Aerodynamic design and aircraft control

3rd assignment

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Contents

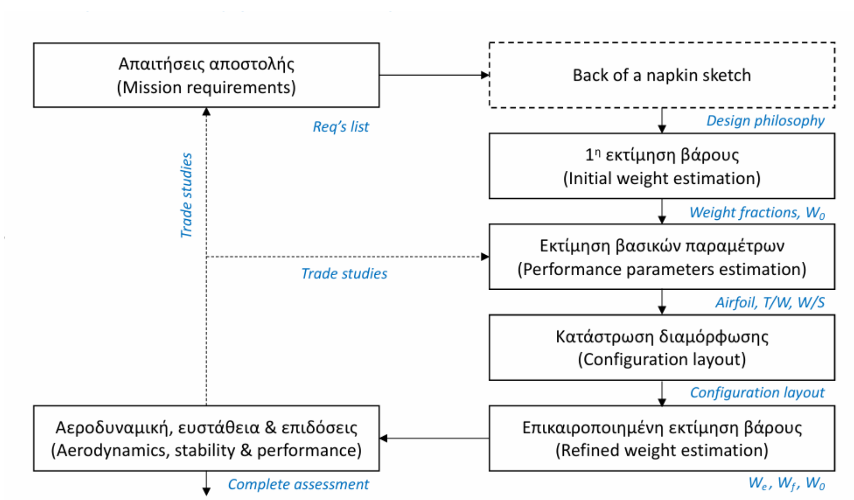
1 Introduction

The objective of the 3rd assignment is the conceptual design of an aerobatic demonstration aircraft. In this project, sizing calculations will be carried out, the airfoil type will be selected, and the basic geometric parameters of the aircraft will be defined. Additionally, aerodynamic and stability analyses will be performed, the aircraft's performance will be estimated, and optimization studies will be conducted regarding the configuration and wing loading.

At the conceptual design level, the mission requirements are first defined, along with a design philosophy that is followed to fulfill them. Then, an initial estimate of the weight and weight fractions is made, and the main geometric features of the aircraft are selected. From these, a preliminary assessment of the aircraft’s capabilities is formed, expressed through appropriate performance metrics (T/W, W/S). Subsequently, the first configuration layout is developed, which leads to a reevaluation of the weight fractions.

With these aircraft characteristics in place, the design team must assess whether the initially defined mission goals are met, and through trade-offs, modify the design or relax some of the mission requirements.

The code for this assignment is provided in additional scripts in this repo, unlike the original Greek report that included it.



Σχήμα 1: Methodology at the conceptual design level.

2 Mission requirements

In this case, the requirements that must be met are defined by the assignment statement. Specifically, with a crew weight of 75 kg, the aircraft must be able to cover 800 kilometers with full fuel. The stall speed must be 50 km/h, while the maximum speed must be 470 km/h (almost ten times higher).

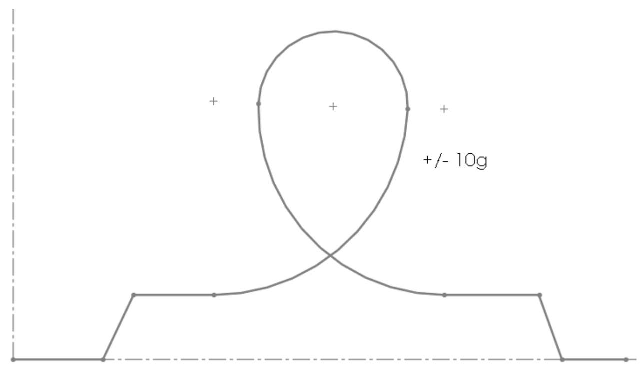
Req no.	Requirement	Value
010	Crew	75 kg
020	Cruise Range	800 km
030	Stall Speed	50 kph
040	Maximum Speed	470 kph
050	Rate of Climb	22 m/s
060	Roll Rate	420 deg/sec
070	Takeoff Runway Length	300 m
080	Landing Runway Length	300 m
090	Service Ceiling	4000 m
100	Load Factor ("g limits")	± 10

Πίνακας 1: Requirements.

Dividing the aircraft's mission into two parts, it must be able to take off with full fuel and payload within 300 meters, achieve a climb rate of 22 m/s, cover 800 km, and land within 300 meters.

Σχήμα 2: Range mission.

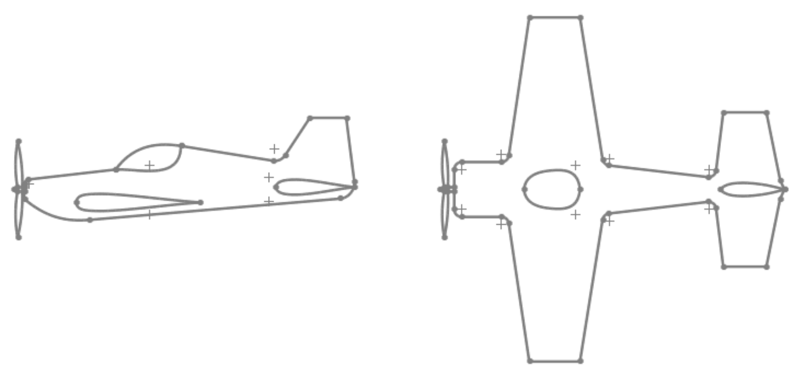
Meanwhile, during the show, with a smaller percentage of full fuel and the pilot's weight, it must be able to withstand ± 10g.



Σχήμα 3: Aerobatic mission.

3 Initial Design

For the first estimate of the parameters, one of the existing aerobatic aircraft is considered as a reference, with an increased main wing area to meet the requirement for a lower V_{stall}. Furthermore, in order to increase the maximum speed, it is expected that a reduction of L/D will be necessary. Because of this, retractable gears are used.



Σχήμα 4: Back of a napkin design.

The use of retractable gears leads to a low wing design in order to provide space for the front wheels (taildragger landing gear). Additionally, this allows for maximizing the control surface areas. The tail section follows a conventional approach, with large control surfaces to enable aerobatic maneuvers. It is expected that there will be a need for a more powerful engine, increased S_{wet}, and greater required thrust.

4 Weight

To estimate the total weight, the mission is divided into different segments: taxi, climb to 4000 meters, cruise, loiter at destination, and finally landing.

$$\frac{W_f}{W_0} = 1.06 \cdot \left(1 - \frac{W_5}{W_0}\right) = 1.06 \cdot \left(1 - \frac{W_1}{W_0} \cdot \frac{W_2}{W_1} \cdot \frac{W_3}{W_2} \cdot \frac{W_4}{W_3} \cdot \frac{W_5}{W_4}\right)$$

where the weight fractions are formed as:

- $W_1/W_0 = 0.97$ (taxi)
- $W_2/W_1 = 0.985$ (climb)
- $W_3/W_2 = 0.95$ (cruise)
- $W_4/W_3 = 1$ (loiter)
- $W_5/W_4 = 0.995$ (landing)

Substituting yields: $\frac{W_f}{W_0} = 0.1$

From the equation:

$$W_0 = \frac{W_{\text{payload}} + W_{\text{crew}}}{1 - W_f/W_0 - W_e/W_0}$$

The only missing element to determine the initial takeoff weight is the ratio W_e/W_0, i.e., the weight of the aircraft without fuel and payload divided by the maximum takeoff weight. This quantity is computed using an iterative process with the above equation combined with the weight ratio derived from Table 3.1 of Raymer.

A mission requirement of the designed aircraft is to withstand ±10g loads. This, combined with the weight of more complex systems like retractable gears and flaps, creates the need for a significantly more robust support structure than that of a conventional Cessna. Furthermore, there is essentially no need to carry payload beyond the crew, meaning the mission is more similar to that of a trainer aircraft where the payload consists solely of the pilot. For these reasons, it is considered that the closest category in the table is that of a jet trainer, despite the presence of a MEKprop instead of a jet engine. In fact, the presence of a MEK, which is heavier than a jet for comparable performance, if anything, makes the need for a sturdier internal structure even more pressing.

$$\frac{W_e}{W_0} = 1.59 \cdot W_0^{-0.1}$$

As a reference point, the Zivko Edge 540 has an empty weight of 531 kg and a maximum takeoff weight of around 816 kg. Here, with a more powerful (and thus heavier) engine, a larger wing, and more complex systems like retractable gears and flaps, the estimated maximum takeoff weight is 1000 kg.

Running the iterative process yields the following for the first weight estimate:

Weight Guess	**We**	**Wf**	**Wtotal**
1000	796.8877 (79.7%)	109.6897	963.6
970	775.3391 (80%)	106.3990	989.5
980	782.5293 (79.8%)	107.4959	980.2

Πίνακας 2: Iterative process for initial weight estimation.

Therefore, the first estimate for the full takeoff weight is 980 kg.

5 Design Path

5.1 Initial Design Evolution

To simultaneously achieve $V_{stall} = 50$ kph and $V_{max} = 470$ kph, approximately 9.5 times the stall speed, the following reasoning process is set in motion.

First of all, the requirements for low V_{stall} lead to the need for a large wing equipped with slats and flaps so that a sufficiently large C_L can be achieved. Such a configuration not only adds extra weight to the wings—requiring stronger spars for structural support—but also increases the overall wetted surface area, which in turn increases parasitic drag. This ultimately demands a more powerful engine.

The necessary calculations lead to an engine requiring around 850 kW of power. For such high output and a good power-to-weight ratio, a turboprop engine is unavoidably selected. However, this choice is clearly overdimensioned and results in a massive climb rate and a service ceiling nearly five times higher than the requirement. Essentially, this creates something that behaves more like an RC aerobatic plane, where the engine's power allows the pilot to do anything, creating an almost square operating envelope limited by structural constraints.

5.2 Relaxing Requirements

A second option is to relax some of the requirements. With V_{stall} still clearly lower than any comparable aerobatic aircraft—around 55 kph (with flaps and slats extended)—and a maximum speed of 460 kph, a smaller wing can be used, leading to reduced wetted surface area and ultimately lower required power. Due to the nonlinear nature of the governing relationships, this relaxation leads to the need for an engine with a nominal output close to 340 kW, something achievable with a turbocharged internal combustion engine. This configuration still includes retractable gears to reduce L/D as much as possible.

A third and final option is a significant relaxation of the stall speed constraint while keeping the other constraints. The resulting aircraft ends up with characteristics similar to existing aerobatic planes, albeit with retractable gears and a slightly more slender design, aiming to reduce S_{wet}/S_{ref} and hence parasitic drag, in order to meet the higher maximum speed target.

5.3 Trade-off

To choose one of the above options, a trade-off must be implemented, taking into account the performance of each design solution. Given the mission nature of the aircraft, some parameters—such as structural robustness, climb rate, and roll rate—are considered non-negotiable. Moreover, given that the aircraft carries only the pilot and has enough fuel for an 800 km range, changes to these figures would not significantly alter the final design.

For the trade-off, the following parameters are selected: V_{stall}, V_{max}, $P_{required}$, and engine cost.

Stall speed is proportional to weight and inversely proportional to $C_{L_{max}}$ and reference area. Having selected an airfoil, and given a value for weight (from prior estimation), V_{stall} effectively depends only on the main wing surface area (assuming the tail does not contribute significantly to lift).

Maximum speed depends on engine power and wetted surface area, which in turn is largely determined by the wing area. A larger wing leads to significantly higher required power. Clearly, not all requirements can be met at once. In effect, a lower V_{stall} leads to a relatively lower V_{max}. These two requirements are antagonistic for any reasonable engine power.

Following the second option, neither requirement is strictly met; however, the deviation from both is minor. The maximum speed is achieved—but at an altitude of 5 km instead of 4. The stall speed without flaps and slats is around

89 kph—lower than the 95 kph of the Zivko, but quite far from the target.

The constraints related to takeoff/landing distance and service ceiling are relatively inactive (analyzed in detail later).

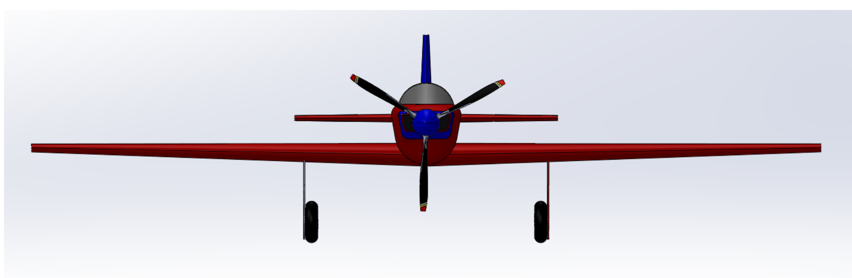
The table below summarizes the performance of each option:

Parameter	1st Option	2nd Option	3rd Option
Stall Speed (kph)	50	55	98
Maximum Speed (kph)	470	460	470
Power Required (kW)	850	350	280

Πίνακας 3: Trade-off Parameters

It is worth noting that in this analysis, design solutions that deviate significantly from the original design philosophy—such as tandem or delta wings—were not considered.

6 Configuration Layout



Σχήμα 5: Front view with landing gear.

6.1 Surface Estimation

Having now established the weight, the value of the ratio $\frac{S_{wet}}{S_{ref}}$ is selected based on Figure 3.5 of "Aircraft Design – A Conceptual Approach" by Daniel P. Raymer. The estimation is $\frac{S_{wet}}{S_{ref}} = 3.1$. Then, assuming an Aspect Ratio of 6, we use Figure 3.6 of the same book for a first estimate of

$$\left(\frac{L}{D}\right)_{\text{max}} = 17$$

At this point, considering the rather low stall speed as a constraint, we opt for double-slotted Fowler flaps combined with high-lift devices at the leading edge (slats). Thus, we consider a maximum lift coefficient of

$$C_{L_{\max}} = 3.06$$

Proceeding to compute the reference wing area:

$$S_{\text{ref}} = \frac{W \cdot g}{0.5 \cdot \rho_\infty \cdot C_{L_{\max}} \cdot V_{\text{stall}}^2}$$

This gives us the wing area under the assumption that only the main wing contributes to lift. Using the ratio $\frac{S_{wet}}{S_{ref}} = 3.1$, we also get the wetted surface area:

$$S_{\text{wet}} = 66.1980\,\text{m}^2$$
$$S_{\text{ref}} = 22.0660\,\text{m}^2$$

And finally, we calculate wingspan and mean aerodynamic chord:

$$\text{b} = \sqrt{\text{AR} \cdot S_{\text{ref}}} = 11.5063\,\text{m}$$
$$\bar{c} = \frac{S_{\text{ref}}}{\text{b}} = 1.9177\,\text{m}$$

6.2 Engine Selection

Next, based on Table 12.3 from the same book, we use a wetted area friction coefficient of $C_{fe} = 0.005$, assuming retractable gears and a slender design as discussed earlier. This allows us to calculate the parasite drag coefficient:

$$C_{D_o} = \frac{S_{\text{wet}}}{S_{\text{ref}}} \cdot C_{fe}$$

Then, calculating the induced drag factor:

$$K = \frac{1}{\pi \cdot AR \cdot e}$$

Assuming a taper ratio of 0.35, we get $e = 0.98$ (to be justified later in the aerodynamic analysis section).

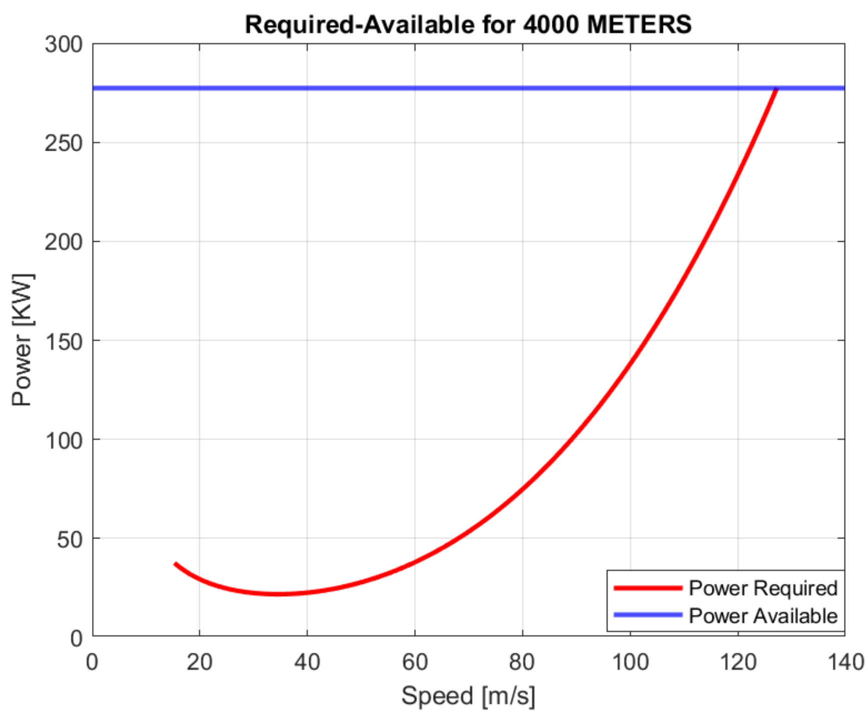
We choose the symmetric airfoil NACA 64(2)-015A, prioritizing inverted flight capability for aerobatics, with minimal differences from the standard configuration. After correcting the lift-curve slope for this airfoil, we obtain the C_L-AoA curve for the aircraft.

With this, we calculate the required power to meet the max speed constraint $V_{max} = 460\,\text{km/h}$. Specifically, we compute the drag as a function of speed and hence determine the net power required. Note: the aircraft is assumed to reach this speed during cruise at **4000 m** altitude.

$$\text{Drag} = 0.5 \cdot \rho_\infty \cdot (C_{D_o} + K \cdot C_L^2) \cdot S_{\text{ref}} \cdot V^2$$

$$P_{\text{required}} = D \cdot V$$

Most aircraft in this category use either turboprop or ICE (internal combustion engines), so the team opted for a propeller-driven solution. The diagram below shows required and available power at 4000 m, highlighting what's needed to reach 460 km/h.

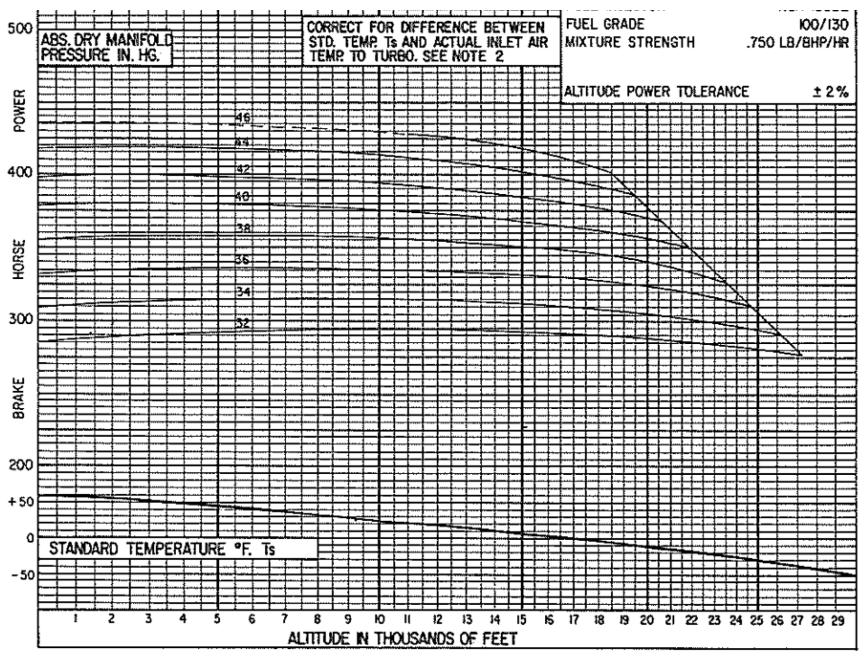


Σχήμα 6: Required and available power.

From the diagram, we need an available power of 277.43 kW at 4000 m. Accounting for propeller efficiency $\eta = 0.85$, the engine-propeller system must provide:

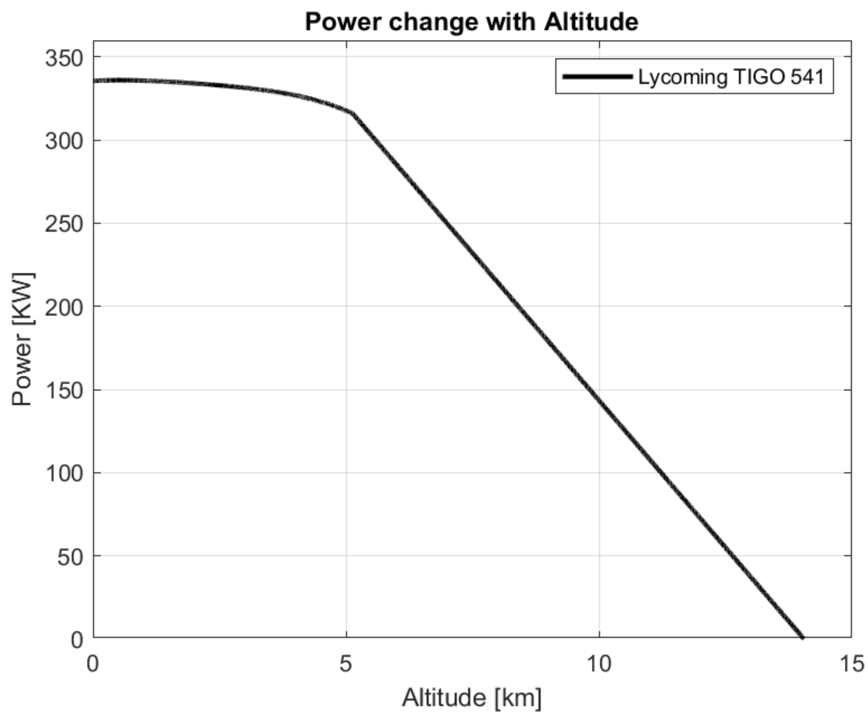
$$\text{Power} = \frac{\text{Power\_net}}{\eta} = 326.39\,\text{kW}$$

This requirement is met by the **Lycoming TIGO-541-D1A**, a turbocharged internal combustion engine with nominal power 450 HP at 48 inHg manifold pressure. Below is the power curve of the slightly derated TIGO-541-E1A (425 HP @ 45 inHg):



Σχήμα 7: Power curve of Lycoming TIGO-541-E1A.

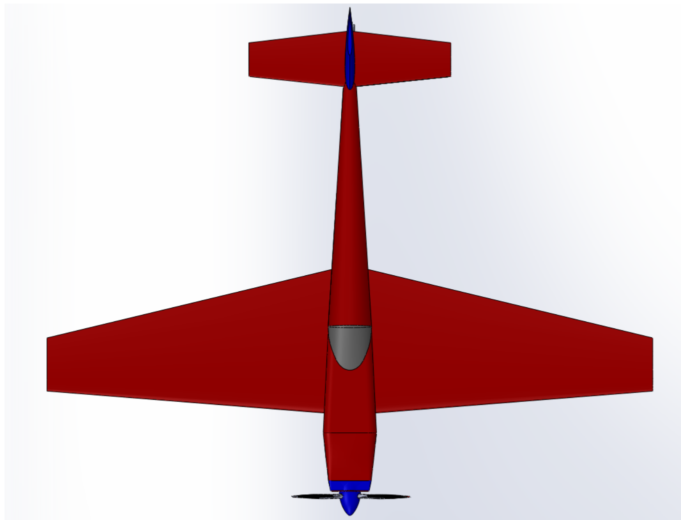
Interpolating gives the available power vs altitude curve for the D1A, shown below:



Σχήμα 8: Available power of Lycoming TIGO-541-D1A.

Additionally, this engine is based on the IO-540, which has an aerobatic version (AEIO-540), so it can be easily adapted for aerobatic use. The engine configuration is **tractor**.

6.3 Wing Characteristics



Σχήμα 9: Top view.

Airfoil Type	b	$\bar{c}$	A	TR	AR
NACA 64(2)-015A	11.4 m	1.91 m	21.77 m^2	0.355	5.97

- The aircraft has 2 low wings for easier maintenance, refueling, pilot visibility, increased internal space, and structural strength.
- No dihedral is applied.
- There is no geometric or aerodynamic twist along the span; wingtips are cut-off.
- Wing incidence angle $i_w = 0°$ so that the wing aligns with the main inertial axis, minimizing trim effort during fast rolls.
- The angle between the waterline and the wing's leading edge is smaller than that with the trailing edge, aligning the quarter-chord line with the waterline.

6.4 Tail Characteristics

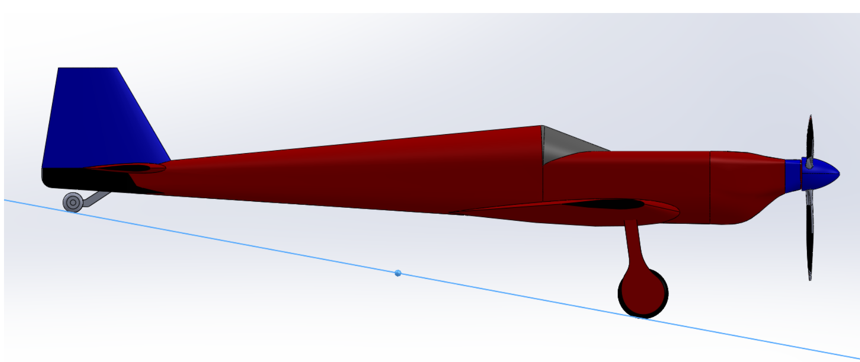
Airfoil Type	b	$\bar{c}$	A	TR	AR
NACA 64-012A	3.8 m	0.845 m	3.21 m^2	0.6	4.51

- The tail is of conventional layout.
- Elevator spans 30% of the chord; about 30% of the vertical tail is blanketed during deflection.
- Horizontal tail arm is 6 m.
- Downwash velocity ratio $\eta = 0.85$.
- Vertical tail area is $1.68\,\mathrm{m}^2$.

6.5 Control Surfaces

On the wing, ailerons cover 70% of the span and 30% of the local chord, also capable of acting as flaperons. High-lift devices deploy via mechanisms on the wing's upper surface. Elevators and rudder each span 30% of their respective surfaces.

6.6 Fuselage – Landing Gear



Σχήμα 10: Side view with landing gear.

Metric	Landing Gear
Configuration	Tail dragger
Front wheel position	0.3 m ahead of CG, 1.85 m from centerline
Rear wheel position	Under fin, 6.7 m behind front wheels
Retractable mechanism	Yes

Πίνακας 4: Landing gear configuration.

7 Αναλύσεις

7.1 Αεροδυναμική

Όπως αναφέρθηκε προηγουμένως, επιλέχθηκε η αεροτομή **64(2)-015A** της σειράς **NACA 6** που εμφανίζει μέγιστο πάχος στο 0.4 της χορδής, έτσι ώστε να διατηρείται στρωτή η ροή για μεγαλύτερο κομμάτι της πτέρυγας, καθώς και να εμφανίζεται μικρότερο skin friction όπως φαίνεται και από το pocket στο διάγραμμα C_L v C_D.

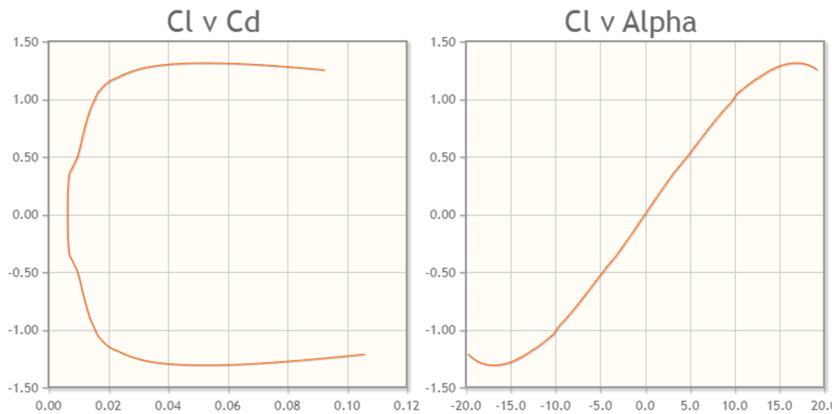
Για **μέση χορδή 1.91 μέτρα**, και κινηματικό ιξώδες 1.42×10^{-5} έως 3.5×10^{-5} m^2/s για υψόμετρο 0–10 km και ταχύτητα ≥ 14 m/s για χαμηλό υψόμετρο και ≥ 20 m/s για υψηλό υψόμετρο, ο αριθμός Reynolds είναι $Re \geq 10^6$ για κάθε πιθανή συνθήκη πτήσης του αεροσκάφους. Άρα η επίδραση της διακύμανσης του αριθμού Reynolds είναι μικρή. Πιο συγκεκριμένα, το $C_{l_{\max}} = 1.31$, και για $AR > 4$ θεωρείται $C_{L_{\max}} = 0.9 \cdot C_{l_{\max}} = 1.18$.

Το αεροσκάφος σχεδιάστηκε με fowler flaps με επέκταση 0.3 της χορδής στο 0.3 του διατάματος από τη βάση, slotted flaperons στο υπόλοιπο 0.7 και slats στο 0.9 της πτέρυγας με επέκταση 0.15 της χορδής. Επειδή το φτερό είναι τραπέζιο με taper ratio $\lambda = 0.355$, το 0.3 του διατάματος από τη βάση αντιστοιχεί στο 0.4 της επιφάνειας. Άρα, από τον Raymer Table 12.2 υπολογίζεται:

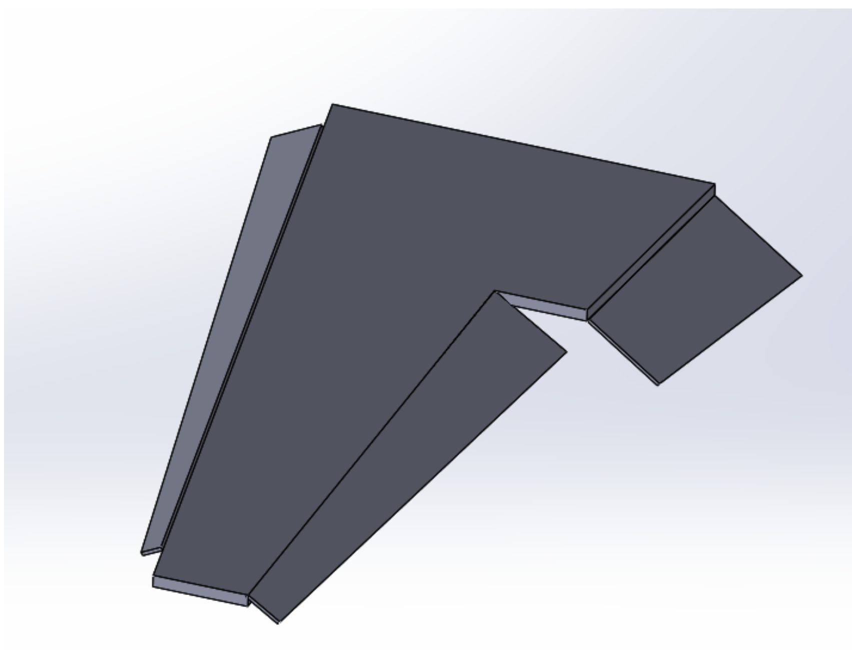
$$\Delta C_{L_{\max}} = 0.4 \cdot 1.3 \cdot (1 + 0.3) + 0.6 \cdot 1.3 + 0.9 \cdot 0.4 \cdot 1.15 = 1.87$$

άρα

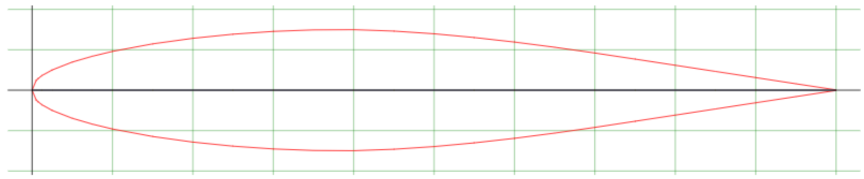
$$C_{L_{\max}} = 1.18 + 1.87 = 3.05$$



Σχήμα 11: Καμπύλες NACA 64(2)-015A



Σχήμα 12: Σχεδιάγραμμα της πτέρυγας με όλα τα high lift devices



Σχήμα 13: Προφίλ NACA 64(2)-015A

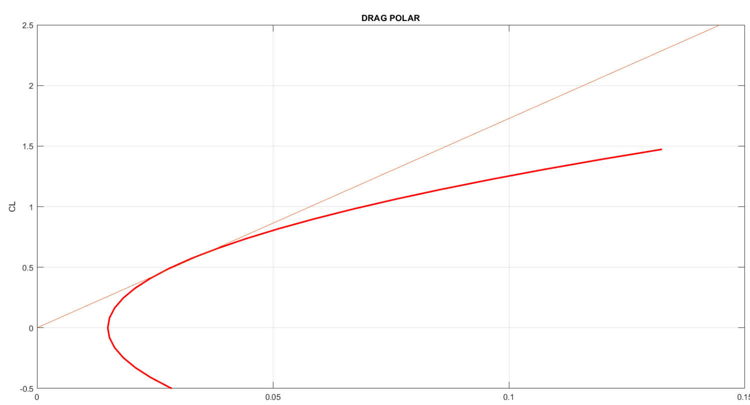
Για πτέρυγα με taper ratio $\lambda = 0.355$ και $AR = 5.97$, η κατανομή lift προσεγγίζει την ελλειπτική. Επομένως:

$$C_{D_i} = \frac{C_L^2}{\pi \cdot AR} = 0.053 \cdot C_L^2$$

και επειδή το fuselage είναι μικρό και λεπτό, θεωρείται $K = 0.053$.

Από το σχέδιο στο CAD προκύπτει $\frac{S_{\text{wet}}}{S_{\text{ref}}} = 2.889$. Για ελαφρύ μονοκινητήριο αεροσκάφος, $C_{fe} = 0.0055$ αλλά λόγω retractable landing gear και laminar airfoil προτιμάται $C_{fe} = 0.0052$. Άρα:

$$C_{D_0} = C_{fe} \cdot \frac{S_{\text{wet}}}{S_{\text{ref}}} = 0.015$$



Σχήμα 14: Drag Polar

7.2 Επιφάνεια οριζόντιου ουραίου πτερυγίου

Για την ευστάθεια του αεροσκάφους τα δύο βασικά μεγέθη είναι C_{m_α} και το static margin. Με $C_{L_{\alpha w}} = 2\pi \cdot \frac{AR}{AR+2} = 4.71$. Ορίζονται:

- $C_{L_{\alpha_t}} = 4.35$ (για $AR_t = 4.5$)
- $V_h = \frac{l_t \cdot S_t}{S \cdot c}$
- $n = 0.85$ (ταχύτητα ουράς)

Το static margin είναι:

$$sm = -\left(\frac{x_{cg}}{c} - \frac{x_{ac}}{c}\right) + nV_h \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}}\left(1 - \frac{d\epsilon}{d\alpha}\right)$$

με $\frac{d\epsilon}{d\alpha} = \frac{2C_{L_{\alpha_w}}}{\pi AR}$. Θέτοντας:

- $sm = 0.05$
- $x_{cg} = 0.7275$ m
- $x_{ac}/c = 0.25$
- $l_t = 6$ m

προκύπτει:

$$S_t = 3.21\, m^2$$

7.3 Ευστάθεια

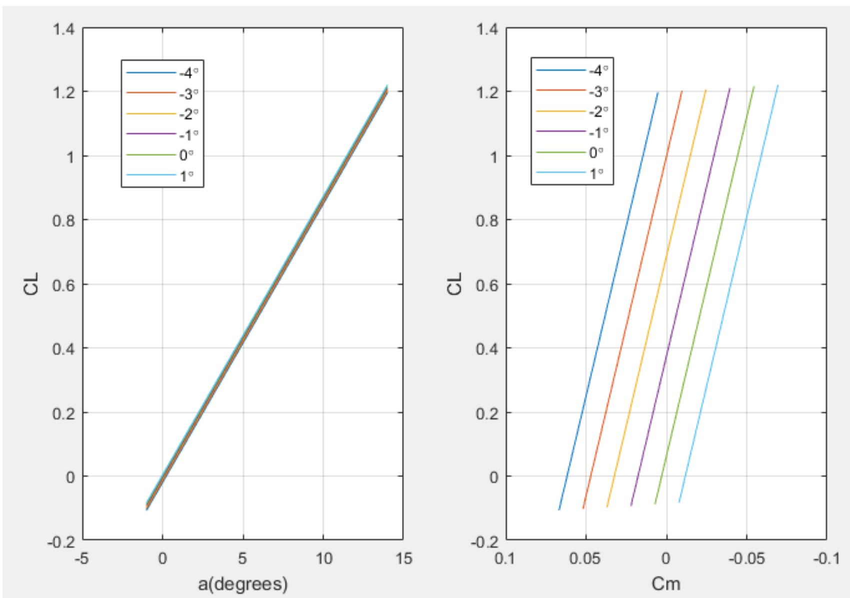
Υπολογίζονται οι σταθεροί όροι και οι κλίσεις:

- $C_{L_0} = C_{L_{0_w}} + C_{L_{\alpha_t}}(\epsilon_0 + i_w - i_t)$, με $\epsilon_0 = 0$
- $C_{m_0} = C_{m_{ac}} + C_{L_{0_w}}(\frac{x_{cg}}{c} - \frac{x_{ac}}{c}) + nV_h C_{L_{\alpha_t}}(\epsilon_0 + i_w - i_t)$
- $C_{L_\alpha} = C_{L_{\alpha_w}} + n\frac{S_t}{S}C_{L_{\alpha_t}}(1 - \frac{d\epsilon}{d\alpha})$
- $C_{m_\alpha} = C_{L_{\alpha_w}}(\frac{x_{cg}}{c} - \frac{x_{ac}}{c}) - nV_h C_{L_{\alpha_t}}(1 - \frac{d\epsilon}{d\alpha})$
- $C_{L_{\delta_e}} = n\frac{S_t}{S}\tau C_{L_{\alpha_t}}, \quad \tau = 0.5$
- $C_{m_{\delta_e}} = -nV_h \tau C_{L_{\alpha_t}}$

Με $i_w = 0^\circ$ και $i_t = -0.1^\circ$ προκύπτουν:

- $C_{L_0} = -9.51 \times 10^{-4}$

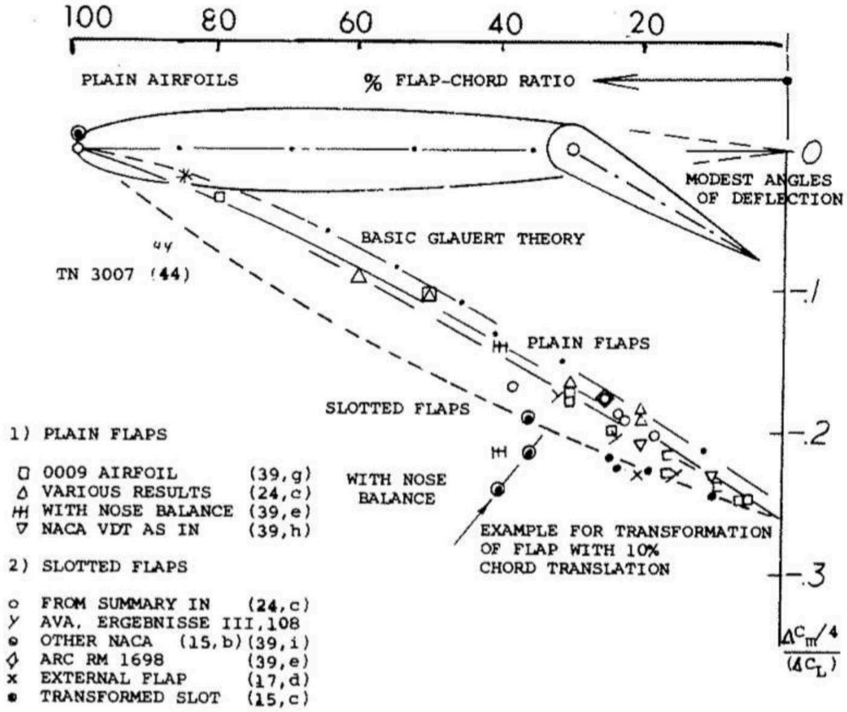
- $C_{L_\alpha} = 4.978$
- $C_{m_0} = 0.003$
- $C_{m_\alpha} = -0.2353$
- $C_{L_{\delta_e}} = 0.2725$
- $C_{m_{\delta_e}} = -0.8562$



Σχήμα 15: Trim diagram no flaps

Με flaps:

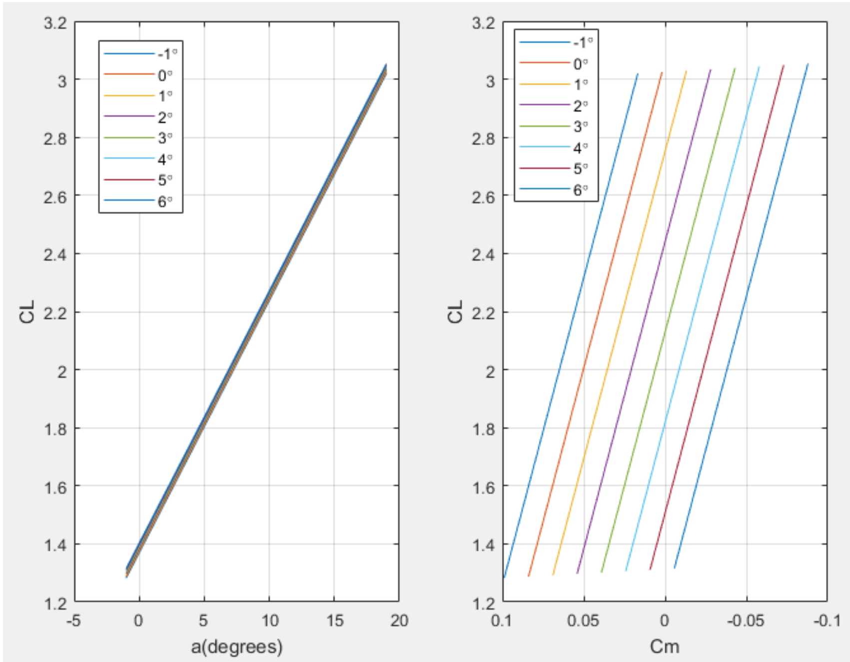
- $\Delta C_L = 1.46 \Rightarrow C_{L_{0_w}} = 1.46$
- $C_{m_{ac}} = -0.2 \cdot \Delta C_L = -0.292$
- Νέο $x_{ac}/c = 0.28$



Σχήμα 16: Flaps pitching moment diagram

Νέοι συντελεστές:

- $C_{L_0} = 1.374$
- $C_{L_\alpha} = 4.978$
- $C_{m_0} = 0.0798$
- $C_{m_\alpha} = -0.2353$
- $C_{L_{\delta_e}} = 0.2725$
- $C_{m_{\delta_e}} = -0.8562$



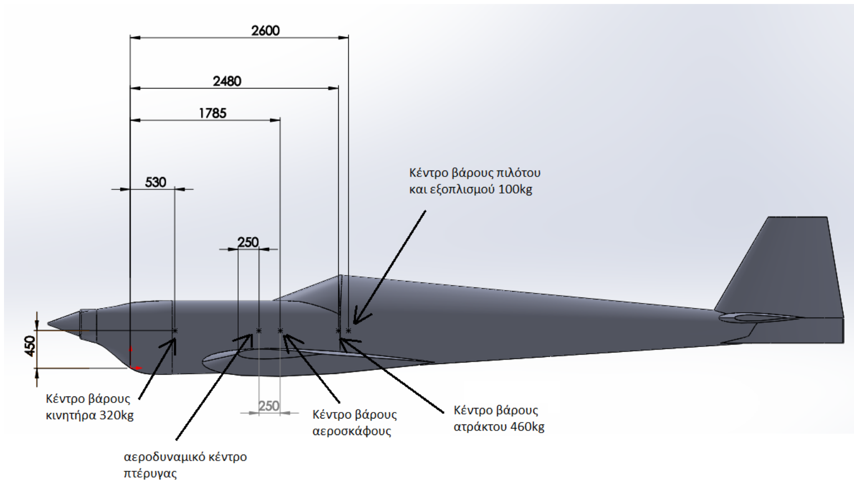
Σχήμα 17: Trim diagram full flaps

δ_e	-4	-3	-2	-1	0	1
C_{Lt}	0.0464	0.0511	0.0559	0.0606	0.0654	0.0701

Πίνακας 5: No Flaps

δ_e	-1	0	1	2	3	4	5	6
C_{Lt}	-0.0006	0.0042	0.0089	0.0137	0.0185	0.0232	0.028	0.0327

Πίνακας 6: Full Flaps



Σχήμα 18: Θέση x_{cg}

Το αεροσκάφος μπορεί να πετάξει σε trimmed κατάσταση για λογικές τιμές δ_e ανεξαρτήτως flaps. Αυτό και η συνεισφορά του tail στο lift οδήγησαν στην επιλογή $x_{cg} = 0.7275$ m ώστε το x_{ac} να βρίσκεται 0.25 m μπροστά από το κέντρο βάρους.

8 Performance

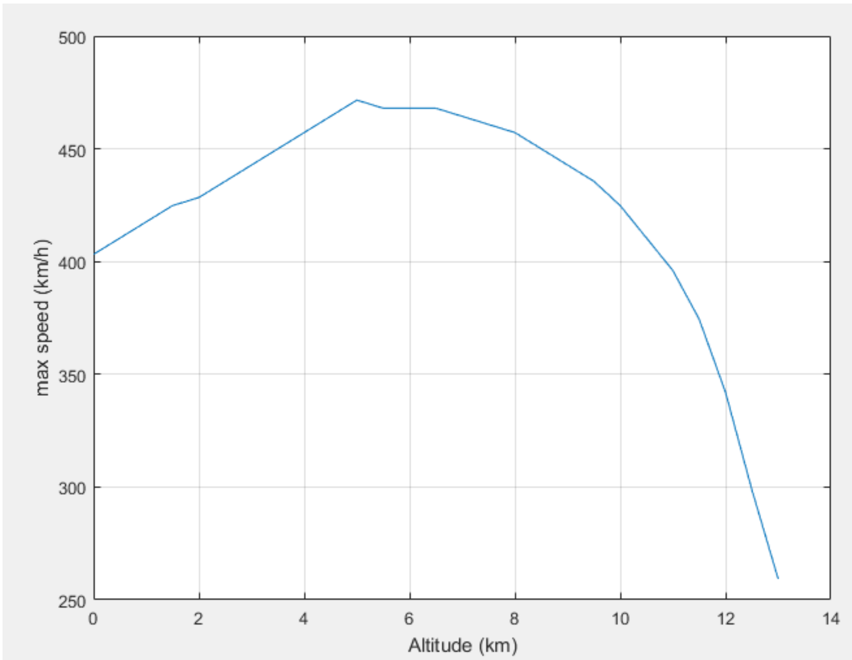
Οι επιδόσεις του αεροπλάνου που ενδιαφέρουν είναι η μέγιστη ταχύτητα, η ελάχιστη ταχύτητα πτήσης, ο μέγιστος ρυθμός climb, το service ceiling, ο ρυθμός περιστροφής και ο συντελεστής φόρτισης.

8.1 Μέγιστη ταχύτητα

Η απαιτούμενη ισχύς του αεροσκάφους οποιαδήποτε στιγμή εκφράζεται από τη σχέση

$$P_r = \frac{1}{2}\rho V^3 S \cdot C_{D_0} + \frac{K \cdot W^2}{\frac{1}{2}\rho V \cdot S}$$

Η ταχύτητα στην οποία η απαιτούμενη ισχύς ισούται με την μέγιστη διαθέσιμη ισχύ είναι η μέγιστη ταχύτητα. Για τον κινητήρα που επιλέχθηκε, υπερτροφοδοτούμενος εμβολοφόρος, η μέγιστη ισχύς είναι ανεξάρτητη της ταχύτητας του αεροσκάφους και μεταβάλλεται με το υψόμετρο σύμφωνα με το παρακάτω διάγραμμα. Έτσι υπολογίζεται η μέγιστη ταχύτητα για κάθε υψόμετρο, στο επίπεδο της θάλασσας είναι 403.2 km/h, στα 4 km είναι 457.2 km/h, ενώ η μέγιστη ταχύτητα εντοπίζεται στα 5 km και είναι 471.6 km/h.



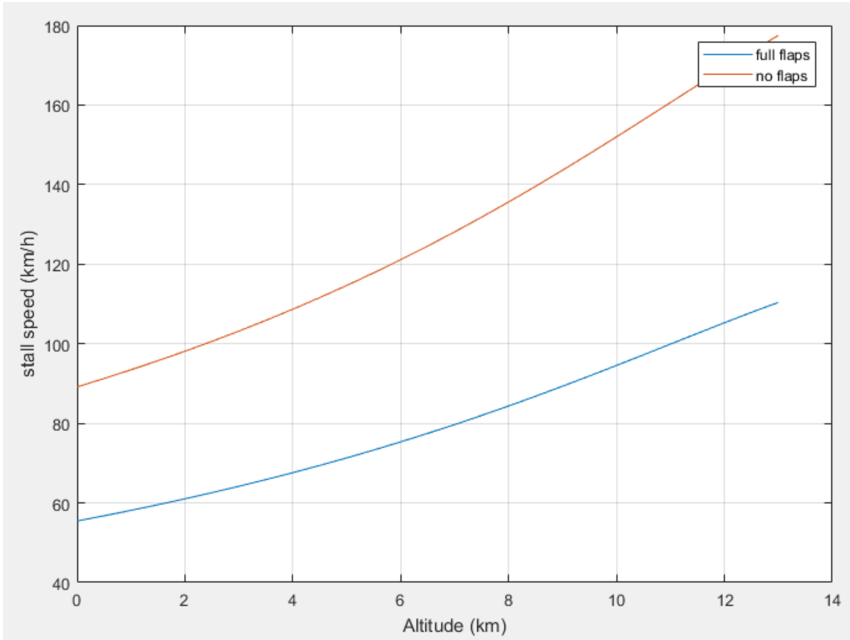
Σχήμα 19: Μέγιστη ταχύτητα

8.2 Ελάχιστη ταχύτητα

Η ελάχιστη ταχύτητα πτήσης είναι η ελάχιστη ταχύτητα στην οποία το lift εξισορροπεί το βάρος:

$$W = L = \frac{1}{2}\rho V_{\text{stall}}^2 S \cdot C_{L_{\max}} \Rightarrow V_{\text{stall}} = \sqrt{\frac{2W}{\rho S C_{L_{\max}}}}$$

Για βάρος 980 kg = 9614 N και την αντίστοιχη πυκνότητα, υπολογίζεται η ταχύτητα stall σε κάθε υψόμετρο. Στο επίπεδο της θάλασσας είναι 15.4 m/s ή 55.4 km/h.



Σχήμα 20: Ελάχιστη ταχύτητα

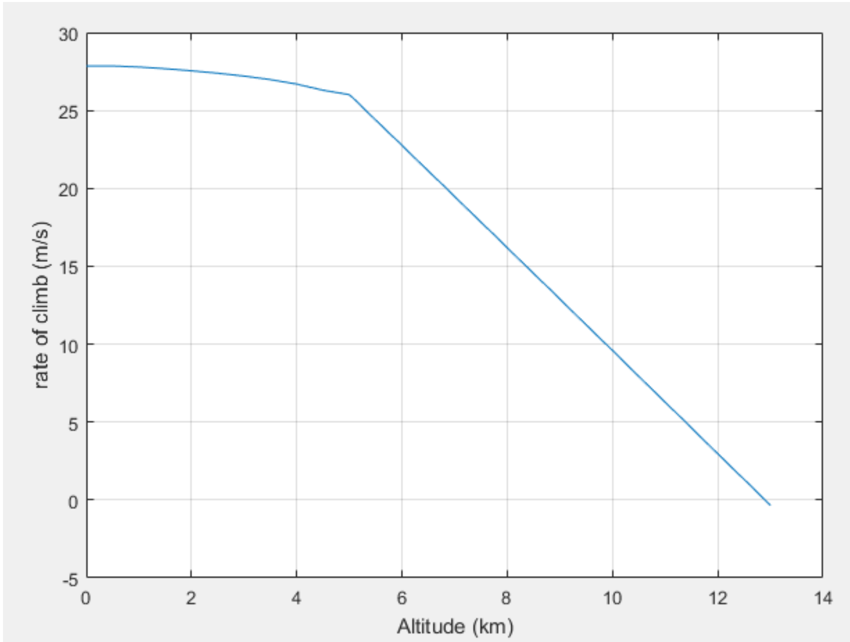
8.3 Ρυθμός climb

Ο ρυθμός climb υπολογίζεται ως εξής:

$$V_c = \left(\frac{4K}{3C_{D_0}}\right)^{1/4} \cdot \sqrt{\frac{W}{\rho S}}$$

$$RoC = \frac{P_{\max}}{W} - \frac{2^{2.5}}{3^{0.75}} \cdot \sqrt{\frac{W}{\rho S}} \cdot (K^3 C_{D_0})^{1/4}$$

Ο μέγιστος ρυθμός εντοπίζεται στην επιφάνεια της θάλασσας και είναι 27.8 m/s. Το υψόμετρο στο οποίο μηδενίζεται είναι το service ceiling, εδώ στα 13 km.



Σχήμα 21: Ρυθμός climb

8.4 Ρυθμός περιστροφής

Ο συντελεστής ροπής απόσβεσης είναι:

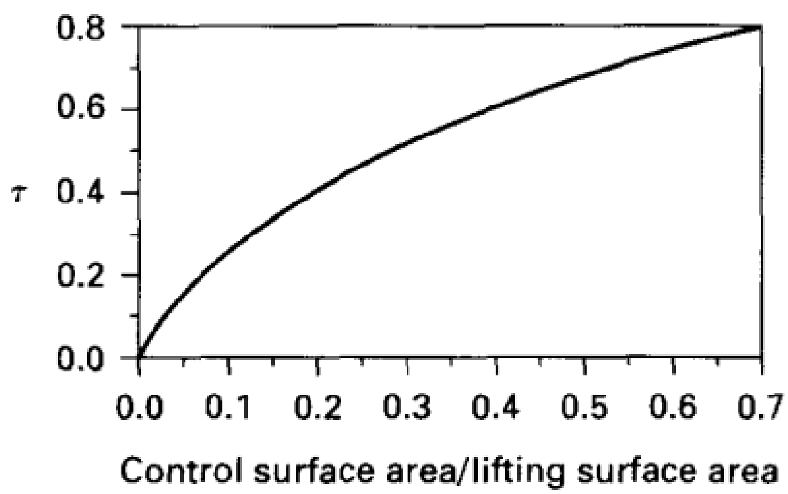
$$C_{l_p} = -\frac{C_{L_\alpha}(1+3\lambda)}{12(1+\lambda)}$$

Ο συντελεστής ροπής ελέγχου για taper ratio λ, ποσοστό aileron $a = 0.7$, και $\tau = 0.5$ είναι:

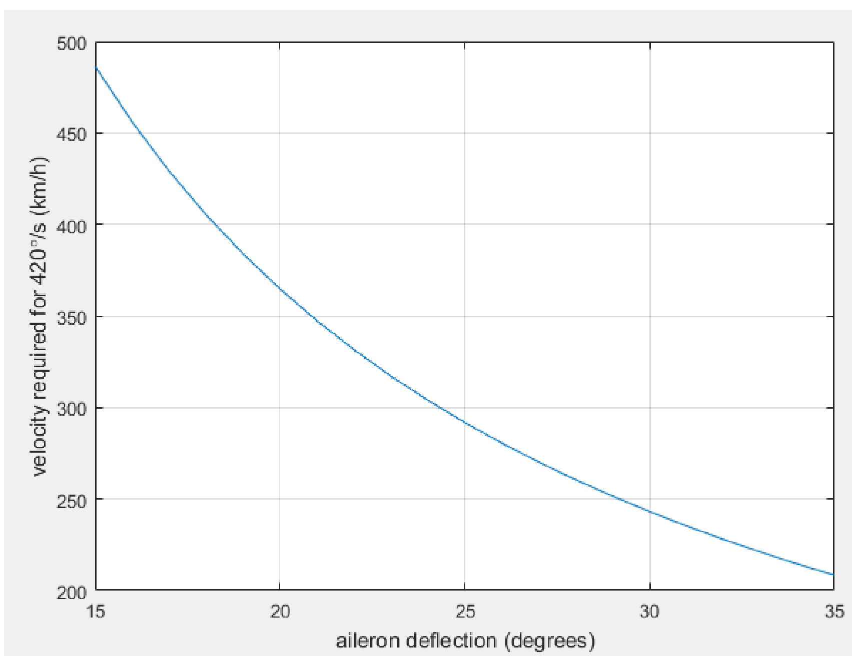
$$C_{l_{\delta_a}} = \tau C_{L_\alpha} \cdot \frac{\lambda a + \frac{a^2}{2}(1-\lambda)}{1+\lambda}$$

Ο σταθερός ρυθμός περιστροφής δίνεται από:

$$p = \frac{2u_0}{b} \cdot 12\tau \cdot \frac{\lambda a + \frac{a^2}{2}(1-\lambda)}{1+3\lambda}$$



Σχήμα 22: Συντελεστής τ



Σχήμα 23: Ρυθμός περιστροφής

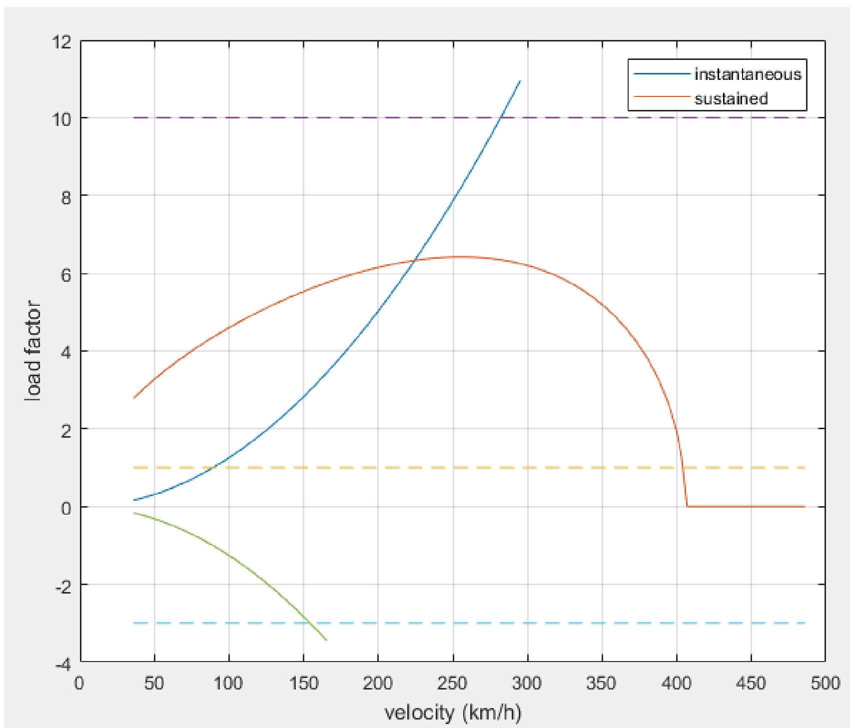
8.5 Συντελεστής φόρτισης

Ο στιγμιαίος συντελεστής φόρτισης:

$$n_{\text{inst}} = \frac{1}{2}\rho V^2 S C_{L_{\max}} \cdot \frac{1}{W}$$

Ο συνεχής συντελεστής φόρτισης:

$$n_{\text{sust}} = \frac{1}{W}\sqrt{\left(P_{\max} - \frac{1}{2}\rho V^3 S C_{D_0}\right) \cdot \frac{1}{2}\rho V S \cdot \frac{1}{K}}$$



Σχήμα 24: Συντελεστής φόρτισης

8.6 Απογείωση

Κατά το Ground Roll, ισχύει:

$$V_{\text{stall\_TO}} = \sqrt{\frac{2Wg}{\rho C_{L_{\text{max\_TO}}} S_{\text{ref}}}} = 19.25\,\text{m/s}$$

$$V_{\text{LO}} = 1.1 \cdot V_{\text{stall\_TO}} = 21.1730\,\text{m/s}$$

$$\text{Thrust}_{\text{TO}} = \frac{P_{\text{TO}}}{0.7 \cdot V_{\text{LO}}}$$

$$S_g = 1.21 \cdot \frac{(Wg/S_{\text{ref}})}{g\rho C_{L_{\text{max\_TO}}} \cdot (\text{Thrust}_{\text{TO}}/gW)}$$

Προκύπτει $S_g = 40.7$ m.

8.7 Προσγείωση

Ταχύτητα προσέγγισης:

$$V_f = 1.15 \cdot V_{\text{stall}}$$

$$R = \frac{V_f^2}{g \cdot 0.2}$$

$$h_f = R \cdot (1 - \cos\theta_{\text{approach}}) + 1$$

$$S_{\text{approach}} = \frac{15 - h_f}{\tan\theta_{\text{approach}}} = 164\,m$$

$$S_{\text{flare}} = R \cdot \sin\theta_{\text{approach}} = 13.7\,m$$

Απόσταση τροχοδρόμησης:

$$S_{\text{ground\_roll\_landing}} = N \cdot V_{\text{TD}} + \frac{WgV_{\text{TD}}^2}{2g} \cdot \frac{1}{\text{Drag}_{\text{groundroll}} + m_r(Wg - L_{\text{landing}})} = 80\,m$$

Συνολικά: $S_{\text{landing}} = 257$ m.

9 Βελτιστοποίηση

Για την βελτιστοποίηση των μεγεθών του διατάματος (Aspect Ratio) και πτερυγικού φόρτου (wing loading) δημιουργήθηκαν 9 σενάρια με διαφορετικές τιμές Wing loading και Aspect Ratio. Έχοντας τους συντελεστές παρασιτικής και επαγώμενης οπισθέλκουσας,

$$\text{Parasitic Drag} = \frac{S_{\text{wet}}}{S_{\text{ref}}} \cdot C_{f_e}, \quad \text{και} \quad K = \frac{1}{\pi \cdot AR \cdot e}$$

υπολογίζεται το μέγεθος:

$$L/D_{\max} = \left(\frac{1}{\text{Parasitic Drag} \cdot 4K} \right)^{0.5}$$

Κατόπιν, υπολογίζεται το νέο βάρος. Έπειτα βρίσκουμε την επιφάνεια αναφοράς:

$$S_{\text{ref}} = \frac{W}{W/S}$$

και ακολουθούν οι υπολογισμοί εκπετάσματος, μέσης χορδής και περιβρεχόμενης επιφάνειας:

$$b = \sqrt{AR \cdot S_{\text{ref}}}, \quad \bar{c} = \frac{S_{\text{ref}}}{b}, \quad S_{\text{wet}} = \left(\frac{S_{\text{wet}}}{S_{\text{ref}}} \right) \cdot S_{\text{ref}}$$

Έχοντας όλα τα παραπάνω μπορούμε να βρούμε τις τιμές των V_{stall}, $V_{\max}$ και Roll Rate:

$$V_{\text{stall}} = \left(\frac{Wg}{0.5 \cdot \rho \cdot S_{\text{ref}} \cdot C_{L_{\max}}} \right)^{0.5}$$

$$\text{Max\_speed} = \left(\frac{P_{\max}}{0.5 \cdot \rho_{4\text{km}} \cdot S_{\text{ref}} \cdot \text{Parasitic Drag}} \right)^{\frac{1}{3}}$$

Ο παρακάτω πίνακας δείχνει τις τιμές των μεγεθών W/S και AR για κάθε σενάριο:

Scenarios		Parameters	
Scenario	Περιγραφή	W/S [kPa]	AR
1	Low W/S, Low AR	35	4
2	Low W/S, Medium AR	35	6
3	Low W/S, High AR	35	8
4	Medium W/S, Low AR	45	4
5	Medium W/S, Medium AR	45	6
6	Medium W/S, High AR	45	8
7	High W/S, Low AR	55	4
8	High W/S, Medium AR	55	6
9	High W/S, High AR	55	8

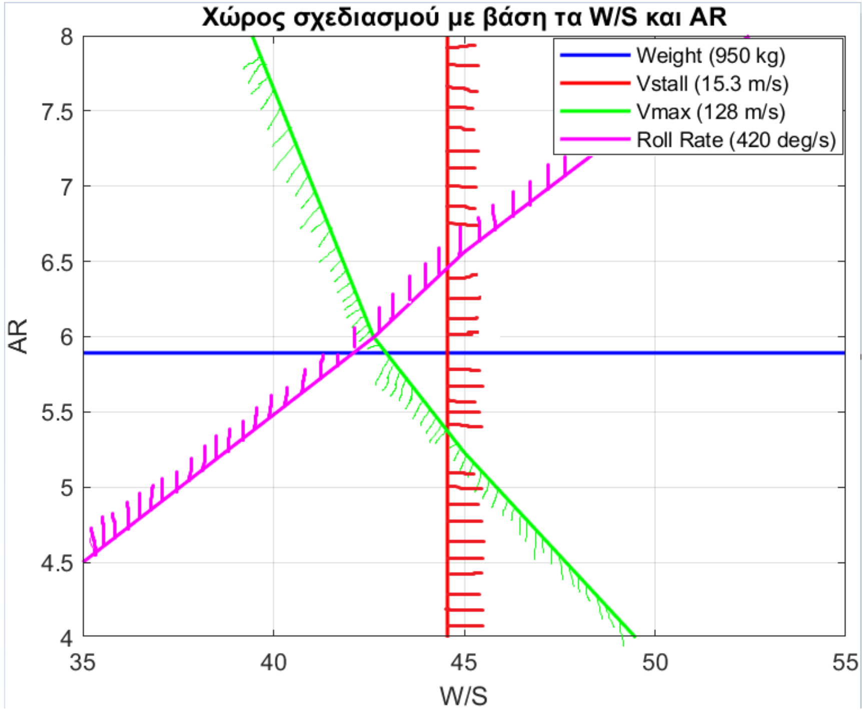
Πίνακας 7: Scenarios for Optimization Process with Different W/S and AR Values

Τα αποτελέσματα φαίνονται στον επόμενο πίνακα:

Scenario	V_{stall} [m/s]	V_{max} [m/s]	Roll Rate [deg/s]	Weight [kg]
1	13.56	114	434	1090
2	13.56	120	381	942
3	13.56	123	343	870
4	15.4	124	492	1090
5	15.4	131	432	942
6	15.4	134	389	870
7	17	133	544	1090
8	17	139	478	942
9	17	143	430	870

Πίνακας 8: Αποτελέσματα σεναρίων

Βλέποντας τις τάσεις των τιμών των περιορισμών, εξάγεται το σχετικό διάγραμμα που δείχνει το design space στο οποίο μπορεί κανείς να κινηθεί επιλέγοντας τις αντίστοιχες τιμές W/S και AR, εξασφαλίζοντας τις απαιτήσεις της αποστολής.



Σχήμα 25: Design Space

Από το σχήμα φαίνεται πως ο χώρος σχεδιασμού είναι αρκετά στενός καθώς τόσο οι περιορισμοί της αποστολής είναι αρκετά απαιτητικοί όσο και οι επιλογές που έγιναν κατά το conceptual design — όπως ο κινητήρας — οδήγησαν σε αυτό το design space. Δεδομένου ότι σε αυτή την τριγωνική περιοχή όλοι οι περιορισμοί της αποστολής ικανοποιούνται, θεωρείται βέλτιστη αυτή η επιλογή με το μικρότερο βάρος. Συγκριτικά με την δική μας επιλογή, μία ακόμη εναλλακτική θα ήταν η αύξηση του AR έως και την τιμή 6.5.

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