Randomised Algorithms
Winter term 2022/2023, Exercise Sheet No. 9

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Exercise 1.

(a) *Let's say w.l.o.g that $a^\star$ satifies the first literal (and only the first) of every clause, then $\neg a$ would satify the second literal of each clause, and hence, $\neg a$ would also be a satisfied assigment.*

Exercise 2.
(a) *The new recurrence relations could be summed up as follows:*

$$\begin{cases} E_0 &= 0 \\ E_i &= 1 + \frac{1}{2}(E_{i+1} + E_{i-1}) \quad \textit{for: } i \in \{1, \ldots, n-1\} \\ E_n &= 1 + \frac{1}{2}(E_n + E_{n-1}) \end{cases}$$

The last implies that: $E_n = 2 + E_{n-1} \implies D_n = 2$.
The second line implies that:

$$D_k = 2 + D_{k-1} \textit{ for } k \in \{1, \ldots, n-1\}$$

By summing each equation from $k = 1$ up to $k = i \leq n-1$, we get:

$$D_1 = 2k + D_{k+1}$$

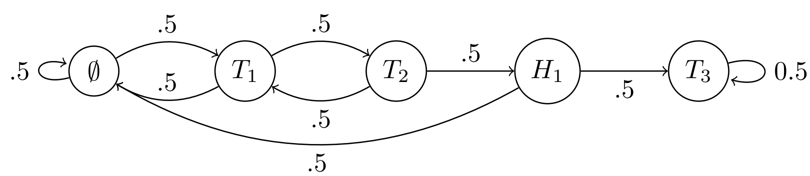
Particularly when $k = n-1$

$$D_1 = 2(n-1) + D_n = 2n = E_1$$

And hence, $D_{k+1} = 2(n-k)$, for $k \in \{1, \ldots, n-1\}$
(b) *For $k \in \{1, \ldots, n\}$, we have:*

$$\begin{aligned} E_k &= \sum_{i=1}^{k} D_i \\ &= \sum_{i=1}^{k} 2(n-(k-1)) \\ &= k(2n-k) + k \end{aligned}$$

Exercise 3.
(a) *The figure below represents the Markov Chain of interest:*
(b)



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T_1
T_2
H_1
T_3