Problem 1

a) The stiffness of the hydraulic-mechanical system can be computed from:

$$k_{eff}(x) = \frac{\beta \cdot A}{x \cdot A + V_{L1}} + \frac{\beta \cdot A}{(h-x) \cdot A + V_{L2}}$$

The minimal effective stiffness is observed at $x = \frac{h}{2}$:

$$k_{min} = k_{eff}(x = \frac{h}{2}) = \frac{\beta \cdot A^2}{0.5 \cdot h \cdot A + V_{L1}} + \frac{\beta \cdot A^2}{0.5 \cdot h \cdot A + V_{L2}}$$

$$V_{L1} = V_{L2} = 1\,dm^3 \quad \beta = 1000\ MPa \quad h = 2.4\ m$$

$$A = \frac{\pi}{4}(D^2 - D_r^2) = \frac{\pi}{4}(40^2 - 25^2) = 765.8\ mm^2$$

$$k_{min} = 2 \cdot \frac{1000 \cdot [10^6] \cdot \left(\frac{765.8}{[10^6]}\right)^2}{0.5 \cdot 2.4 \cdot \frac{765.8}{[10^6]} + \frac{1}{[10^3]}} = 6.112 \cdot 10^5\ \frac{N}{m} = 611.2\ \frac{N}{mm}$$

b) The minimum eigenfrequency is computed from the minimum effective stiffness:

$$\omega_{n,min} = \sqrt{\frac{k_{min}}{m}} = \sqrt{\frac{611.2 \cdot [10^3]}{1200}} = 22.6\ \frac{rad}{s}$$

c) The ramp time is computed from the minimum eigenfrequency:

$$t_R = \frac{6}{\omega_{n,min}} = 0.266\ s$$

The maximum velocity is computed from:

$$v_0 = \frac{1}{1-\sigma} \cdot \frac{\Delta s}{\Delta t} = \frac{1}{1 - \frac{t_R}{\Delta t}} \cdot \frac{\Delta s}{\Delta t} = \frac{1}{1-0.133} \cdot \frac{1.2}{2} = 0.69\ \frac{m}{s}$$

d) The maximum load pressure is during the ramp up:

$$p_{L,max} = \frac{m \cdot \ddot{y} + F}{A} = \frac{m \cdot \frac{v_0}{t_R} + F}{A} = \frac{1200 \cdot 2.60 + 400}{765.8} \cdot [10] = 46.0\ bar$$

The minimum load pressure is during the ramp down:

$$p_{L,min} = \frac{m \cdot \ddot{y} + F}{A} = \frac{m \cdot \left(-\frac{v_0}{t_R} \right) + F}{A} = \frac{-1200 \cdot 2.60 + 400}{\frac{765.8}{[10^6]}} = -3.56 \cdot 10^6 \ Pa = -35.6 \ bar$$

e) The maximum flow requirement is computed from:

$$Q_{L,max} = v_0 \cdot A = 0.69 \cdot \frac{765.8}{[10^6]} = 5.30 \cdot 10^{-4} \ \frac{m^3}{s} = 31.8 \ \frac{l}{min}$$

The maximum load pressure and the maximum flow requirement appear simultaneously at the end of the ramp up, i.e., at $t = t_R = 0.266 \ s$.

f) The maximum value of the no load flow is at $t = 0.266 \, s$:

$$Q_{NL,max} = Q_{NL}(t = 0.266s) = Q_L \cdot \sqrt{\frac{p_S}{p_S - p_L}} = 31.8 \cdot \sqrt{\frac{60}{60 - 46.0}} = 65.8 \ \frac{l}{min}$$

g) The bandwidth requirement is formulated as an inequality:

$$\omega_v \geq 3 \cdot \omega_n \Rightarrow 2 \cdot \pi \cdot 15 \geq 3 \cdot 22.6 \Rightarrow 94.2 \ \frac{rad}{s} \geq 67.7 \ \frac{rad}{s}$$

Hence, all the valves are fast enough. The required rated flow is computed from the no-load flow requirement:

$$Q_{r,min} = Q_{NL,max} \cdot \sqrt{\frac{p_r}{p_S}} = 65.8 \cdot \sqrt{\frac{10}{60}} = 26.9 \ \frac{l}{min}$$

Hence, only valve #3 is large enough.

h)

Linearize around two steady state situations: maximum speed, $v_0 = 0.69 \ \frac{m}{s}$ and $v_0 = 0.0 \ \frac{m}{s}$ at the beginning (no load pressure and no load flow) by solving for the steady state variables: $Q_L^{(ss)} = Q_v^{(ss)}$, $p_L^{(ss)}$ and $u^{(ss)}$.

Initially, we compute the cylinder ring area

$$A = \frac{\pi}{4} \cdot \left(d^2 - d_r^2 \right) = \frac{\pi}{4} \cdot \left(40^2 - 25^2 \right) = 766 \ mm^2$$

Also, we compute the maximum discharge area of the spool valve using Eq. (7.3) or the similar ss-version Eq. (7.19) in combination with the rated data on the valve:

$$Q_v^{(ss)} = C_d \cdot A_{d0} \cdot u^{(ss)} \cdot \sqrt{\frac{1}{\rho} \cdot (p_s - \frac{u^{(ss)}}{|u^{(ss)}|} \cdot p_L^{(ss)})} \Rightarrow Q_r = C_d \cdot A_{d0} \cdot \sqrt{\frac{1}{\rho} \cdot p_r} \Rightarrow$$

$$A_{d0} = \frac{Q_r}{C_d \cdot \sqrt{\frac{1}{\rho} \cdot p_r}} = \frac{\frac{40}{[6 \cdot 10^4]}}{0.6 \cdot \sqrt{\frac{1}{875} \cdot 10 \cdot [10^5]}} = 3.287 \cdot 10^{-5}\ m^2 = 32.9\ mm^2$$

First we examine the steady state situation where $v_0 = 0.69\ \frac{m}{s}$. To compute the steady state flow we use Eq. (7.1) or the ss-version Eq. (7.17) because we know the steady state speed, $\dot{y}^{(ss)} = v_0$:

$$Q_L^{(ss)} = A \cdot \dot{y}^{(ss)} = \frac{766}{[10^6]} \cdot 0.69 = 5.284 \cdot 10^{-4}\ \frac{m^3}{s} = 31.7\ \frac{l}{min}$$

To compute the steady state load pressure we use Eq. (7.2) or the ss-version Eq. (7.18) because we know the external force, $F^{(ss)}$.

$$p_L^{(ss)} = \frac{F^{(ss)}}{A} = \frac{400}{\frac{766}{[10^6]}} = 5.224 \cdot 10^5\ \frac{N}{m^2} = 5.22\ bar$$

To compute the steady state normalized spool travel we use Eq. (7.19) assuming that $u^{(ss)} > 0$ and knowing that $Q_v^{(ss)} = Q_L^{(ss)}$ we get:

$$u^{(ss)} = \frac{Q_v^{(ss)}}{C_d \cdot A_{d0} \cdot \sqrt{\frac{1}{\rho} \cdot (p_s - p_L^{(ss)})}} = \frac{\frac{31.7}{[6 \cdot 10^4]}}{0.6 \cdot \frac{32.9}{[10^6]} \cdot \sqrt{\frac{1}{875} \cdot (60 - 5.22) \cdot [10^5]}} = 0.339$$

Next, we examine the steady state situation where $v_0 = 0.0\ \frac{m}{s}$

To compute the steady state flow we use Eq. (7.1) or the ss-version Eq. (7.17) because we know the steady state speed, $\dot{y}^{(ss)}$:

$$Q_L^{(ss)} = A \cdot \dot{y}^{(ss)} = \frac{766}{[10^6]} \cdot 0.0 = 0.0\ \frac{m^3}{s} = 0.0\ \frac{l}{min}$$

The steady state load pressure is directly given:

$$p_L^{(ss)} = 0.0\ bar$$

To compute the steady state normalized spool travel we use Eq. (7.19) assuming that $u^{(ss)} \geq 0$ and knowing that $Q_v^{(ss)} = Q_L^{(ss)}$ we get:

$$u^{(ss)} = \frac{Q_v^{(ss)}}{C_d \cdot A_{d0} \cdot \sqrt{\frac{1}{\rho} \cdot (p_s - p_L^{(ss)})}} = \frac{0}{0.6 \cdot \frac{32.9}{[10^6]} \cdot \sqrt{\frac{1}{875} \cdot 60 \cdot [10^5]}} = 0.0$$

i)
Set up a position servo to drive this system by computing the proportional gain, K_p, using $K_p = 0.7079 \cdot \frac{2 \cdot \zeta_{mh} \cdot \omega_{vmh}}{K_{mh}}$.

We have to compare the two situations and, for conservative reasons, pick the one that yields the smallest proportional gain. Initially, we compute the capacitance according to Eq. (7.32) as:

$$C = \frac{V_0}{4 \cdot \beta} = \frac{A \cdot h + V_{L1} + V_{L2}}{4 \cdot \beta} = \frac{\frac{766}{[10^6]} \cdot 2.4 + \frac{1}{[10^3]} + \frac{1}{[10^3]}}{4 \cdot 1000 \cdot [10^6]} = 9.59 \cdot 10^{-13} \ \frac{m^3}{Pa}$$

We examine the situation where $v_0 = 0.69 \ \frac{m}{s}$. Initially, the flow gain and the pressure-flow gain are computed according to Eq. (7.21) and Eq. (7.22):

$$K_{qu} = C_d \cdot A_{d0} \cdot \sqrt{\frac{1}{\rho} \cdot (p_s - \frac{u^{(ss)}}{\left|u^{(ss)}\right|} \cdot p_L^{(ss)})} = 0.6 \cdot \frac{32.9}{[10^6]} \cdot \sqrt{\frac{1}{875} \cdot (60 - 5.22) \cdot [10^5]} = 1.56 \cdot 10^{-3} \ \frac{m^3}{s}$$

$$K_{qp} = \frac{C_d \cdot A_{d0} \cdot \left|u^{(ss)}\right|}{2 \cdot \sqrt{\rho \cdot (p_s - \frac{u^{(ss)}}{\left|u^{(ss)}\right|} \cdot p_L^{(ss)})}} = \frac{0.6 \cdot \frac{32.9}{[10^6]} \cdot 0.339}{2 \cdot \sqrt{875 \cdot (60 - 5.22) \cdot [10^5]}} = 4.82 \cdot 10^{-11} \ \frac{m^3}{s \cdot Pa}$$

The parameters of the hydraulic-mechanical model, G_{mh} are computed from Eqs. (7.34), (7.35) and (7.36):

$$K_{mh} = \frac{K_{qu}}{A} = \frac{1.56 \cdot 10^{-3}}{\frac{766}{[10^6]}} = 2.04 \ \frac{m}{s}$$

$$\omega_{mh} = \frac{A}{\sqrt{m \cdot C}} = \frac{\frac{766}{[10^6]}}{\sqrt{1200 \cdot 9.59 \cdot 10^{-13}}} = 22.6 \ \frac{rad}{s}$$

$$\zeta_{mh} = \frac{K_{qp}}{2 \cdot A} \cdot \sqrt{\frac{m}{C}} = \frac{4.82 \cdot 10^{-11}}{2 \cdot \frac{766}{[10^6]}} \cdot \sqrt{\frac{1200}{9.59 \cdot 10^{-13}}} = 1.11$$

Note, that ω_{mh} also could have been computed according to the more general expression Eq. (6.28) yielding the same result. The eigenfrequency is adjusted according to Eq. (7.53):

$$\omega_{vmh} = 0.9 \cdot \omega_{mh} = 0.9 \cdot 22.6 = 20.3 \ \tfrac{rad}{s}$$

Finally, we can insert in the expression for the proportional gain, Eq. (7.58), and get:

$$K_p = 0.7079 \cdot \frac{2 \cdot \zeta_{mh} \cdot \omega_{vmh}}{K_{mh}} = 0.7079 \cdot \frac{2 \cdot 1.11 \cdot 20.3}{2.04} = 15.7 \ m^{-1}$$

Now we examine the 0-situation where $Q_L^{(ss)} = 0.0 \ \tfrac{l}{min}$, $p_L^{(ss)} = 0.0 \ bar$ and $u^{(ss)} = 0.0$. For this situation we need to introduce the leakage dependant minimum spool travel $u_{lim}^{(ss)}$. It is taken to be the maximum value from the interval given just below Eq. (7.26), $u_{lim}^{(ss)} = u_\varepsilon = 0.05$. This value may be entered in the modified expressions for the flow gain and the flow-pressure gain in Eq. (7.27):

$$K_{qu} = K_{qu}^{(0)} = C_d \cdot A_{d0} \cdot \sqrt{\frac{1}{\rho} \cdot p_s} = 0.6 \cdot \frac{32.9}{[10^6]} \cdot \sqrt{\frac{1}{875} \cdot 60 \cdot [10^5]} = 1.63 \cdot 10^{-3} \ \tfrac{m^3}{s}$$

$$K_{qp} = K_{qp}^{(0)} = \frac{C_d \cdot A_{d0} \cdot u_{lim}^{(ss)}}{2 \cdot \sqrt{\rho \cdot p_s}} = \frac{0.6 \cdot \frac{32.9}{[10^6]} \cdot 0.05}{2 \cdot \sqrt{875 \cdot 60 \cdot [10^5]}} = 6.80 \cdot 10^{-12} \ \tfrac{m^3}{s \cdot Pa}$$

The new parameters of the hydraulic-mechanical model, G_{mh} are computed from Eqs. (7.34) and (7.36):

$$K_{mh} = \frac{K_{qu}}{A} = \frac{1.63 \cdot 10^{-3}}{\frac{766}{[10^6]}} = 2.13 \ \tfrac{m}{s}$$

$$\zeta_{mh} = \frac{K_{qp}}{2 \cdot A} \cdot \sqrt{\frac{m}{C}} = \frac{6.80 \cdot 10^{-12}}{2 \cdot \frac{766}{[10^6]}} \cdot \sqrt{\frac{1200}{9.59 \cdot 10^{-13}}} = 0.157$$

Finally, we can insert in the expression for the gain, Eq. (7.58), and get:

$$K_p = 0.7079 \cdot \frac{2 \cdot \zeta_{mh} \cdot \omega_{vmh}}{K_{mh}} = 0.7079 \cdot \frac{2 \cdot 0.157 \cdot 22.6}{2.13} = 2.12 \ m^{-1}$$

This is easily the lowest proportional gain, and therefore this should be chosen as guideline for the tuning of the proportional controller.

Problem 2

a) Find a suitable valve specification for controlling this system. Assume: $p_r = 70\,bar$ and $C_d = 0.6$.

To find a suitable valve we must find the bandwidth, ω_v of the valve and the rated flow Q_r. The bandwidth is computed from Eq. (5.40):

$$\omega_v = 3 \cdot \omega_n = 3 \cdot \omega_{mh}$$

Hence, we need the eigenfrequency. For that purpose we compute the capacitance:

$$C = \frac{V_0}{4 \cdot \beta} = \frac{D + V_{L1} + V_{L2}}{4 \cdot \beta} = \frac{\frac{225}{[10^6]} + \frac{0.15}{[10^3]} + \frac{0.15}{[10^3]}}{4 \cdot 1000 \cdot [10^6]} = 1.31 \cdot 10^{-13}\ \tfrac{m^3}{Pa}$$

Nw we get ω_{mh} from Eq. (7.48):

$$\omega_{mh} = \frac{D_\omega}{\sqrt{J \cdot C}} = \frac{D}{2 \cdot \pi \cdot \sqrt{J \cdot C}} = \frac{\frac{225}{[10^6]}}{2 \cdot \pi \cdot \sqrt{18 \cdot 1.31 \cdot 10^{-13}}} = 23.3\ \tfrac{rad}{s}$$

Note, we would have gotten exactly the same result by using Eq. (6.17).
The necessary bandwidth of the valve is $\omega_v = 3 \cdot \omega_{mh} = 69.9\ \tfrac{rad}{s}$

The rated flow is computed from Eq. (5.39):

$$Q_r = 1.1 \cdot Q_{NL,max} \cdot \sqrt{\frac{p_r}{p_s}}$$

Hence, we need to compute the maximum no-load flow. In this case our only information comes from a situation where:

$$Q_L = n \cdot D = \frac{1800}{[60]} \cdot \frac{225}{[10^6]} = 6.75 \cdot 10^{-3}\ \tfrac{m^3}{s} = 405\ \tfrac{l}{min}$$

The corresponding no-load flow is computed from Eq. (5.37):

$$Q_{NL,max} = Q_{NL} = Q_L \cdot \sqrt{\frac{p_s}{p_s - p_L}} = 405 \cdot \sqrt{\frac{240}{240 - 50}} = 455\ \tfrac{l}{min}$$

This value is the maximum no-load flow and it is now possible to compute the necessary rated flow of the valve:

$$Q_r = 1.1 \cdot Q_{NL,max} \cdot \sqrt{\frac{p_r}{p_s}} = 1.1 \cdot 455 \cdot \sqrt{\frac{70}{240}} = 270\ \tfrac{l}{min}$$

In total, we need a valve with a rated flow of at least $Q_r = 270\ \tfrac{l}{min}$ @ $p_r = 70\,bar$ and a bandwidth of at least $\omega_v = 69.9\ \tfrac{rad}{s}$.

b)

Linearize around the 0-position by solving for the steady state variables: $Q_L^{(ss)} = Q_v^{(ss)}$, $p_L^{(ss)}$ and $u^{(ss)}$.

This is easily done, since it is stated that we are supposed to work on the 0-position:

$$Q_L^{(ss)} = Q_v^{(ss)} = 0 \ \tfrac{l}{min}$$

$$p_L^{(ss)} = 0 \ bar$$

$$u^{(ss)} = 0$$

c)

Set up a position servo to drive this system by computing the proportional gain, K_p, using

$$K_p = 0.7079 \cdot \frac{2 \cdot \zeta_{mh} \cdot \omega_{vmh}}{K_{mh}}$$

Initially, we compute the maximum discharge area of the spool valve using Eq. (7.3) or the similar ss-version Eq. (7.19) in combination with the rated data on the valve:

$$Q_v^{(ss)} = C_d \cdot A_{d0} \cdot u^{(ss)} \cdot \sqrt{\frac{1}{\rho} \cdot \left(p_s - \frac{u^{(ss)}}{\left|u^{(ss)}\right|} \cdot p_L^{(ss)}\right)} \Rightarrow Q_r = C_d \cdot A_{d0} \cdot \sqrt{\frac{1}{\rho} \cdot p_r} \Rightarrow$$

$$A_{d0} = \frac{Q_r}{C_d \cdot \sqrt{\frac{1}{\rho} \cdot p_r}} = \frac{\frac{270}{[6 \cdot 10^4]}}{0.6 \cdot \sqrt{\frac{1}{875} \cdot 70 \cdot [10^5]}} = 8.40 \cdot 10^{-5} \ m^2 = 84.0 \ mm^2$$

Since we are examining the 0-situation where $u^{(ss)} = 0.0$ we need to introduce the leakage dependant minimum spool travel $u_{lim}^{(ss)}$. It is taken to be the maximum value from the interval given just below Eq. (7.26), $u_{lim}^{(ss)} = u_\varepsilon = 0.05$. This value may be entered in the modified expressions for the flow gain and the flow-pressure gain in Eq. (7.27):

$$K_{qu} = K_{qu}^{(0)} = C_d \cdot A_{d0} \cdot \sqrt{\frac{1}{\rho} \cdot p_s} = 0.6 \cdot \frac{84.0}{[10^6]} \cdot \sqrt{\frac{1}{875} \cdot 240 \cdot [10^5]} = 8.35 \cdot 10^{-3} \ \tfrac{m^3}{s}$$

$$K_{qp} = K_{qp}^{(0)} = \frac{C_d \cdot A_{d0} \cdot u_{lim}^{(ss)}}{2 \cdot \sqrt{\rho \cdot p_s}} = \frac{0.6 \cdot \frac{84.0}{[10^6]} \cdot 0.05}{2 \cdot \sqrt{875 \cdot 240 \cdot [10^5]}} = 8.69 \cdot 10^{-12} \ \tfrac{m^3}{s \cdot Pa}$$

The remaining unknown parameters of the hydraulic-mechanical model, G_{mh} are computed from Eqs. (7.48) and (7.50):

$$K_{mh} = \frac{K_{qu}}{D_\omega} = \frac{8.35 \cdot 10^{-3}}{\frac{1}{2 \cdot \pi} \cdot \frac{225}{[10^6]}} = 233 \ \tfrac{rad}{s}$$

$$\zeta_{mh} = \frac{K_{qp}}{2 \cdot D_\omega} \cdot \sqrt{\frac{J}{C}} = \frac{8.69 \cdot 10^{-12}}{2 \cdot \frac{1}{2 \cdot \pi} \cdot \frac{225}{[10^6]}} \cdot \sqrt{\frac{18}{1.31 \cdot 10^{-13}}} = 1.42$$

The adjusted eigenfrequency is obtained from Eq. (7.53):

$$\omega_{vmh} = 0.9 \cdot \omega_{mh} = 0.9 \cdot 23.3 = 21.0 \ \frac{rad}{s}$$

So here we have a heavily damped system where it will be possible to set up a system with a bandwidth close to ω_{vmh}. We insert in the expression for the proportional gain, Eq. (7.58), and get:

$$K_p = 0.7079 \cdot \frac{2 \cdot \zeta_{mh} \cdot \omega_{vmh}}{K_{mh}} = 0.7079 \cdot \frac{2 \cdot 1.42 \cdot 21.0}{233} = 0.181 \, rad^{-1}$$