Aerodynamic design and aircraft control

1st assignment

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Contents

1 Introduction

In the second assignment, we are tasked with performing calculations related to the stability of the F-104, both in approach conditions and during cruise. Then, we are asked to develop a control logic to mitigate some undesirable characteristics.

This work was completed through the collaborative effort of the author and Pavlos Vichas. The code for the assignment is provided in two separate files, one for stability (`stability.m`) and one for control (`control.m`).

The aircraft has the following geometric characteristics:

Parameter	Value	Units
Wing		
Reference area	18.22	[m^2]
Wingspan	6.7	[m]
Semi-chord sweep angle	8	[deg]
Taper ratio	0.36	[-]
Airfoil	NACA 0006	[-]
Mean aerodynamic chord	2.91	[m]
Horizontal Tail		
Reference area	4.2	[m^2]
Wingspan	3.6	[m]
Semi-chord sweep angle	0	[deg]
Taper ratio	0.15	[-]
Airfoil	NACA 0006	[-]
Tail arm	5.3	[m]

2 Stability During Approach

We first examine the stability of the aircraft under the following conditions:
Approach Mass: 6,400 [kg]
Approach Speed: 315 [km/h]
Altitude: Sea level
Configuration: Full flaps and gear down.

According to the problem statement, only the main wing and the horizontal tail contribute to longitudinal stability.

Furthermore, the aircraft speed during approach is found to be 0.264 Mach.

2.1 C_{m_α} and Static Margin

The total pitching moment experienced by the aircraft at its center of gravity is the superposition of the moments generated by the main wing and the horizontal tail. Consequently, the total pitching moment coefficient slope, $C_{m\alpha_{\text{total}}}$, is also the sum of the individual contributions:

$$C_{m\alpha_{\text{total}}} = C_{m\alpha_{\text{wing}}} + C_{m\alpha_{\text{tail}}} = 0.044 - 0.1671 = -0.31$$

Knowing that the airfoil is symmetric (NACA 0006), the lift curve slope a_0 is 2π. Taking into account the wing sweep angle Λ and a flight Mach number of 0.264, the effective lift-curve slope of the wing becomes:

$$C_{L\alpha_{\text{wing}}} = \alpha = \frac{a_0 \cdot \cos(\Lambda)}{\sqrt{1 + \left(\frac{a_0 \cdot \cos(\Lambda)}{\pi \cdot AR}\right)^2} + \frac{a_0 \cdot \cos(\Lambda)}{\pi \cdot AR}} = 2.9815$$

From Nelson's Equation 2.9, the moment slope due to the wing is:

$$C_{m\alpha_{\text{wing}}} = C_{L\alpha_{\text{wing}}} \cdot \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) = 0.044$$

For the tail (with no sweep), we compute:

$$C_{L\alpha_{\text{tail}}} = \frac{a_0}{1 + \left(\frac{a_0}{\pi \cdot e \cdot AR_{\text{tail}}}\right)} = 3.81$$

Then from Nelson's Equation 2.27:

$$C_{m\alpha_{\text{tail}}} = -\eta \cdot V_H \cdot C_{L\alpha_{\text{tail}}} \cdot \left(1 - \frac{d\epsilon}{d\alpha}\right) = -0.354$$

with

$$V_H = \frac{S_{\text{tail}} \cdot l_{\text{tail}}}{S_{\text{wing}} \cdot \bar{c}}, \quad \frac{d\epsilon}{d\alpha} = \frac{2C_{L\alpha_{\text{wing}}}}{\pi \cdot \epsilon \cdot AR}$$

where ϵ is the Oswald efficiency factor:

$$\epsilon = \frac{1}{\delta + 1}$$

The value of δ is obtained using Figure 5.20 from Anderson's book. Here, ϵ is slightly less than 1 for both wing and tail. The efficiency factor η is assumed to be 1, since the tail is mounted high enough to receive undisturbed flow — a reasonable assumption.

The static margin is a dimensionless measure that expresses the fraction of the mean aerodynamic chord between the center of gravity and the neutral point (the point at which $C_{m\alpha} = 0$):

$$\frac{x_{np}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \eta \cdot V_H \cdot C_{L\alpha_{\text{tail}}} \cdot \left(1 - \frac{d\epsilon}{d\alpha}\right)$$

$$\text{Static Margin} = \frac{x_{np}}{\bar{c}} - \frac{x_{cg}}{\bar{c}} = 0.104$$

2.2 C_{m_0} and C_{L_0}

Since the aircraft is in a steady approach (constant descent rate), both force and moment equilibrium apply. Therefore:

$$C_m = 0, \quad \text{Lift} = \text{Weight}$$

$$C_{L_{\text{approach}}} = \frac{\text{Lift}}{0.5 \cdot 1.2 \cdot V^2 \cdot S}$$

From the previous section, we know the slope of the pitching moment curve. The total lift slope is:

$$C_{L\alpha_{\text{total}}} = C_{L\alpha_{\text{wing}}} + C_{L\alpha_{\text{tail}}}$$

The problem also provides the correction terms of both the moment and lift curves as functions of the elevator deflection δs. Therefore, the following general forms apply:

$$C_L = C_{L_0}^* + C_{L\alpha} \cdot \alpha + C_{L_{\delta s}} \cdot \delta s$$

$$C_m = C_{m_0}^* + C_{m\alpha} \cdot \alpha + C_{m_{\delta s}} \cdot \delta s$$

Note that $C_{L_0}^* = 0$ due to the symmetric geometry of both wing and tail airfoils.

Substituting values:

$$C_{m_0} = C_{m_0}^* + C_{m_{\delta s}} \cdot \delta s = -0.1685 + 0.1809 = 0.0124$$

$$C_{L_0} = C_{L_0}^* + C_{L_{\delta s}} \cdot \delta s = 0 - 0.0843 = -0.0843$$

2.3 C_m vs Angle of Attack

Knowing from the previous questions both the slope of the pitching moment curve and C_{m_0}, i.e., the point where it intersects the y-axis, for $\delta s = -7.1$ degrees, the following diagram of C_m vs α arises:

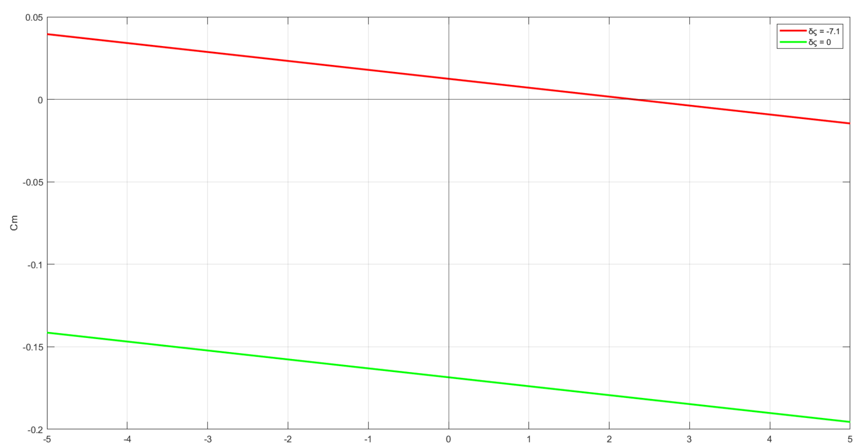


Figure 1: C_m vs α trimmed (red), untrimmed (green).

It is apparent that for an angle of attack of 2.3 degrees, we have $C_m = 0$, i.e., the aircraft is in a trimmed condition. Changing

δs would shift this curve up or down, thus also changing the y-intercept and, naturally, the trim point (x-intercept). Indicatively, the diagram also shows the curve for $\delta s = 0$. It is evident that under these conditions the aircraft cannot reach equilibrium.

3 Stability during Cruise

For the cruise conditions, at high speeds, the slopes of the lift and pitching moment coefficients, as well as the changes in lift and pitching moment coefficients due to elevator deflection, are functions of the Mach number based on the following relation:

$$C_x = A \cdot M^4 + B \cdot M^3 + C \cdot M^2 + D \cdot M + E$$

with the coefficients given in the following table.

Coefficient	A	B	C	D	E
C_{L_a}	-5.524	34.344	-78.940	76.698	-21.539
C_{m_a}	-4.485	23.523	-41.934	29.286	-8.085
$C_{L_{\delta s}}$	0.00331	0.878	-3.694	4.432	-0.760
$C_{m_{\delta s}}$	0.086	-1.912	6.986	-7.934	1.115

Additionally, the following data apply:
Cruise Mass: 7400 [kg]
Cruise speed: 961 [km/h]
Altitude: 35,000 [ft]
Configuration: No Flaps & Gears up

3.1 C_{m_0} and C_{L_0}

If the aircraft were flying in steady level flight at a speed of 961 km/h at 35,000 feet, with a mass of 7400 kg, the C_L arises from the force balance:

$$C_{L_{trimmed}} = \frac{\text{Lift}}{0.5 \cdot 1.2 \cdot V^2 \cdot S} = 0.2943$$

For the C_L curve, we know that the term C_{L_0} will be zero due to the geometry of the airfoils of the main wing and tail. Also, we know that $C_{L_{\delta s}} \cdot \delta s = -0.023$ (which is the constant term of the lift coefficient curve), and therefore, for $\alpha = 3°$, it follows that $C_{L_{3°}} = 0.237$, which is lower than the required value for cruise. It is evident that the aircraft is not trimmed for $\alpha = 3°$ and $\delta s = -1.5°$.

The angle of attack at which the aircraft must be for force and moment equilibrium to exist at $\delta s = -1.5°$ follows from:

$$C_L = C_{L_0} + C_{L\alpha} \cdot \alpha + C_{L_{\delta s}} \cdot \delta s$$

with $C_{L_{\delta s}}$ given from the flight speed. Solving, it results in the corresponding angle being $\alpha_{trimmed} = 3.66°$.

Now the constant term of the moment coefficient can also be calculated by simple substitution in the corresponding formula and with $C_m = 0$ for $\alpha_{trimmed}$. It results in $C_{m_0} = 0.095$. It is worth noting here that this result is the sum of C_{m_0} and $C_{m_{\delta s}} \cdot \delta s$, since this is the y-axis intercept. Otherwise, C_{m_0}, as it appears in the above equation, is equal to 0.0506 (this is not asked for in this question but will be needed in the next one). From now on, this will be denoted as $C^*_{m_0}$.

Substituting into the formula for C_m at $\alpha = 3°$ results in $C_{m_{3°}} = -0.017$, showing the presence of a positive pitching moment at this point. Previously, in the corresponding approach question, the moment coefficient was zero at trim. To return the aircraft to a trim condition at this angle of attack, the elevator deflection must be changed.

3.2 δs and α for Cruise

The goal here is to find the appropriate δs and α, i.e., the corresponding trim points, for a range of speeds from Mach 0.7 to 2.

From the formulas:

$$C_L = C_{L_0} + C_{L\alpha} \cdot \alpha + C_{L_{\delta s}} \cdot \delta s$$

and

$$C_m = C_{m_0} + C_{m\alpha} \cdot \alpha + C_{m_{\delta_s}} \cdot \delta s$$

it results that for trim, we know $C_m = 0$, and just like before, $C_{L_{trim}} = (7400 \cdot g)/(0.5 \cdot 1.2 \cdot V^2 \cdot S)$. Substituting above and using $C_{L_0} = 0$ and, from the previous question, $C^*_{m_0} = 0.0506$, it results in:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta_s}} \\ C_{m_\alpha} & C_{m_{\delta_s}} \end{bmatrix} \begin{bmatrix} \alpha \\ \delta_s \end{bmatrix} = \begin{bmatrix} C_{L_{trim}} \\ -C^*_{m0} \end{bmatrix}$$

Using an iterative procedure, this system is solved for each speed, updating the coefficient matrix of the unknowns $x = [\alpha, \delta s]^T$. The results are summarized in the following diagram:

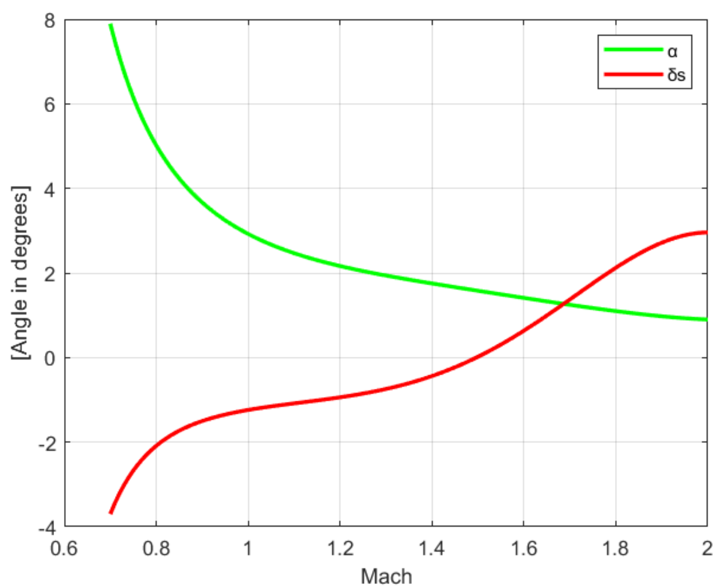


Figure 2: Trim point as a function of Mach.

3.3 -700 kg, 2 M

Knowing the flight speed, altitude, and weight defines the mission parameters, and therefore the required Lift and C_L.

The problem that arises here is that both the elevator deflection and the angle of attack are unknown quantities. To compute them, we use the trim diagram—or rather the logic derived from using it.

Initially, we assume $\delta s = 0$. We find an angle of attack for which force equilibrium exists. Then, from the moment equilibrium, we find the corresponding $\delta s'$ for which the aircraft is in trim at the α obtained from the force balance. We solve again the force balance for a new α', and repeat until the absolute value of the difference $\delta s' - \delta s$ is less than a predefined tolerance. The iterative process resulted in the optimal value for the elevator deflection being $\delta s = 3.84°$, while the angle of attack is equal to 0.11 degrees.

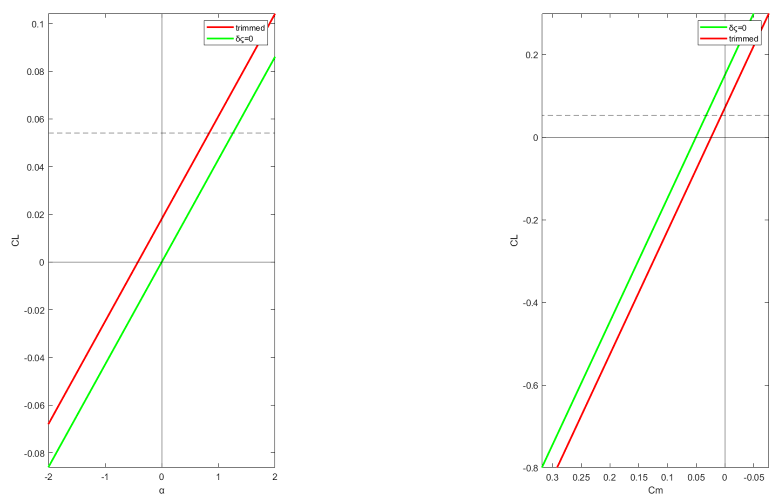


Figure 3: Trim diagram.

4 Control

The longitudinal motion of the aircraft can be described by a system with 4 state variables: $x_1 = u$, the axial velocity, $x_2 = w$, the vertical velocity, $x_3 = q$, the pitch rate, and $x_4 = \theta$, the pitch angle. The manipulated variable for this motion is the deflection of the horizontal tail, $u = \eta$.

From the assignment statement, it is given that the linearized state-space system can be expressed as:

$$\dot{x} = A \cdot x + B \cdot u$$

with,

$$A = \begin{bmatrix} -0.0352 & 0.1070 & 0 & -32.2 \\ -0.2140 & -0.4400 & 305 & 0 \\ 1.198 \times 10^{-4} & -0.0154 & -0.4498 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -22.1206 \\ -4.6580 \\ 0 \end{bmatrix}$$

Additionally, the outputs are defined as x_1 through x_4, with the output matrix C being the identity matrix. Since there are no external disturbances, the matrix D is equal to zero.

4.1 Transfer Functions

To convert the state-space model to a transfer function model between the output variables and the manipulated variable, i.e., elevator movement, we first apply the Laplace transform to the state equations (assuming zero initial conditions):

$$sX(s) = AX(s) + BU(s)$$
$$Y(s) = CX(s) + DU(s)$$

Rewriting the first equation and solving for $X(s)$:

$$(sI - A)X(s) = BU(s)$$
$$X(s) = (sI - A)^{-1}BU(s)$$

Substituting $X(s)$ into the output equation:

$$Y(s) = C(sI - A)^{-1}BU(s) + DU(s)$$
$$Y(s) = \left[C(sI - A)^{-1}B + D\right]U(s)$$

The transfer function matrix $G(s)$ relates the output $Y(s)$ to the input $U(s)$:

$$G(s) = C(sI - A)^{-1}B + D$$

Using MATLAB, all these calculations are performed with the appropriate command. The resulting transfer functions are the following:

$$\frac{u(s)}{\eta(s)} = \frac{-2.367s^2 - 3.091s + 55.03}{s^4 + 0.925s^3 + 4.949s^2 + 0.1825s + 0.1078}$$

$$\frac{w(s)}{\eta(s)} = \frac{-22.12s^3 - 1431s^2 - 50.36s - 32.18}{s^4 + 0.925s^3 + 4.949s^2 + 0.1825s + 0.1078}$$

$$\frac{q(s)}{\eta(s)} = \frac{-4.658s^3 - 1.873s^2 - 0.1671s}{s^4 + 0.925s^3 + 4.949s^2 + 0.1825s + 0.1078}$$

$$\frac{\theta(s)}{\eta(s)} = \frac{-4.658s^2 - 1.873s - 0.1671}{s^4 + 0.925s^3 + 4.949s^2 + 0.1825s + 0.1078}$$

4.2 Dynamic Characteristics of the State-Space Model

The poles arise from solving the characteristic polynomial of the transfer functions:

$$s^4 + 0.925s^3 + 4.949s^2 + 0.1825s + 0.1078$$

Solving this yields the poles at:

- $s = -0.445871 \pm 2.16761i$
- $s = -0.0166285 \pm 0.14743i$

However, to extract more information, the characteristic polynomial must be separated into two quadratics, one that contains the short-period dynamic behavior (denoted with subscript s) and one the phugoid (subscript p). It results in:

$$(s^2 + 2\zeta_p \omega_p s + \omega_p^2)(s^2 + 2\zeta_s \omega_s s + \omega_s^2) = 0$$

Solving this now yields:

- $\omega_p = 0.148$ rad/s
- $\omega_s = 2.210$ rad/s
- $\zeta_p = 0.112$
- $\zeta_s = 0.201$

In the two figures below, the root loci for the 4 equations are summarized.

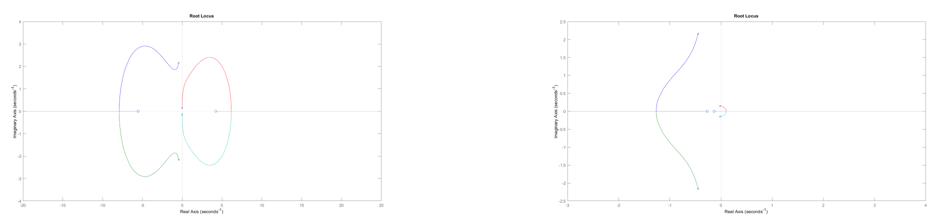
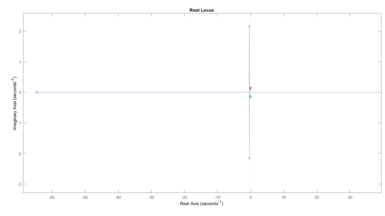


Figure 4: Root loci for u, θ.

It can be seen from the location of the zeros and poles that the short-period characteristics are limited by the presence of zeros very close to the short-period poles.

On the other hand, here it can be seen that the zeros near the phugoid poles drastically reduce their influence on the transfer function. Finally, a zero appears in the right half-plane for w and for θ, which invert the output (make it negative).



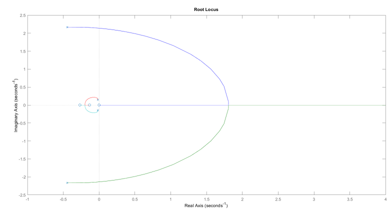


Figure 5: Root loci for w, q.

4.3 Comparison with MIL-F-8785C

As shown in the table below, the short-period damping ζ_s does not meet the required criteria and therefore it is appropriate to use a feedback loop so that the MIL-F-8785C standard is satisfied.

Parameter	Limits	Value	Satisfied
ζ_p	$\zeta_p \geq 0.04$	0.11	Yes
ζ_s	$\zeta_s \geq 0.5$	0.20	No
ω_s	$0.8 < \omega_s < 3$	2.21	Yes

4.4 Simple Feedback Loop

Before the undesirable characteristics are corrected, it is crucial to investigate whether the system is controllable. The controllability matrix $\mathbf{C}_c$ is defined as:

$$\mathbf{C}_c = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \cdots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix}$$

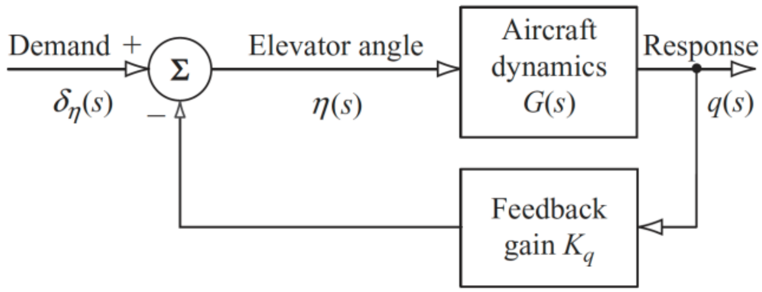
The system is controllable if rank($\mathbf{C}_c$) $= n$.

Since the system is controllable, we can proceed with developing a closed-loop with gain feedback to correct the undesirable characteristics.

with $G(s)$,

$$\frac{q(s)}{\eta(s)} = G(s) = \quad \frac{-4.658s^3 - 1.873s^2 - 0.1671s}{s^4 + 0.925s^3 + 4.949s^2 + 0.1825s + 0.1078}$$

The transfer function that results for the closed loop is:



$$G_{\text{closed loop}}(s) = \frac{G(s)}{1 + K_q \cdot G(s)}$$

The goal is to keep the phugoid dynamic characteristics as close as possible to the existing ones and to increase the short-period damping above 0.5. After several trials, it seems that this is achieved for $K_q = -0.4$. The closed-loop transfer function is:

$$G_{\text{closed loop}}(s) = \frac{-4.658s^3 - 1.873s^2 - 0.1671s}{s^4 + 2.788s^3 + 5.698s^2 + 0.2493s + 0.1078}$$

The characteristics are now within the desired limits.

- $\omega_p = 0.139$ rad/s
- $\omega_s = 2.360$ rad/s
- $\zeta_p = 0.126$
- $\zeta_s = 0.583$

Below is the step response of q to a step input, both in open and closed loop. It can be seen that the short-period behavior of the closed loop is clearly improved compared to the open loop. Almost all oscillations have been eliminated. However, the initial undershoot remains, albeit reduced.

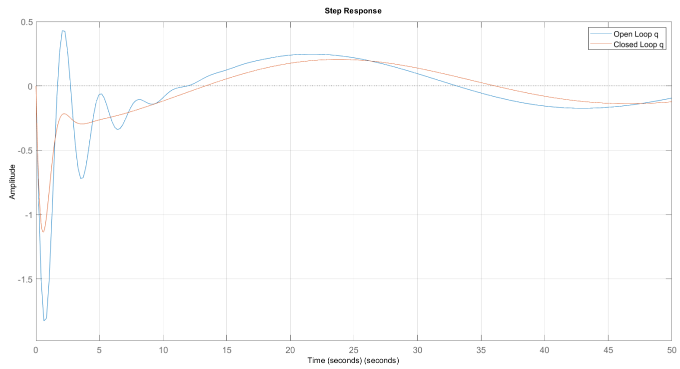


Figure 6: Comparison of step response in open and closed loop with $K_q = -0.4$.

4.4.1 $\zeta = 0.6$ and $\omega_s = 2$ rad/s

To define specific characteristics, it is enough to select the appropriate gain so that the characteristic polynomial changes accordingly. Here, the main interest lies in the short-period quadratic, which must take the form $s^2 + 2.4s + 4$. To achieve this, we choose $K_q = -0.42$ for which we approach the desired characteristics most closely. ζ_s is exactly equal to the target, while $\omega_s = 2.37$ rad/s. The response is shown below:

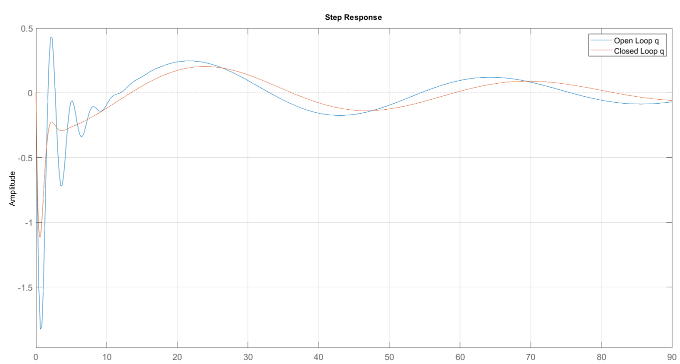


Figure 7: Comparison of step response in open and closed loop with $K_q = -0.42$.

4.5 State Feedback

For the state feedback model, we can divide the state space into two reduced models: one with the phugoid characteristics and one with the short-period. Since q is part of the short-period characteristics, the control logic will first be developed on the reduced short-period model and then generalized to the full one.

The reduced model has the following form:

$$\dot{x}_{\text{red}} = \begin{pmatrix} -0.44 & 305 \\ -0.0154 & -0.4498 \end{pmatrix} \cdot x_{\text{red}} + \begin{pmatrix} -22.12 \\ -4.658 \end{pmatrix} \cdot u$$

and

$$y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot x$$

with $x = [w, q]^T$

To verify whether the behavior of the reduced model is close to the full one, we compare the step responses of the two open-loop systems for the variable of interest, q.

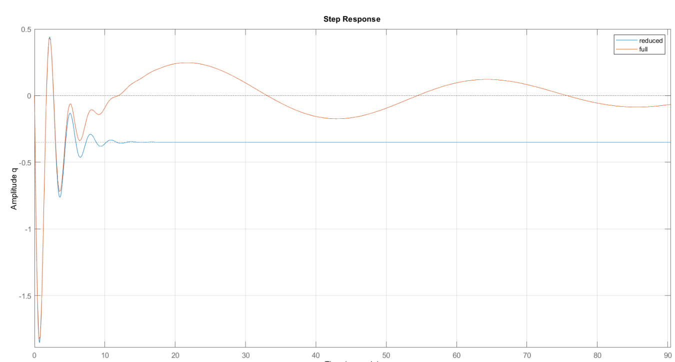


Figure 8: Comparison of step response of full and reduced model in open loop.

As shown, the two responses are quite close during the first seconds, i.e., during the short-term dynamics. After that, the reduced model, not modeling the phugoid dynamics, stays at a fixed value, while the full model exhibits a damped oscillation.

Placing the poles where we want for $\zeta = 0.6$ and $\omega_s = 2$ rad/s, i.e., at $p_{1,2} = -1.2000 \pm 1.6000i$, the resulting reduced controller is $K_{\text{red}} = [0.0010, -0.3291]$.

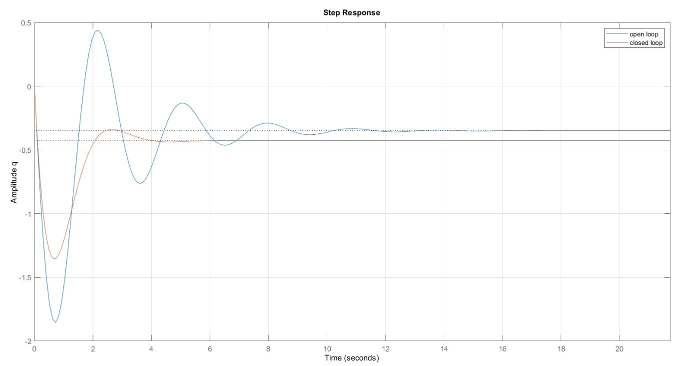


Figure 9: Step response in open and closed loop for q in the reduced system.

As shown, the desired characteristics are imposed exactly; however, the response does not differ drastically from the one in the previous question.

Finally, generalizing to the full system, the overall K is $K_{\text{full}} = [0, 0.0010, -0.3291, 0]$. The step response of the full system in open and closed loop for q is shown below.

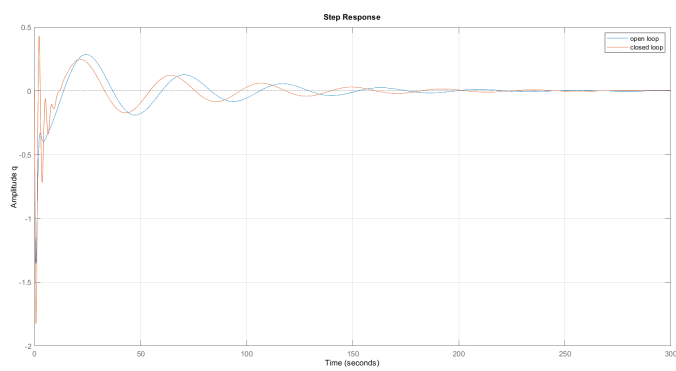


Figure 10: Step response in open and closed loop for q in the full system.