

Randomised Algorithms
Winter term 2022/2023, Exercise Sheet No. 3

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Exercise 1.

(a) Comparisons between 15 and 8:

- *Pivot is 7: Decision will be postponed.*
- *Pivot is 15: They will be compared.*
- *Pivot is 10: They will be immediately separated and hence never be compared.*

(b) The probability of 8 and 15 being compared:

$$\frac{2}{|\{\text{Numbers in the input between 8 and 15}\}|} = \frac{2}{|\{8,11,19,10,15,18\}|} = \frac{2}{6} = \frac{1}{3}$$

Exercise 2.

- (a)*
- (b)*
- (c)*

Exercise 3.

(a) We start from Top to Bottom, we assign 1 to the root, and follow these two strategies to assign the levels below until we reach the leaves:

If the parent is \vee :

- First child: 0
- Second child: Parent Value

If the parent is \wedge :

- First child: 1
- Second child: Parent Value

(b) The following figures captures the algorithm:

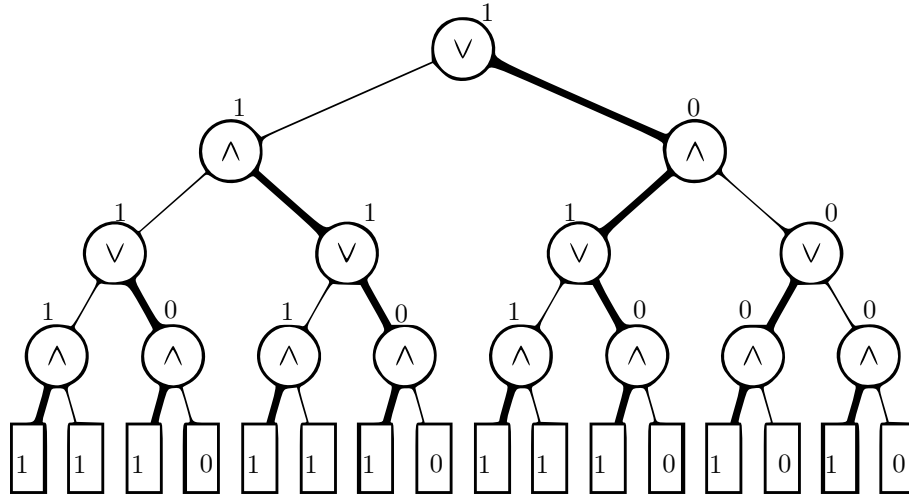


Figure 1: graph-incscape

Exercise 4.

We have for $i, j \in \{1, \dots, n\}$:

$$\min_i M_{i,j} \leq M_{i,j} \leq \max_j M_{i,j}$$

Hence, for $i \in \{1, \dots, n\}$:

$$\max_j \min_i M_{i,j} \leq \max_j M_{i,j} \text{ (The RHS is independent of } j\text{)}$$

Notice that the LHS is a constant (independent of both i and j), and the past inequality is verified $\forall i$. We finally get:

$$\max_j \min_i M_{i,j} \leq \min_i \max_j M_{i,j}$$