Problem 1

a) We have a function

$$y = A \cdot \sin(2 \cdot \pi \cdot f \cdot t)$$

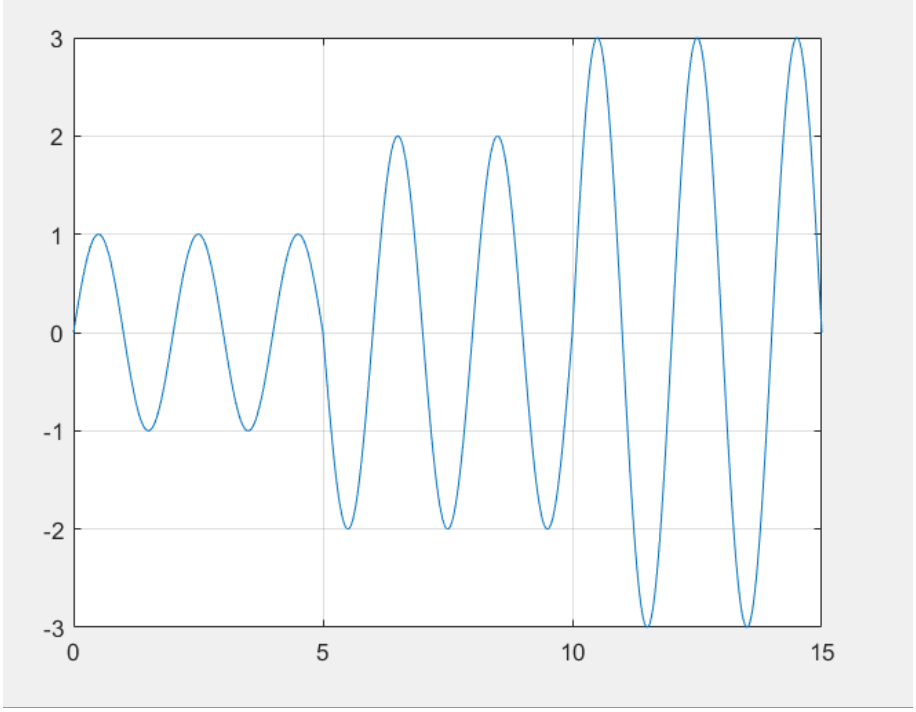
Where

$$f = 0.5Hz$$

$$A = \begin{cases} 1 & t \leq 5s \\ 2 & 5s < t \leq 10s \\ 3 & t > 10s \end{cases}$$

Plot y as a function of time, t, for a total time of 15 s. Report the maximum magnitude of y. Also report the RMS value of this quantity.

Solution as plot of y vs time in seconds.



b) We have a function

$$y = A \cdot t^2 + \mathrm{B}$$

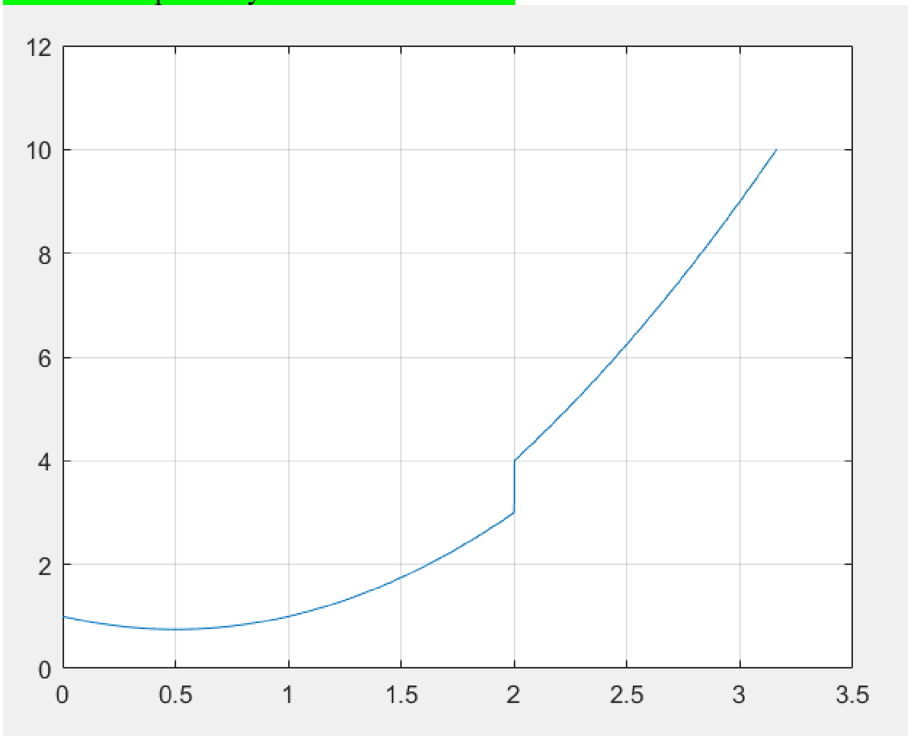
Where

$$A = 1$$

$$B = \begin{cases} 1 - t & t \leq 2s \\ 0 & t > 2s \end{cases}$$

Plot y as a function of time, t, until y becomes larger than 10.

Solution as plot of y vs time in seconds.



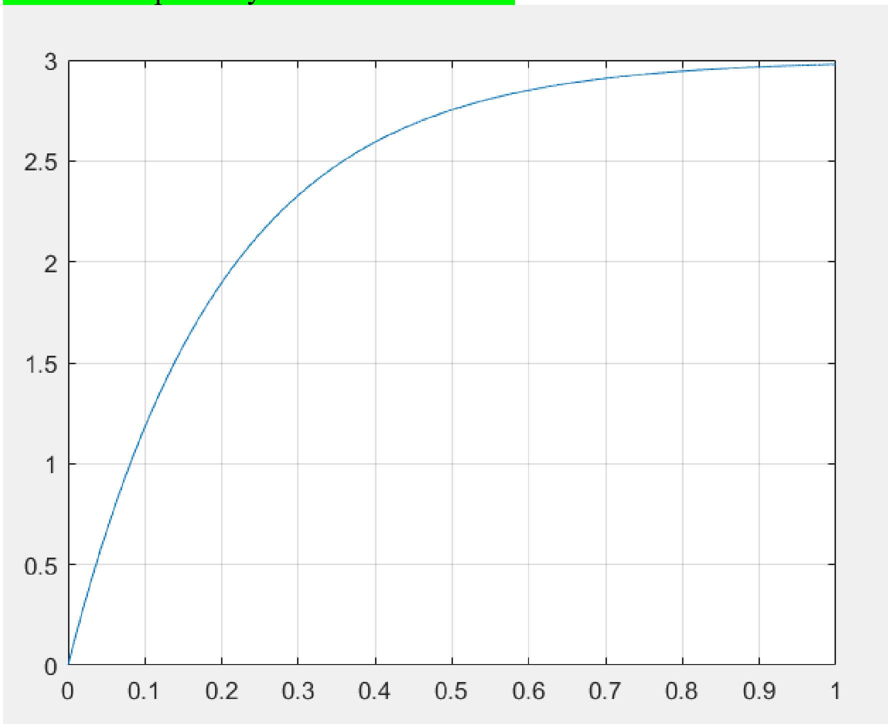
Problem 2

a) We have a differential function

$$\dot{y} = \frac{3 - y}{0.2}$$

Solve the differential function numerically using Forward-Euler time integration scheme. Plot y as a function of time for 1.0 s. The initial value is $y = 0$.

Solution as plot of y vs time in seconds.

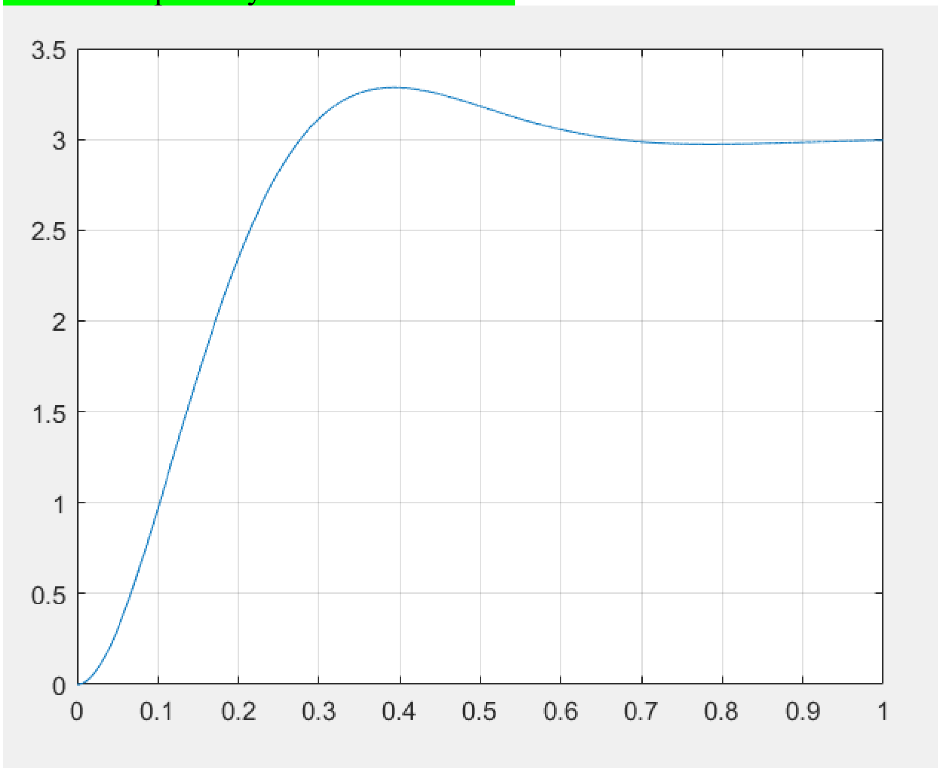


b) We have a differential function

$$\ddot{y} = 100 \cdot (3 - y) - 12 \cdot \dot{y}$$

Solve the differential function numerically using Forward-Euler time integration scheme. Plot y as a function of time for 1.0 s. The initial values are $y = 0$ and $\dot{y} = 0$. Report the maximum magnitude of y and $\dot{y}$. Also report the RMS value of these two quantities.

Solution as plot of y vs time in seconds.



Problem 3
An ideal mechanical pendulum is shown in Fig. 1.

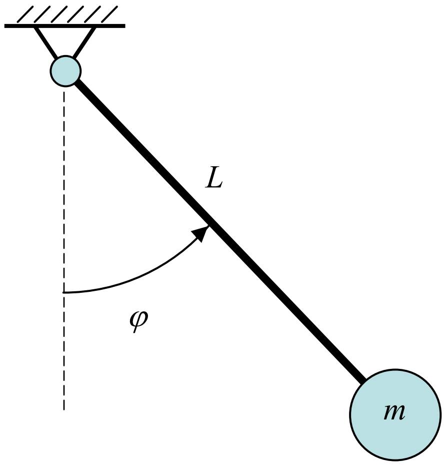
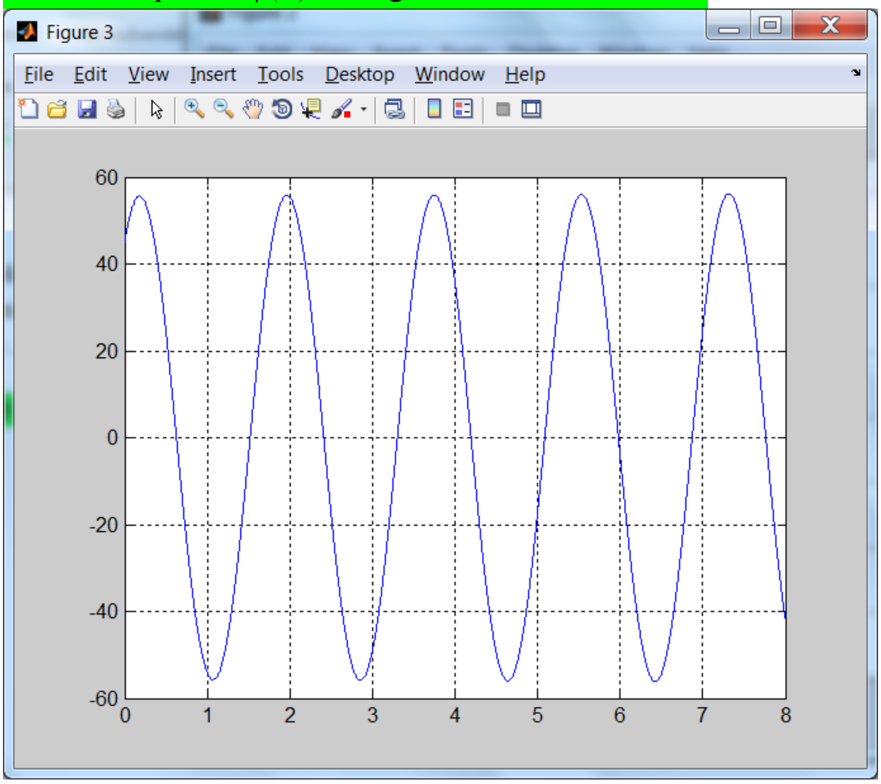


Figure 1 Simple pendulum with concentrated mass.

The following data is given: $L = 0.7\ m$ and $m = 24\ kg$. Simulate the motion of the pendulum with the following initial conditions: $\varphi(t=0) = 45°$ and $\dot{\varphi}(t=0) = 2\ \frac{rad}{s}$. Simulate the motion for a total time of $T = 8\ s$.

$$\ddot{\varphi} + \frac{g}{L}\sin\varphi = 0$$

Solution as plot of $\varphi(t)$ in degrees vs time in seconds.



Problem 4

A spring-damper-mass system as shown in Fig. 2 is to be examined. The mass is subjected to forces from gravity, spring and damper as well as an applied force, F , that varies with time as follows:

$$F(t) = F_0 \cdot sin(\omega_p \cdot t)$$

The spring stiffness is $k = 40\ kN/m$, the damping constant is $c = 500\ Ns/m$ and the mass is $m = 100\ kg$.

The position of the mass is to be plotted for a period of 2 seconds. A total of four different situations are to be analyzed. In each situation the mass starts from rest in a position where the spring is undeformed, i.e., $y = 0$.

The only thing that varies in the four situations is the applied force, *F:*

Situation	Size of applied force, F_0, [N]	Frequency, ω_p, [rad/s]
a	0	-
b	500	10
c	500	20
d	500	30

Recommended time step: 0.0001 seconds. Report the maximum magnitude of y and $\dot{y}$. Also report the RMS value of these two quantities.

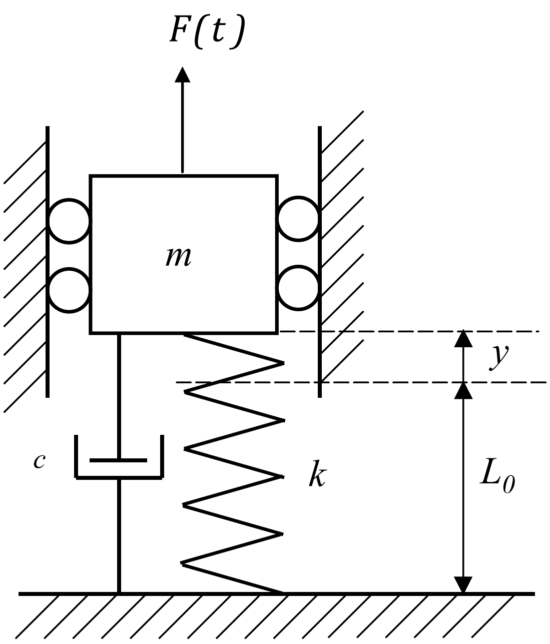
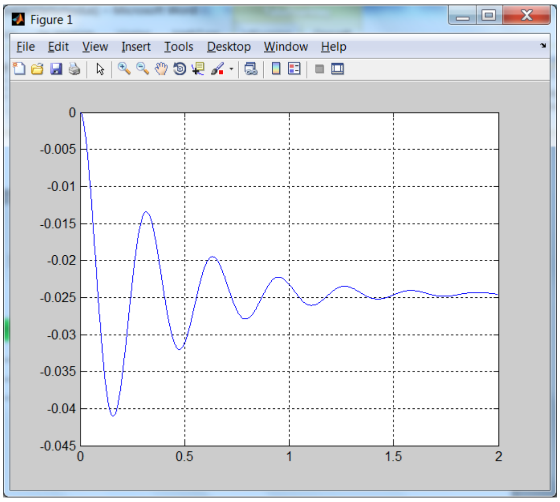


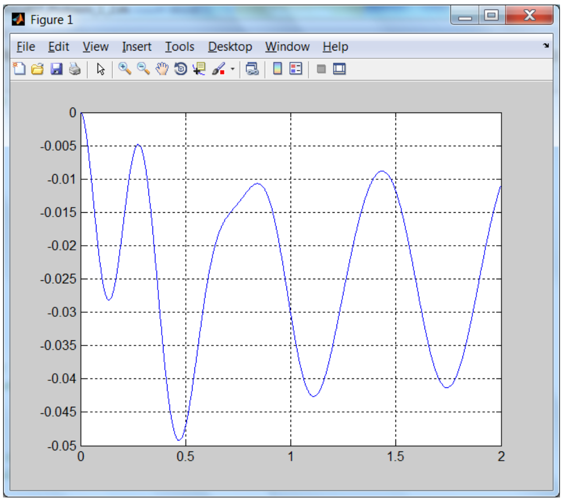
Figure 2 Mass-spring-damper system. The undeformed length of the spring, L_0, is shown.

Solution as plot of $y(t)$ in meter vs time in seconds.

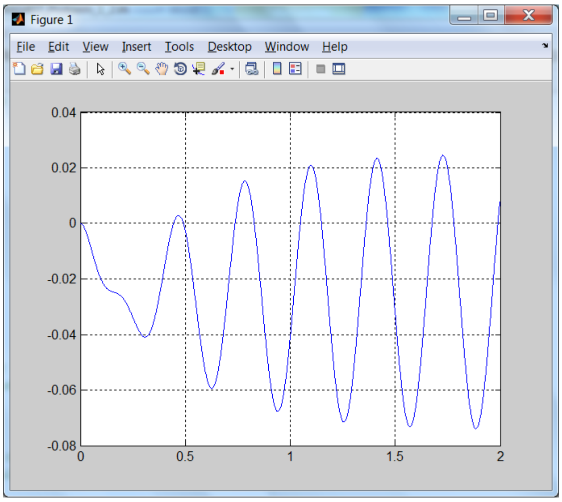
Situation a



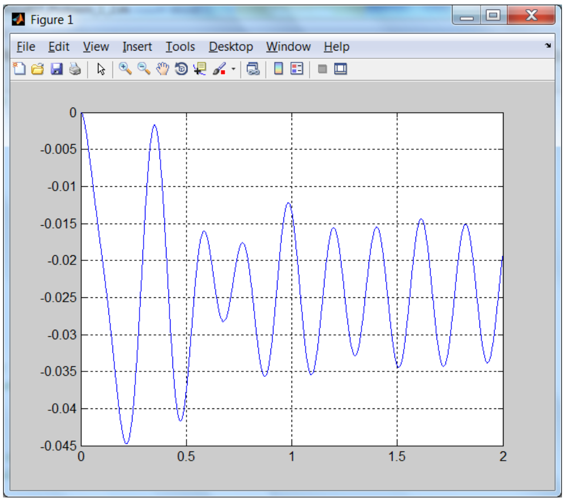
Situation b



Situation c



Situation d



Problem 5
In Fig. 3 is shown a body that can only rotate. It is connected to the ground via a rotational spring and a rotational damper.

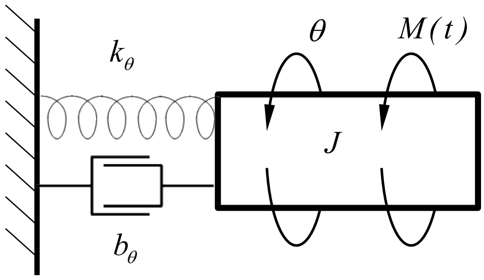


Figure 3 Rotating spring-damper inertia.

The following data is given: $J = 2\ kg \cdot m^2$, $k_\theta = 1800\ \frac{N \cdot m}{rad}$ and $b_\theta = 8\ \frac{N \cdot m \cdot s}{rad}$. The rotational spring is undeformed when $\theta = 0\ rad$. Four different situations are to be investigated. They are characterized by the external applied moment:

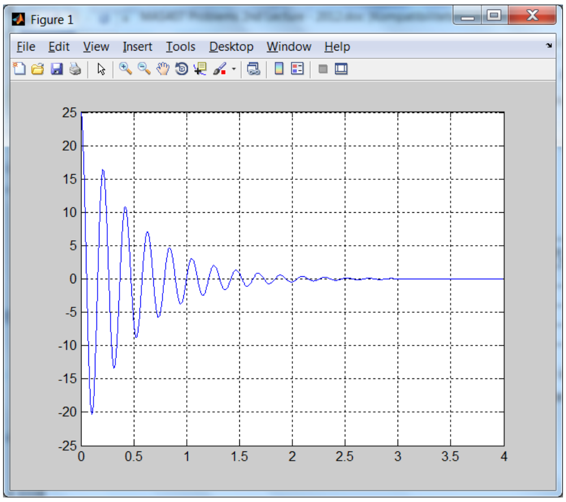
$$M(t) = M_0 \cdot \sin\left(\omega_p \cdot t\right)$$

Situation	Size of applied moment, $M_0\ [N \cdot m]$	Frequency, $\omega_p\ \left[\frac{rad}{s}\right]$
a	0	-
b	75	20
c	75	30
d	75	40

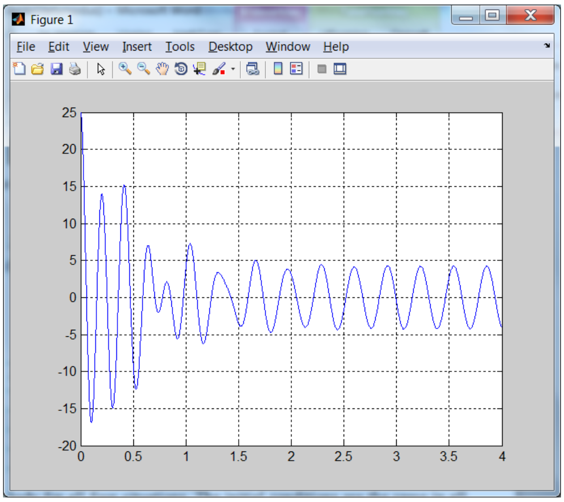
Simulate the motion of the rotating body for all four situations. The initial conditions are the same in all situations: $\theta(t=0) = 25°$ and $\dot{\theta}(t=0) = 0\ \frac{rad}{s}$. Simulate the motion for a total time of $T = 4\ s$.

Solution as plot of $\theta(t)$ in degree vs time in seconds.

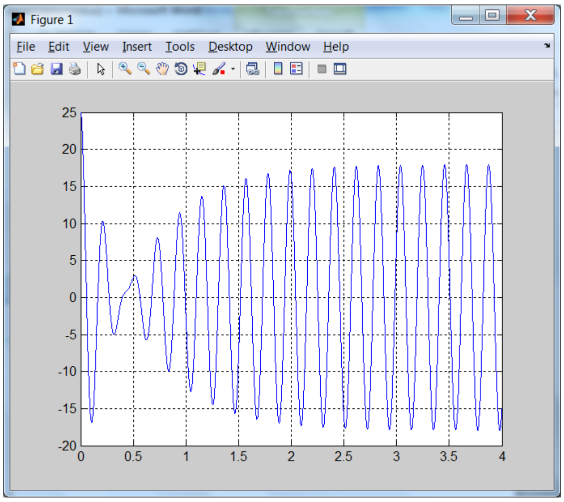
Situation a



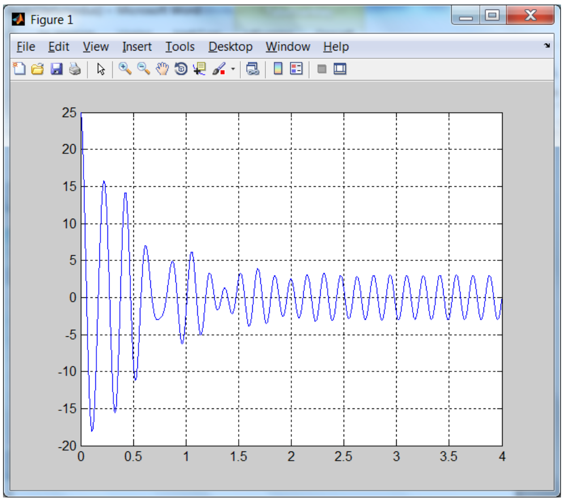
Situation b



Situation c



Situation d



Problem 6 (Previous exam, 15%)
A mechanical system consists of a mass, an incline, and a linear tension spring, see Figure 4. The mass slides on the incline and there is a friction force between the incline and the mass.
The end of the spring is subjected to a prescribed motion as a function of time:

$$x_0(t) = A + B \cdot t$$

where $A = 0.2\,m$ and $B = 0.1\,\frac{m}{s}$. The undeformed length of the spring is $L_0 = 100\,mm$ and the spring stiffness is $k = 1000\,N/m$. The spring can only provide a tension force:

$$F_k = \begin{cases} k \cdot \Delta & \Delta \geq 0 \\ 0 & \Delta < 0 \end{cases}$$

where $\Delta = x_0 - x - L_0$.

The friction force acts against the motion and is given as:

$$F_{fr} = F_0 \cdot tanh\left[\frac{\dot{x}}{v_0}\right]$$

where $F_0 = 500\,N$ and $v_0 = 0.01\,\frac{m}{s}$.

The mass is $m = 300\,kg$ and the incline angle is $\theta = 30°$

At time $t = 0\,s$ the position of the mass is $x = 100\,mm$ and the velocity is $\dot{x} = 0\,\frac{m}{s}$.

Make a simulation model of the mechanical system and simulate from $t = 0\,s$ to $t = 25\,s$.
Plot the position x of the mass as a function of time.

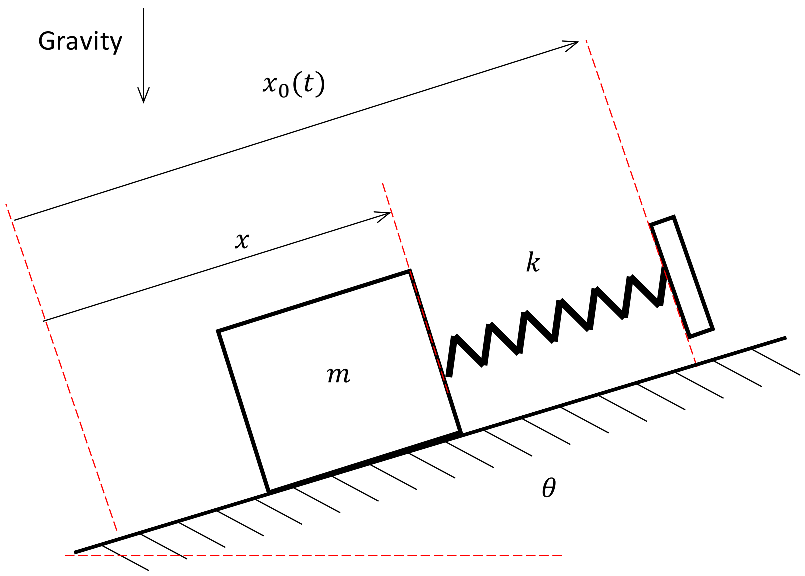
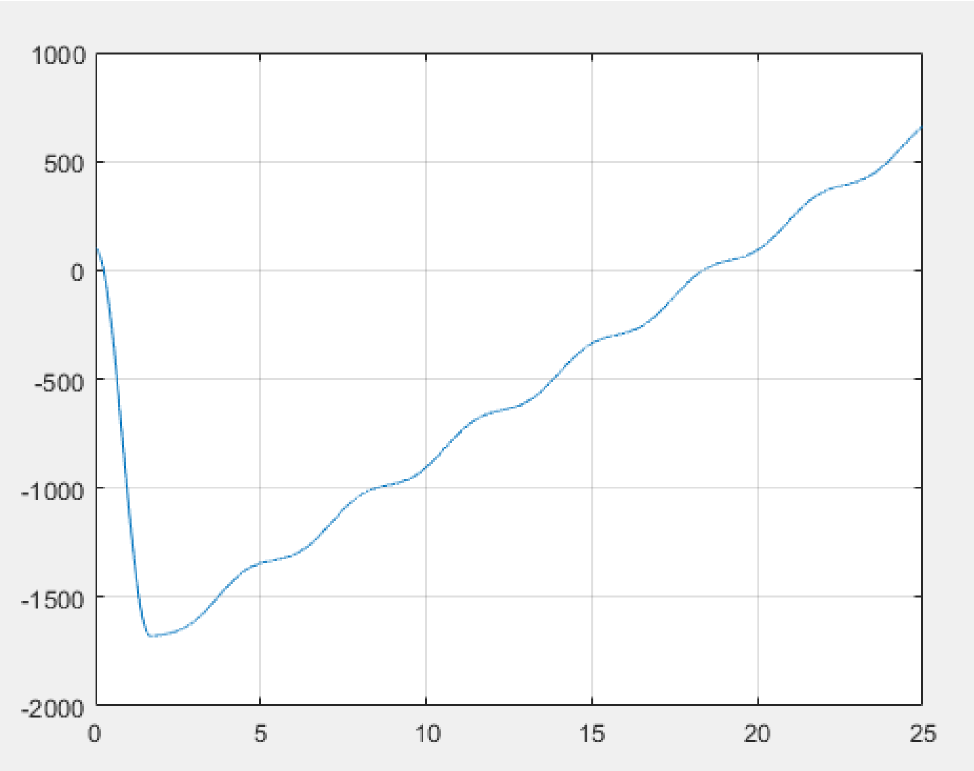


Figure 4 Mass on incline.

Solution as plot of $x(t)$ in mm vs time in seconds.



Problem 7 (Previous exam, 30%)
A mechanical system is shown in Figure 5. It consists of two masses, three linear springs and three linear dampers.

The lower mass has a value of $m_1 = 2000$ kg and the upper mass has a value of $m_2 = 400$ kg. Both masses translate frictionless in the vertical direction.

All spring are linear and can be both in compression and tension. They are identical and have the following data: undeformed length of spring $L_0 = 500\ mm$ and spring stiffness $k = 100\ kN/m$.

The three linear dampers are identical and has a damping constant $b = 1500\ \frac{N\cdot s}{m}$.

Further geometrical dimensions are: $h = 800\ mm$ and $h_0 = 3100\ mm$.

At time $t = 0\ s$ the positions of the masses are $x_1 = 500\ mm$ and $x_2 = 1800\ mm$. Also, at time $t = 0\ s$ the masses are at rest, $\dot{x}_1 = \dot{x}_2 = 0\ \frac{m}{s}$.

Make a simulation model of the mechanical system and simulate from $t = 0\ s$ to $t = 5\ s$.
Plot the position of both masses, x_1 and x_2, as a function of time.

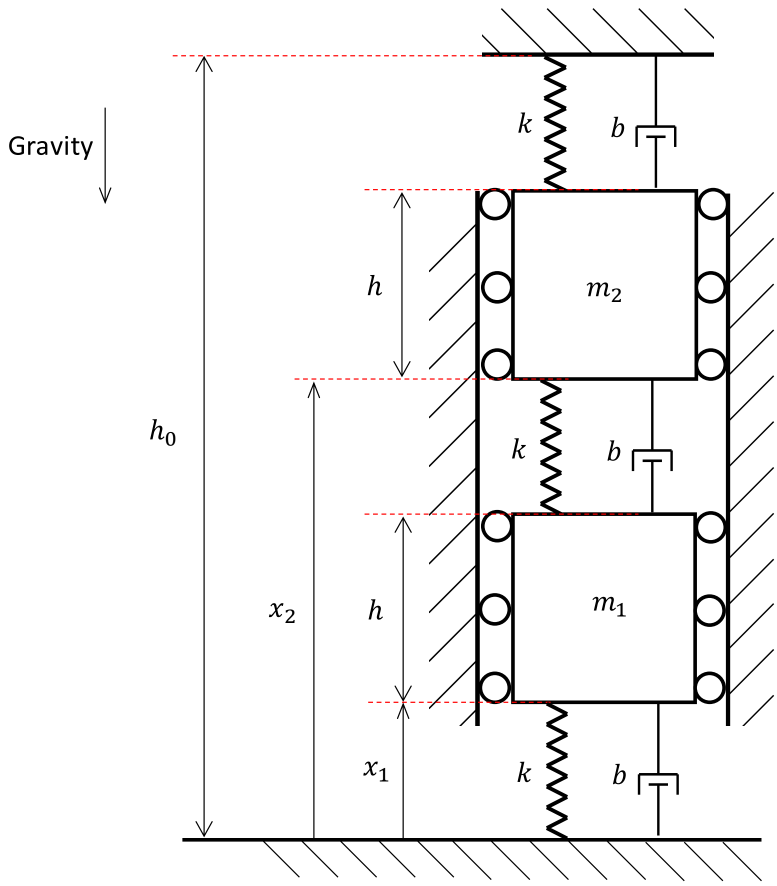


Figure 5 Mechanical system.

Solution as plot of $x_1(t)$ (blue) and $x_2(t)$ (red) in mm vs time in seconds.