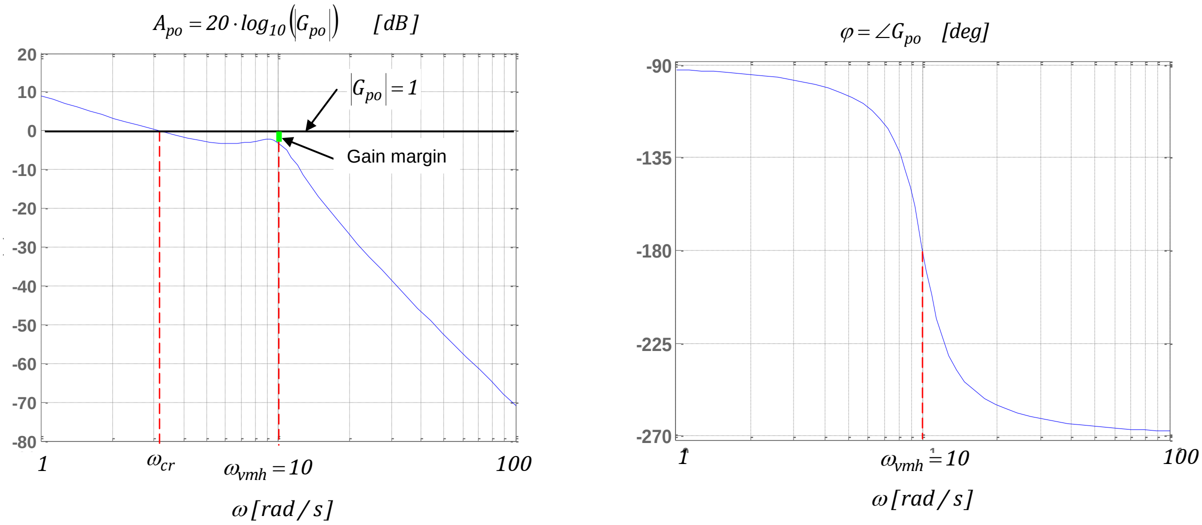


Figure 7.5 Bode plot of position servo open loop transfer function, G_{po}. The parameters of the transfer function are: $K_p \cdot K_{mh} = 2.8$, $\omega_{vmh} = 10\ \frac{rad}{s}$ and $\zeta_{mh} = 0.2$.

In order for the system to be stable then the gain must be less than unity for $\omega = \omega_{vmh}$. Based on this, the maximum value of the controller gain can be derived from (7.54) as:

$$\begin{aligned} & \left| G_{po}(s = j \cdot \omega_{vmh}) \right| \leq 1 \Rightarrow \\ & \frac{K_p \cdot K_{mh}}{\left| j \cdot \omega_{mh} \cdot \left(\frac{(j \cdot \omega_{vmh})^2}{\omega_{vmh}^2} + \frac{2 \cdot \zeta_{mh} \cdot j \cdot \omega_{vmh}}{\omega_{vmh}} + 1 \right) \right|} \leq 1 \Rightarrow \\ & K_p \leq \frac{2 \cdot \zeta_{mh} \cdot \omega_{vmh}}{K_{mh}} \end{aligned} \tag{7.55}$$

The result in (7.55) is quite general within servo systems. In practice, a certain safety factor must be introduced, mostly because ζ_{mh} is difficult to assess exactly without experimental work. This safety factor is normally expressed in dB (deciBell) based on the typical presentation of transfer function gain

$$A_{po}(\omega) = 20 \cdot log_{10}\left(\left|G_{po}(s = j \cdot \omega)\right|\right) \qquad \textbf{(7.56)}$$

Let the safety factor be $A_{po} = -A_{vmh}$. In that case the maximum value of the controller gain can be derived from:

$$\begin{aligned} &20 \cdot log_{10}\left(\left|G_{po}(s = j \cdot \omega_{vmh})\right|\right) = -A_{vmh} \Rightarrow \\ &K_p = 10^{\frac{-A_{vmh}}{20}} \cdot \frac{2 \cdot \zeta_{mh} \cdot \omega_{vmh}}{K_{mh}} \end{aligned} \qquad \textbf{(7.57)}$$

For a typical safety factor of $A_{vmh} = 3\,dB$ we get:

$$K_p = 0.7079 \cdot \frac{2 \cdot \zeta_{mh} \cdot \omega_{vmh}}{K_{mh}} \qquad \textbf{(7.58)}$$

In Figure 7.5 the cross frequency, ω_{cr}, is indicated as the frequency at which we have unity gain. An alternative interpretation of Nyquist means that at this frequency the phase lag must not yet have reached $-180°$. The equation for the cross frequency is:

$$\left|G_{po}(s = j \cdot \omega_{cr})\right| = 1 \Rightarrow A_{po}(\omega_{cr}) = 0 \qquad \textbf{(7.59)}$$

For low damping, $\zeta_{mh} \le 0.4$ it is usually a good approximation to state that:

$$\omega_{cr} \approx K_p \cdot K_{mh} \qquad \textbf{(7.60)}$$

The closed loop transfer function becomes:

$$G_{pc} = \frac{1}{\frac{s^3}{K_p \cdot K_{mh} \cdot \omega_{vmh}^2} + \frac{2 \cdot \zeta_{mh} \cdot s^2}{K_p \cdot K_{mh} \cdot \omega_{vmh}} + \frac{s}{K_p \cdot K_{mh}} + 1} \qquad \textbf{(7.61)}$$

In Figure 7.6 the bode plot of a closed loop transfer function corresponding to the open loop transfer function of Figure 7.5 is shown.
The bandwidth of the entire system is normally taken to be the frequency, ω_{bw}, at which we have a closed loop transfer gain of $-3\,dB$:

$$\begin{aligned} &A_{pc}(\omega) = 20 \cdot log_{10}\left(\left|G_{pc}(s = j \cdot \omega)\right|\right) \\ &\Downarrow \\ &A_{pc}(\omega_{bw}) = 20 \cdot log_{10}\left(\left|G_{pc}(s = j \cdot \omega_{bw})\right|\right) = -3 \end{aligned} \qquad \textbf{(7.62)}$$

For low damping, $\zeta_{mh} \le 0.4$ it is usually a good approximation to state that:

$$\omega_{cr} \approx K_p \cdot K_{mh} \approx \omega_{bw} \tag{7.63}$$

This often simplifies analysis of position servos substantially since the main dynamic performance parameter, ω_{bw}, can be estimated as the product of two known gains.

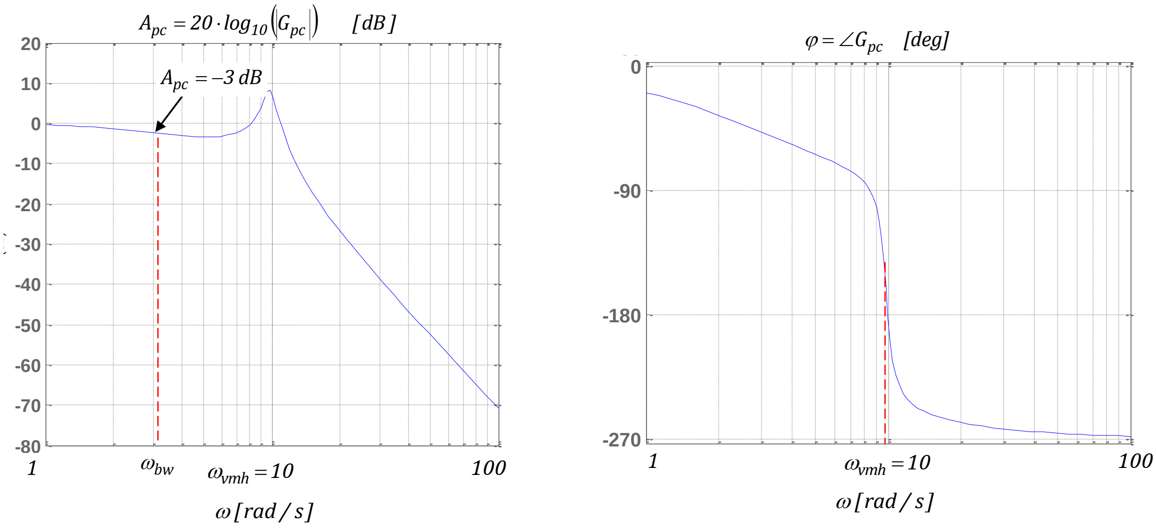


Figure 7.6 Bode plot of position servo closed loop transfer function.

To further investigate the simple proportional controller we examine the steady state error, $e^{(ss)}$, for two situations:

- a step input (constant position reference)
- a ramp input (constant velocity reference)

First, we establish an expression for the steady state error, see also Figure 7.3:

$$\begin{aligned} & e^{(ss)} = e(t \to \infty) = s \cdot e(s)\big|_{s=0} \\ & e(s) = y_{ref} - y = y_{ref} - G_{pc} \cdot y_{ref} = \left(1 - G_{pc}\right) \cdot y_{ref} \Rightarrow \\ & s \cdot e(s) = \frac{\dfrac{s^3}{K_p \cdot K_{mh} \cdot \omega_{vmh}^2} + \dfrac{2 \cdot \zeta_{mh} \cdot s^2}{K_p \cdot K_{mh} \cdot \omega_{vmh}} + \dfrac{s}{K_p \cdot K_{mh}}}{\dfrac{s^3}{K_p \cdot K_{mh} \cdot \omega_{vmh}^2} + \dfrac{2 \cdot \zeta_{mh} \cdot s^2}{K_p \cdot K_{mh} \cdot \omega_{vmh}} + \dfrac{s}{K_p \cdot K_{mh}} + 1} \cdot s \cdot y_{ref} \end{aligned} \tag{7.64}$$

For a step input with the size y_0 we insert $y_{ref} = \dfrac{y_0}{s}$ in (7.64):

$$e^{(ss)} = \left. \frac{\dfrac{s^3}{K_p \cdot K_{mh} \cdot \omega_{vmh}^2} + \dfrac{2 \cdot \zeta_{mh} \cdot s^2}{K_p \cdot K_{mh} \cdot \omega_{vmh}} + \dfrac{s}{K_p \cdot K_{mh}}}{\dfrac{s^3}{K_p \cdot K_{mh} \cdot \omega_{vmh}^2} + \dfrac{2 \cdot \zeta_{mh} \cdot s^2}{K_p \cdot K_{mh} \cdot \omega_{vmh}} + \dfrac{s}{K_p \cdot K_{mh}} + 1} \cdot s \cdot \frac{y_0}{s} \right|_{s=0} = 0 \tag{7.65}$$

Hence, the is no steady state error. However, if we take a ramp input with the speed v_0 we insert $y_{ref} = \frac{v_0}{s^2}$ in (7.64):

$$e^{(ss)} = \left. \frac{\frac{s^3}{K_p \cdot K_{mh} \cdot \omega_{vmh}^2} + \frac{2 \cdot \zeta_{mh} \cdot s^2}{K_p \cdot K_{mh} \cdot \omega_{vmh}} + \frac{s}{K_p \cdot K_{mh}}}{\frac{s^3}{K_p \cdot K_{mh} \cdot \omega_{vmh}^2} + \frac{2 \cdot \zeta_{mh} \cdot s^2}{K_p \cdot K_{mh} \cdot \omega_{vmh}} + \frac{s}{K_p \cdot K_{mh}} + 1} \cdot s \cdot \frac{v_0}{s^2} \right|_{s=0} = \frac{v_0}{K_p \cdot K_{mh}} \qquad \textbf{(7.66)}$$

Hence, whenever we have a constant velocity input of the reference speed, we will have a steady state position error.
This position error can be reduced in different ways. A typical approach is to add an I-term in the controller, thereby changing it into a PI-controller:

$$G_c = K_p \cdot \left(1 + \frac{1}{T_i \cdot s} \right) \qquad \textbf{(7.67)}$$

Also, it is common in servo applications to add a lead-lag term in the controller:

$$G_c = K_p \cdot \frac{1 + \frac{s}{\omega_{LL}}}{1 + \frac{\alpha_{LL} \cdot s}{\omega_{LL}}} \qquad \textbf{(7.68)}$$

This will add open-loop gain for frequencies below ω_{cr} which, in turn, will improve the accuracy (but not the bandwidth) of the position servo, see also Figure 7.7.

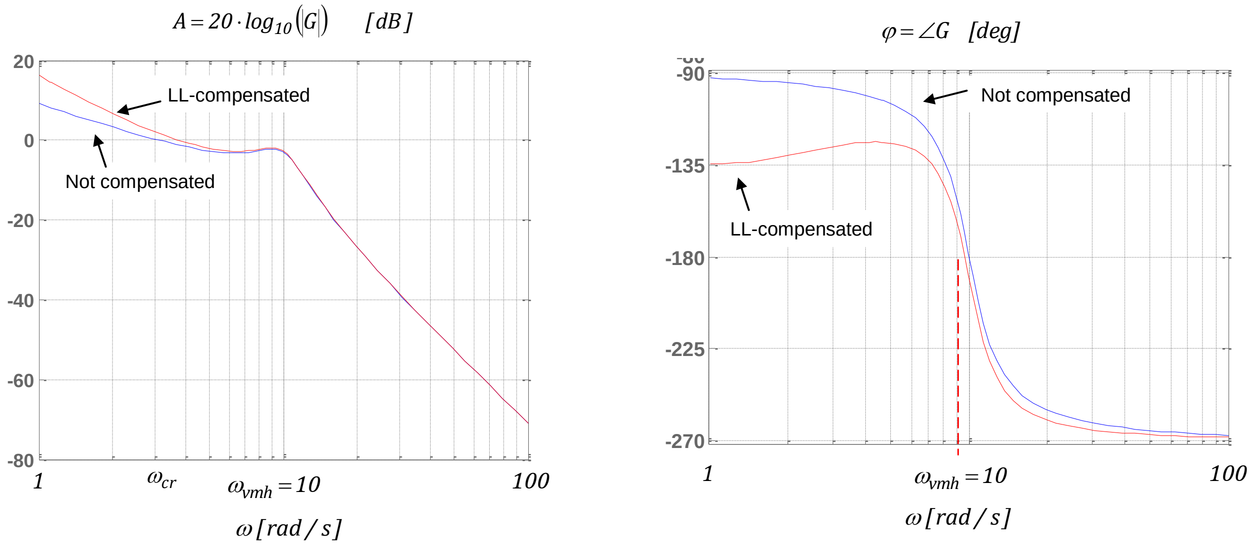


Figure 7.7 Bode plot of position servo open loop transfer function with and without lead-lag compensation. In this case, the lead-lag parameters are $\omega_{LL} = 2.25 \frac{rad}{s}$ (app. 80% of $\omega_{cr} \approx 3.0 \frac{rad}{s}$) and $\alpha_{LL} = 5$.

The lead frequency, ω_{LL}, and the lead-lag frequency ratio, α_{LL}, should preferrably be chosen based on the particular system, however, good results will normally be achieved by ensuring that:

$$\omega_{LL} \leq \omega_{cr} \tag{7.69}$$

$$2 \leq \alpha_{LL} \leq 10 \tag{7.70}$$

----- oo 0 oo -----

APPENDIX

Hydraulic Fluids

Hydraulic System Design

A.1 Introduction

The main purpose of the hydraulic fluid is to transport energy from the pump to the actuators. Secondary purposes involve the lubrication of the moving mechanical parts to reduce wear, noise and frictional losses, protecting the hydraulic components against corrosion and transporting heat away from its sources. The preferred working fluid in most applications is mineral oil, although in certain applications there is a requirement for water-based fluids. Water-based fluids and high water-based fluids provide fire resistance at a lower cost and have the advantage of relative ease of fluid storage and disposal. The recommended classification system is as follows:

HFA – dilute emulsions, i.e. oil-in-water emulsions, typically with 95% water content.

HFB – Invert emulsions, i.e. water-in-oil emulsions, typically with 40% water content.

HFC – Aqueous glycols, i.e. solutions of glycol and polyglycol in water, typically with 40% water content.

HFD – Synthetic fluids containing no water, such as silicone and silicote esters.

The selection of the appropriate fluid will require specialist advice from both the component manufacturer and the fluid manufacturer.

The most commonly used hydraulic fluid is mineral oil and in the following sections it is the physical properties of commercial mineral oils that is discussed.
The purpose of this appendix is to define certain physical properties which will prove useful and to discuss properties related to the nature of fluids. Because the fluid is the medium of transmission of power in a hydraulic system, knowledge of its characteristics is essential.

A.2 Fluid density

The mass density, ρ, of a hydraulic fluid is defined as a given mass divided by its volume, see Equation (A.1).

$$\rho = \frac{m}{V} \tag{A.1}$$

where

ρ	mass density $[kg/m^3]$
m	mass of the fluid $[kg]$
V	volume of the fluid $[m^3]$

The mass density is both temperature and pressure dependant. It decreases with increasing temperature but increases with increasing pressure. A generally accepted empirical expression, the Dow and Fink equation, describes this:

$$\rho(t,p) = \rho_0(t) \cdot \left(1.0 + A_\beta(t) \cdot p - B_\beta(t) \cdot p^2\right) \tag{A.2}$$

where

ρ	mass density $[kg/m^3]$
ρ_0	mass density at atmospheric pressure $[kg/m^3]$
A_β	temperature dependant coefficient $[bar^{-1}]$
B_β	temperature dependant coefficient $[bar^{-2}]$
p	pressure $[bar]$

The density of a hydraulic fluid is normally (DIN 51757) given by the fluid manufacturer as the density at $15\,^oC$ and atmospheric pressure. This reference density lies between 0.85 and 0.91 g/cm^3 (850-910 kg/m^3) for commercial hydraulic fluids.
The reference mass density in Equation (A.2) may be determined by:

$$\rho_0(t) = \frac{\rho_{15}}{\left(1 + \alpha_t \cdot (t - 15)\right)} \tag{A.3}$$

where

ρ_0	mass density at atmospheric pressure $[kg/m^3]$
ρ_{15}	mass density at atmospheric pressure and $15\,^oC$ $[kg/m^3]$
α_t	thermal expansion coefficient $[deg^{-1}]$
t	temperature $[^oC]$

The thermal expansion coefficient is normally regarded as independent of temperature and pressure and lies within the range of 0.00065 to 0.0007 deg^{-1}. The variation of the mass density at atmospheric pressure is shown in Figure A1.

The two coefficients in Equation (A.2) are normally referred to as the Dow and Fink coefficients. They have experimentally been found to:

$$A_\beta = \left(-6.72 \cdot 10^{-4} \cdot T^2 + 0.53 \cdot T - 36.02\right) \cdot 10^{-6} \tag{A.4}$$

$$B_\beta = \left(2.84 \cdot 10^{-4} \cdot T^2 - 0.24 \cdot T + 57.17\right) \cdot 10^{-9} \tag{A.5}$$

where

A_β temperature dependant coefficient $[\mathrm{bar}^{-1}]$

B_β temperature dependant coefficient $[\mathrm{bar}^{-2}]$

T absolute temperature $[\mathrm{K}]$

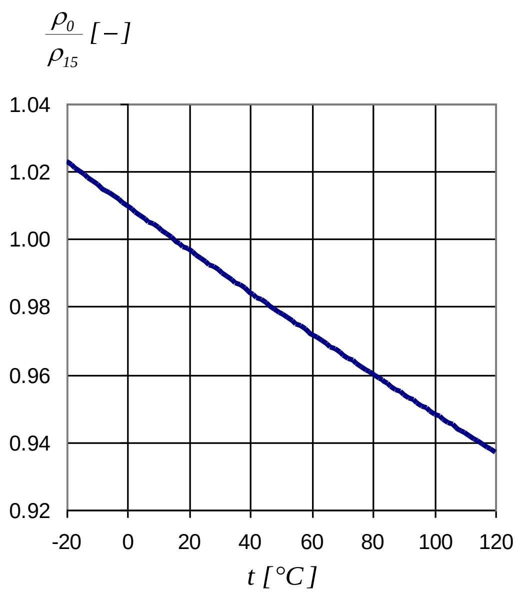


Fig. A1 The variation of the density at atmospheric pressure with temperature

Inserting Equations (A.3)..(A.5) in Equation (A.2) means that the density can be determined by calculations only (no measurements), for any pressure and temperature combination, as long as the reference mass density, ρ_{15}, is known. The variation of the mass density with temperature and pressure is displayed graphically in Figure A2. The mass density is displayed relative to the reference mass density.

A.3 Viscosity

The most important of the physical properties of hydraulic fluids is the viscosity. It is a measure of the resistance of the fluid towards laminar (shearing) motion, and is normally specified to lie within a certain interval for hydraulic components in order to obtain the expected performance and lifetime. The definition of viscosities is related to the shearing stress that appears between adjacent layers, when forced to move relative (laminarly) to each other. For a newtonian fluid this shearing stress is defined as:

$$\tau_{xy} = \mu \frac{d\dot{x}}{dy} \tag{A.6}$$

where

τ_{xy} shearing stress in the fluid, $[\mathrm{N/m^2}]$

μ	dynamic viscosity, [Ns/m^2]
$\dot{x}$	velocity of the fluid, [m/s]
y	coordinate perpendicular to the fluid velocity, [m]

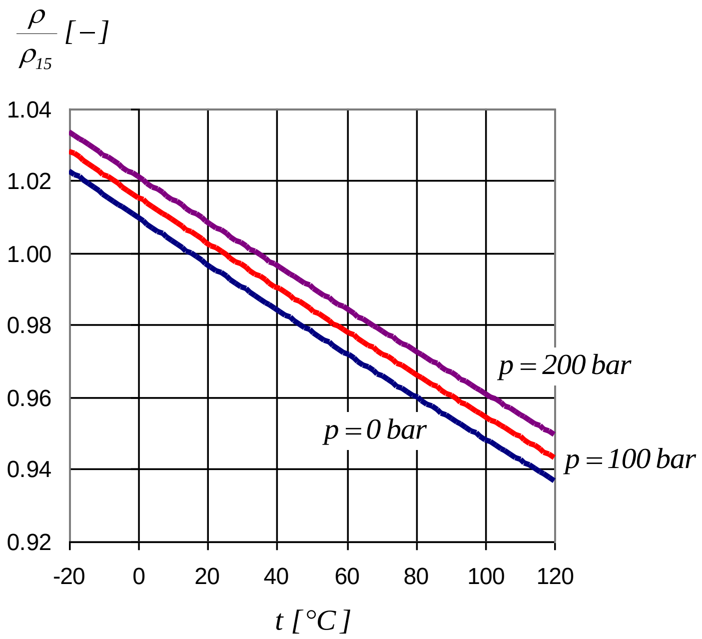


Fig. A2 The variation of the mass density with temperature and pressure

In Figure A3 the variables associated with the definition of the dynamic viscosity are shown.

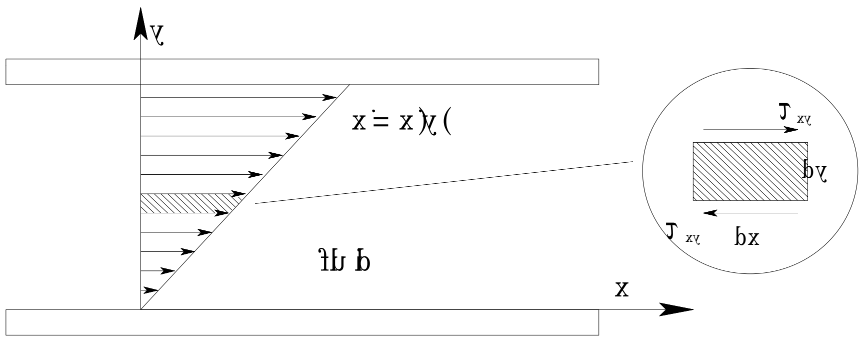


Fig. A3 Shearing stresses in a fluid element

The usual units for the dynamic viscosity are P for Poise or cP for centipoise. Their relation to the SI-units is as follows: $1\text{P} = 100\,\text{cP} = 0.1\,\text{Ns/m}^2$. For practical purposes, however, the dynamic viscosity is seldom used, as compared to the kinematic viscosity that is defined as follows:

$$\nu = \frac{\mu}{\rho} \tag{A.7}$$

where

ν	kinematic viscosity, [m^2/s]
μ	dynamic viscosity, [Ns/m^2]
ρ	density, [kg/m^3]

The usual unit used for ν is centistoke, cSt, and it relates to the SI units as follows:

$$1\,\text{cSt} = 10^{-6}\,\frac{\text{m}^2}{\text{s}} = 1\,\frac{\text{mm}^2}{\text{s}}$$

A low viscosity corresponds to a "thin" fluid and a high viscosity corresponds to a "thick" fluid. The viscosity depends strongly on temperature and also on pressure. The temperature dependency is complex and is normally, DIN51562 and DIN51563 described by the empirical Uddebuhle-Walther equation:

$$log_{10}\, log_{10}(\nu + 0.8) = C_\nu - m_\nu \cdot log_{10} \cdot T \tag{A.8}$$

where

ν	kinematic viscosity, [cSt]
C_ν, m_ν	constants for the specific fluid
T	absolute temperature, [K]

This dependency is normally shown in specially designed charts, where the kinematic viscosity shown as function of the temperature becomes a straight line, see Figure A4.

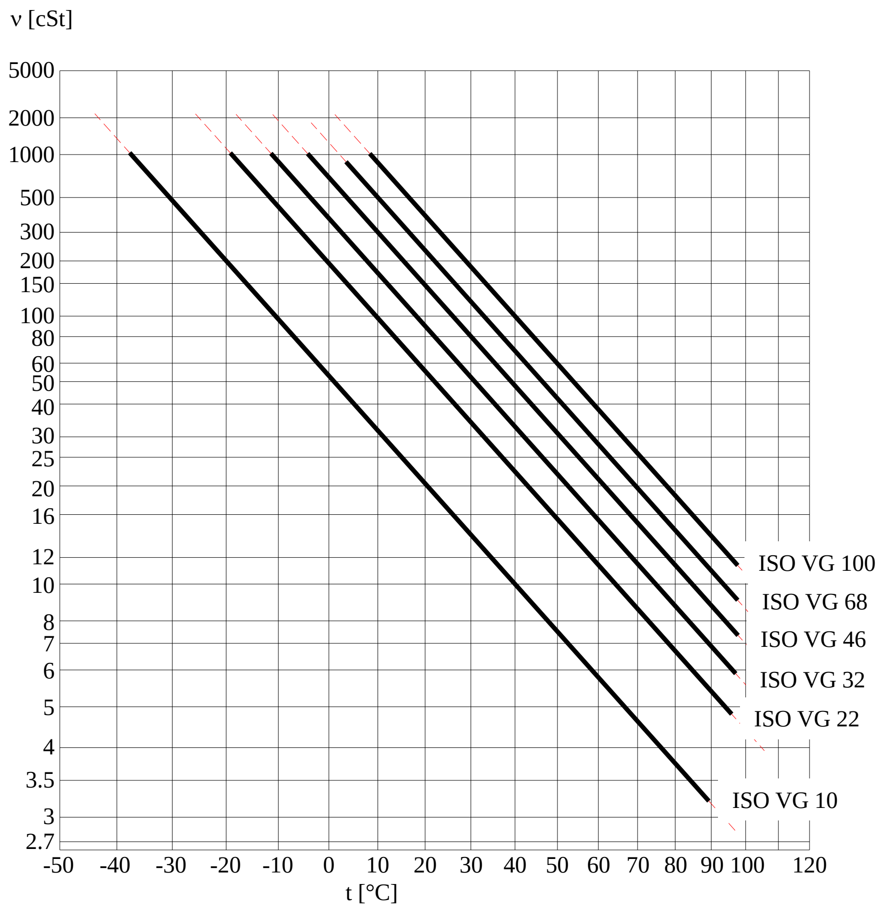


Fig. A4 Uddebuhle-chart: The temperature dependency for some of the most commonly used mineral oils. The ISO VG standard refers ν at 40°C

The vertical axis of an Uddebuhle chart is a mapping of log log(ν+0.8), i.e., approximately a double logarithmic axis (especially at higher values of ν). The

horizontal axis is a mapping of logT, i.e., a logarithmic axis. A hydraulic fluid is, in general, referred to by its kinematic viscosity at 40°C.
A different way of describing a hydraulic fluid is by means of the viscosity index, where the temperature dependency is related to a temperature sensitive fluid and a temperature insensitive fluid. The hydraulic fluid to be indexed and the 2 reference oils must have the same viscosity at a temperature of 210°F. If that is fulfilled, the viscosity index, V.I., may be determined as:

$$VI = \frac{L - U}{L - H} \cdot 100\% \tag{A.9}$$

where

VI	viscosity index
L	kinematic viscosity at 100°F for the temperature sensitive fluid
U	kinematic viscosity at 100°F for the fluid to be indexed
H	kinematic viscosity at 100°F for the temperature insensitive fluid

Different standards, e.g. DIN ISO 2909, offer a list of reference fluids with different kinematic viscosities at 210°F to pick from. The method dates back to 1929 and the improvement in mineral oil distillation and refining means that many hydraulic fluids come out with an index above 100.
Beside the temperature dependency the viscosity also depends on pressure, especially at higher levels. The general accepted expression is as follows:

$$\mu = \mu_0 \cdot e^{B_\eta p} \tag{A.10}$$

where

μ	dynamic viscosity, [Ns/m^2]
μ_0	dynamic viscosity at atmospheric pressure [Ns/m^2]
B_η	temperature dependant parameter, [bar^{-1}]
p	pressure, [bar]

The parameter B_η may, within temperature ranges from 20°C to 100°C, be determined empirically as:

$$B_\eta = 0.0026 - 10^5 \cdot t \tag{A.11}$$

where

B_η	temperature dependant parameter, [bar^{-1}]
t	temperature, [°C]

The pressure dependency may be rewritten to cover kinematics viscosities:

$$\nu = \frac{\mu_0}{\rho} \cdot e^{B_\eta p} \tag{A.12}$$

where

ν	kinematic viscosity, [m^2/s]
μ_0	dynamic viscosity at atmospheric pressure [Ns/m^2]
ρ	density, [kg/m^3]

B_η	temperature dependant parameter, [bar^{-1}]
p	the pressure, [bar]

In the above it should be remembered that the density increases with pressure, thereby making the kinematic viscosity less sensitive to pressure rise.

A.4 Dissolvability

The capability of dissolving air (saturation point) varies strongly for hydraulic fluids with pressure. For pressure levels up to approximately 300 bar, the Henry-Dalton sentence applies:

$$V_a = \alpha_V \cdot V_F \cdot \frac{p_a}{p_{atm}} \tag{A.13}$$

where

V_a	volume of dissolved air in the oil, [m^3]
α_V	Bunsen coefficient, approximately constant at 0.09
V_F	volume of the fluid at atmospheric pressure, [m^3]
p_a	absolute pressure, [bar]
p_{atm}	atmospheric pressure ≈ 1 bar, [bar]

The capability of hydraulic fluids to absorb air is a problem, because the subsequent release of air at lower pressures leads to reduced fluid stiffness.

A.5 Stiffness

When pressurized a hydraulic fluid is compressed causing an increase in density. This is described by means of the compressibility which is defined as

$$\kappa_F = \frac{1}{\rho} \cdot \frac{\partial \rho}{\partial p} \tag{A.14}$$

where

κ_F	compressibility of the fluid, [bar^{-1}]
ρ	mass density, [kg/m^3]
p	pressure, [bar]

The reciprocal of κ_F is defined as the stiffness or bulk modulus of the fluid:

$$\beta_F = \frac{1}{\kappa_F} \tag{A.15}$$

where

κ_F	compressibility, [bar^{-1}]
β_F	bulk modulus, [bar]

Based on the above definition it can be shown that for fixed temperature the stiffness is proportional to the pressure rise caused by a compression of the fluid:

$$dp = \frac{\beta_F \cdot dV}{V_0} \tag{A.16}$$

where

dp	increase in pressure, [bar]
β_F	bulk modulus of the fluid, [bar]
dV	the compression, i.e., decrease in volume, [m^3]
V_0	the volume corresponding to the initial pressure, [m^3]

Just like density the bulk modulus and the compressibility are functions of temperature and pressure. Inserting Equation (A.2) in Equation (A.14) and Equation (A.15) leads to:

$$\beta_F(t,p) = \frac{1.0 + A_\beta(t) \cdot p - B_\beta(t) \cdot p^2}{A_\beta(t) - 2 \cdot B_\beta(t) \cdot p} \tag{A.17}$$

where

β_F	stiffness of the fluid, [bar]
A_β	temperature dependant coefficient, [bar^{-1}]
p	pressure, [bar]
B_β	a temperature dependant coefficient, [bar^{-2}]

Where the temperature dependant coefficients can be determined from Equation (A.4) and Equation (A.5). It should be noted that Equation (A.17) implies that the fluid stiffness may be calculated for any temperature and pressure combination regardless of the specific type of mineral oil. The variation of the fluid stiffness with temperature and pressure is displayed graphically in Figure A5.

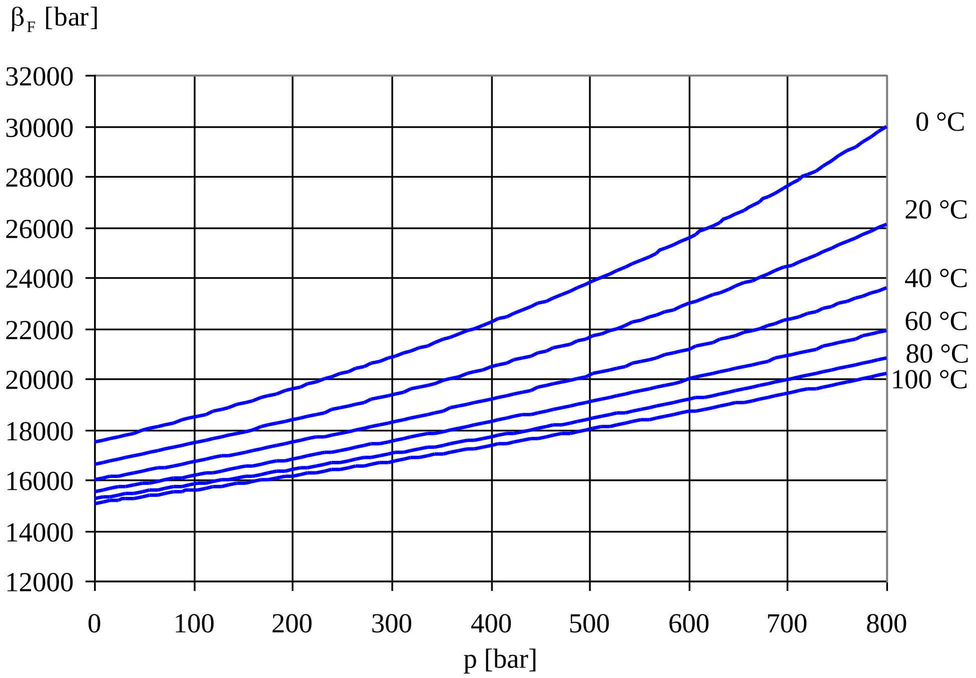


Fig. A5 The variation of the fluid stiffness with temperature and pressure

In real systems air will be present in the fluid. The volume percentage at atmospheric pressure will go as high as 20 %. As air is much more compressible than the pure fluid it has, potentially, a strong influence on the effective stiffness of the air containing fluid. If the air, however, is **dissolved** in the fluid there is **no significant effect** on the compressibility. Hence, it is the amount of **free or entrapped** air in the fluid that **markedly reduces** the effective stiffness. Taking the presence of air into account the effective stiffness of the fluid becomes:

$$\beta_{eff}(t, p, \varepsilon_A) = \frac{1}{\frac{1}{\beta_F} + \varepsilon_A\left(\frac{1}{\beta_A} - \frac{1}{\beta_F}\right)} \approx \frac{1}{\frac{1}{\beta_F} + \frac{\varepsilon_A}{\beta_A}} \tag{A.18}$$

where

β_{eff}	effective stiffness of the fluid-air mixture, [bar]
ε_A	the volumetric ratio of free air in the fluid
β_F	stiffness of the pure fluid according to, [bar]
β_A	the air stiffness according to, [bar]
p	pressure, [bar]

The volumetric ratio is defined as:

$$\varepsilon_A = \frac{V_A}{V_F + V_A} \tag{A.19}$$

where

ε_A	volumetric ratio of free air in the fluid
V_A	the volume of air, [m^3]
V_F	volume of the fluid, [m^3]

Assuming adiabatic conditions the volume and stiffness of the air may be determined as:

$$V_A = V_{A0} \cdot \left(\frac{p_{atm}}{p_a}\right)^{\frac{1}{c_{ad}}} \tag{A.20}$$

$$\beta_A = c_{ad} \cdot p_a \tag{A.21}$$

where

V_A	volume of air, [m^3]
V_{A0}	volume of air at atmospheric pressure, [m^3]
p_{atm}	atmospheric pressure ≈ 1 bar, [bar]
p_a	absolute pressure, [bar]
c_{ad}	adiabatic constant for air, 1.4

The volume of the fluid is determined from:

$$V_F(t, p) = V_{F0} \cdot \frac{\rho_0(t_0)}{\rho(t, p)} \tag{A.22}$$

where

V_F	volume of the fluid, [m^3]
V_{F0}	volume of the fluid at atmospheric pressure and a reference temperature, [m^3]
ρ_0	mass density at atmospheric pressure according to Equation (2.3), [kg/m^3]
ρ	the mass density according to Equation (2.2), [kg/m^3]
t_0	reference temperature, [°C]
t	temperature, [°C]
p	pressure, [bar]

From Equation (A.20) and Equation (A.22) it is clear, that the volumetric ratio varies with both temperature and pressure. A reference volumetric ratio at atmospheric pressure is defined:

$$\varepsilon_{A0} = \frac{V_{A0}}{V_{F0} + V_{A0}} \tag{A.23}$$

where

ε_{A0}	the reference volumetric ratio of free air in the fluid at atmospheric pressure
V_{A0}	volume of air at atmospheric pressure, [m^3]
V_{F0}	volume of the fluid at atmospheric pressure and a reference temperature, [m^3]

Knowing this reference, volumetric ratio together with the reference temperature, t_0, may be rearranged to yield an expression for the volumetric ratio directly obtainable from temperature and pressure:

$$\varepsilon_A(t,p) = \frac{1.0}{\left(\frac{1.0-\varepsilon_{A0}}{\varepsilon_{A0}}\right)\cdot\frac{\rho_0(t_0)}{\rho(t,p)}\cdot\left(\frac{p_{atm}}{p_a}\right)^{\frac{-1}{c_{ad}}}+1.0} \tag{A.24}$$

In Figure A6 the variation of the effective stiffness according to Equation (A.18) is displayed. The variation of the stiffness is dramatic for small pressure levels. The curves in Fig. A6 do not take into account the effect of the Henry-Dalton sentence, Equation (A.13), according to which the free air should dissolve at a few bars pressure and subsequently have no effect on the effective stiffness. The Henry-Dalton sentence, however, is for static conditions and in a hydraulic system the pressure variations outside the tank reservoirs are typically so fast, that the hydraulic fluid does not have time to dissolve the free air. Naturally, some air is dissolved, meaning that the curves shown in Fig. A6 represents worst case, i.e., instantaneously pressure build up.

Stiffness plays a central role w.r.t. to dynamic performance of hydraulic systems and should be determined/predicted as precisely as possible. This is, however, not an easy task. The sum $V_{F0}+V_{A0}$ in Equation (A.23) is relatively easily determined, whereas V_{F0} or V_{A0} are more elusive.

As a rule of thumb, the stiffness under working conditions used for modelling a system should **not be set above 10000 bar**, unless verified by means of testing.

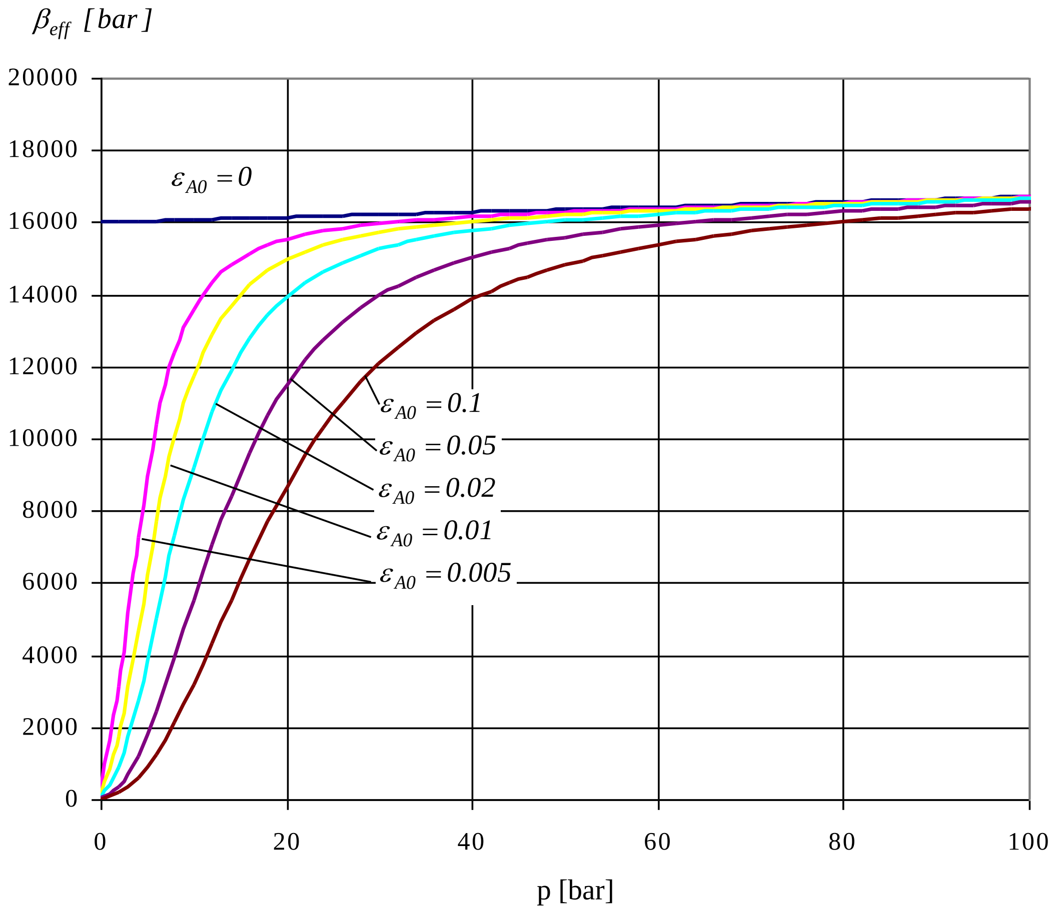


Fig. A6 Variation of effective stiffness of fluid-air mixture with respect to pressure and volume ratio of free air at atmospheric pressure. The temperature of the fluid is 40 °C and the compression of the free air is assumed adiabatic

-----oo0oo-----