

# Randomised Algorithms

## Winter term 2022/2023, Exercise Sheet No. 3

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### Exercise 1.

(a) Comparisons between 15 and 8:

- Pivot is 7: Decision will be postponed.
- Pivot is 15: They will be compared.
- Pivot is 10: They will be immediately separated and hence never be compared.

(b) The probability of 8 and 15 being compared:

$$\frac{2}{|\{\text{Numbers in the input between 8 and 15}\}|} = \frac{2}{|\{8,11,19,10,15\}|} = \frac{2}{5}$$

### Exercise 2.

(a) Based on the lecture notes, for a letter  $x$  of size  $n$ , we would have:  $\mathbb{P}[\{A_t \text{ failing}\}] \leq (1 - \frac{4}{n})^{\frac{t}{2}}$

(b) Hey, well well well

(c) The probability of  $\mathcal{A}$  failing for an input of size  $n$  is the same of the probability of failing in the first case (when the probability is at least  $\frac{1}{2} + \frac{1}{\sqrt{n}}$ ) for an input size of  $n^2$ . Hence, using (b),  $L \in BPP$ .

**Exercise 3.**

(a) We start from Top to Bottom, we assign 1 to the root, and follow these two strategies to assign the levels below until we reach the leaves:

If the parent is  $\vee$ :

- First child: 0
- Second child: Parent Value

If the parent is  $\wedge$ :

- First child: 1
- Second child: Parent Value

(b) The following figures captures the algorithm:

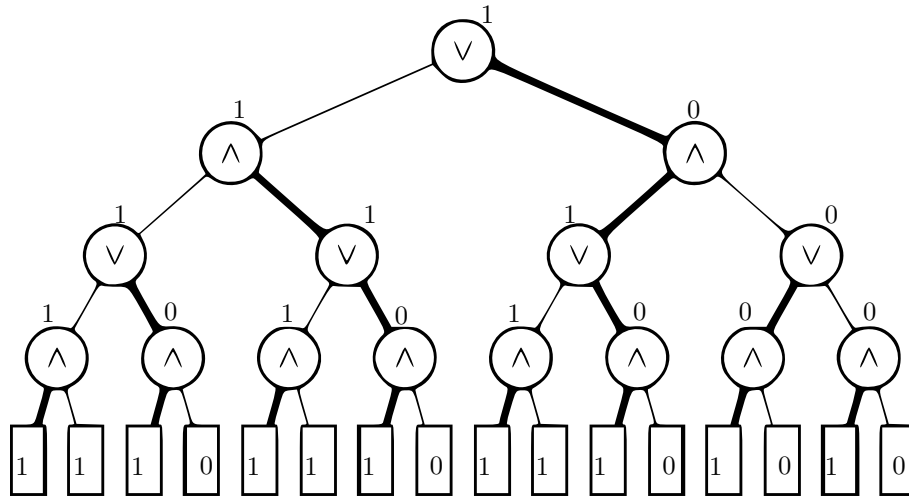


Figure 1: graph-incscape

**Exercise 4.**

We have for  $i, j \in \{1, \dots, n\}$ :

$$\min_i M_{i,j} \leq M_{i,j} \leq \max_j M_{i,j}$$

Hence, for  $i \in \{1, \dots, n\}$ , and as the RHS is independent of  $j$ :

$$\max_j \min_i M_{i,j} \leq \max_j M_{i,j}$$

Notice that the LHS is a constant (independent of both  $i$  and  $j$ ), and the past inequality is verified  $\forall i$ . We finally get:

$$\max_j \min_i M_{i,j} \leq \min_i \max_j M_{i,j}$$