Digital Control

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1 Dynamical System

The initial step in controlling any system involves comprehending it and subsequently constructing a model. In the case of this particular system, a mathematical model has been meticulously developed and documented by Sreenivas Yelneedi, S. Lakshminarayanan, and G.P. Rangaiah in their paper titled 'A Comparative Study of Three Advanced Controllers for the Regulation of Hypnosis.' This model dissects into three distinct components, each encompassing the dynamic model of a distinct process: the breathing system model, the pharmacokinetic model, and the pharmacodynamic model.

Within this framework, the following variables play pivotal roles: C_{insp} represents the inspired drug concentration, C_0 signifies the fresh anesthetic gas concentration (both in $(vol.\%)$), V denotes the volume of the breathing system, C_1 represents the alveolar concentration, f_R stands for the frequency of respiration, V_T signifies the volume of each breath, Δ denotes the physiological dead space, ΔQ accounts for losses through the pressure relief valves, and Q_0 represents the flow rate entering the breathing system. It is noteworthy that C_0 serves as the manipulated variable.

The mathematical model for all these variables is elucidated through the following set of equations. (1) through (4) describe the pharmacokinetic model, whereas (5) and (6) describe the pharmacodynamic model.

$$V \cdot \frac{dC_{\text{insp}}}{dt} = Q_0 \cdot C_0 - (Q_0 - \Delta Q) \cdot C_{\text{insp}} - f_R \cdot (V_T - \Delta) \cdot (C_{\text{insp}} - C_1) \quad (1)$$

$$\frac{dC_1}{dt} = \sum_{j=2}^{5} \left(k_{j1} C_j \frac{V_j}{V_1} - k_{1j} C_1 \right) + \frac{f_R (V_T - \Delta)}{V_1} (C_{\text{insp}} - C_1) \quad (2)$$

$$\frac{dC_j}{dt} = k_1 j \frac{C_1 V_1}{V_j} - k_{j1} C_j, \quad j = 3, 4, 5 \quad (3)$$

$$\frac{dC_2}{dt} = k_{12} C_1 \frac{V_1}{V_2} - k_{21} C_2 - k_{20} C_2 \quad (4)$$

$$\frac{dC_e}{dt} = k_{e0} (C_1 - C_e) \quad (5)$$

$$BIS = 100 - 100 \cdot \frac{(C_e)^\gamma}{(C_e)^\gamma + E \cdot C_{50}} \quad (6)$$

Taking equation (6) and linearizing around the equilibrium point $C_e = EC_50$, it turns out that

$$BIS = k_m \cdot C_e \tag{7}$$

k_m being equal to $k_m = -\frac{100\cdot\gamma}{4\cdot E\cdot C_{50}}$.

For the following parameters, the values taken are the averages of the values found in the paper.

Table 1: Parameter Values

Parameter	Value	Parameter	Value
k_{12}	1.26	k_{13}	0.402
k_{14}	0.243	k_{15}	0.0646
k_{20}	0.0093	k_{21}	0.21
k_{31}	0.23	k_{41}	0.00304
k_{51}	0.0005	k_{e0}	0.3853
V_1	2.31	V_2	7.1
V_3	11.3	V_4	3
V_5	5.1	γ	1.534
$Q_0 = V$	5	Δ	0.15
ΔQ	0.4	f_R	14.5
E	0.7478	V_T	0.75

All these can be expressed in a series of equations using state space variables as follows:

Alternatively, in a state-space model, $\dot{x} = Ax + Bu$ and $\dot{y} = Cx + Du$, where:

$$y = \begin{bmatrix} BIS \\ C1 \end{bmatrix}$$

$$
\begin{aligned}
\dot{C}_{\text{insp}} &= -\frac{Q_0 - \Delta Q + f_R(V_T - \Delta)}{V} C_{\text{insp}} + \frac{f_R(V_T - \Delta)}{V} C_1 + \frac{Q_0}{V} C_0 \\
\dot{C}_1 &= \frac{f_R(V_T - \Delta)}{V_1} C_{\text{insp}} - \frac{k_{12} + k_{13} + k_{14} + k_{15} + f_R(V_T - \Delta)}{V_1} C_1 + \frac{k_{21}V_2}{V_1} C_2 + \frac{k_{31}V_3}{V_1} C_3 + \frac{k_{41}V_4}{V_1} C_4 + \frac{k_{51}V_5}{V_1} C_5 \\
\dot{C}_2 &= \frac{k_{12}V_1}{V_2} C_1 - (k_{21} + k_{20}) C_2 \\
\dot{C}_3 &= \frac{k_{13}V_1}{V_3} C_1 - k_{31} C_3 \\
\dot{C}_4 &= \frac{k_{14}V_1}{V_4} C_1 - k_{41} C_4 \\
\dot{C}_5 &= \frac{k_{15}V_1}{V_5} C_1 - k_{31} C_5
\end{aligned}
$$

and

$$
x = \begin{bmatrix} C_{\text{insp}} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{bmatrix}
$$

The matrices for the model are:

$$
A = \begin{bmatrix}
-2.6600 & 1.7400 & 0 & 0 & 0 & 0 & 0 \\
3.7662 & -4.6189 & 0.6455 & 0.1125 & 0.0039 & 0.0011 & 0 \\
0 & 0.4099 & -0.2193 & 0 & 0 & 0 & 0 \\
0 & 0.0822 & 0 & -0.0230 & 0 & 0 & 0 \\
0 & 0.1871 & 0 & 0 & -0.0030 & 0 & 0 \\
0 & 0.0293 & 0 & 0 & 0 & -0.0005 & 0 \\
0 & 0.3853 & 0 & 0 & 0 & 0 & -0.3853
\end{bmatrix}
$$

$$
B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 51.28 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

It is evident that there are no disturbances and thus the D matrix is a zero matrix. Furthermore, the fact that C_0, the manipulated variable directly affects only C_{insp}. A model of the system without feedback can be found below. BIS has a range between 0 and 100 and thus the first output amplitude is a mere indication of the curve compared to time.

Figure 1: No feedback system.

Below is the step response of the open loop system. It is evident that the time it takes to reach a steady state is quite measurable and thus requires the application of a control technique to ensure that it responds in a more timely manner.

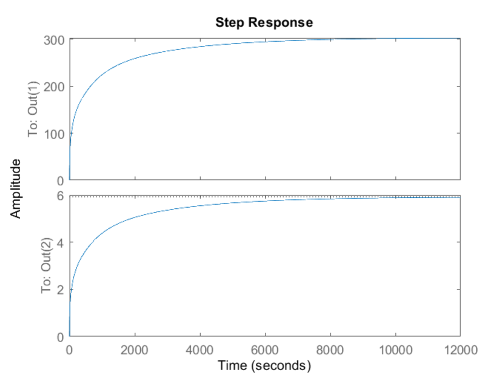

Figure 2: Step Response open loop

2 Closed loop control

To initiate control, we will first implement a straightforward closed-loop control system. In this setup, negative feedback is achieved through the use of a BIS monitor, as illustrated in the figure below. The dynamics of the controller are also accounted for, more in the appendix.

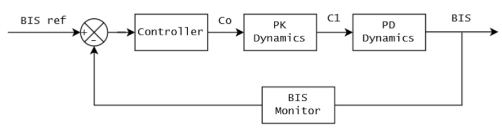

Figure 3: Negative feedback closed loop.

To gain an understanding of the response of the negative feedback closed loop system, a step response is obtained. It is evident that the system is unstable and exhibits chaotic behaviour.

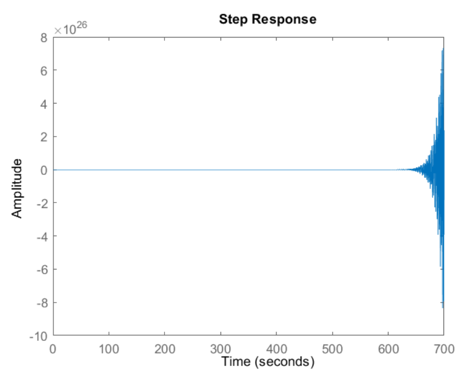

Figure 4: Closed loop step response.

Iteratively, a PID controller for this system, with a proportional gain $K_p = 0.2$, an integral time $T_i = 1$, and a derivative time $T_d = 1.1$. The outcomes from this control strategy are depicted in the figure below.

Using this control strategy the outcome is adequate. The overshoot is not more than 20%, while the settling time is about 15 seconds. All the transfer functions can be found in the appendix.

Reducing the system volume V_1 by 30% speeds the system up, requiring 6000 seconds to steady state, while increasing the volume by the same

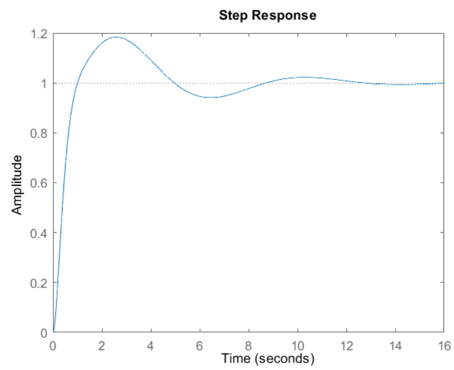

Figure 5: Closed loop controlled step response.

percentage makes the system unstable. In both cases, the implementation of the PID controller is effective and does not deviate significantly from the original problem.

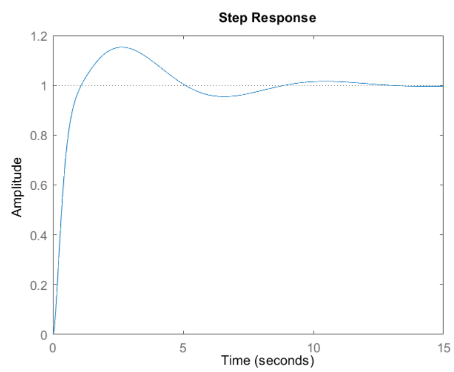

Figure 6: Step response for controlled closed loop for $0.7V_1$

Increasing the hepatic metabolism rate constant k_{20} by 50%, slows the dynamics of the system, as observed by the slower open loop response, requiring 14000 seconds to reach a steady state. Despite that, the step response without changing any parameter of the PID controller is virtually unchanged.

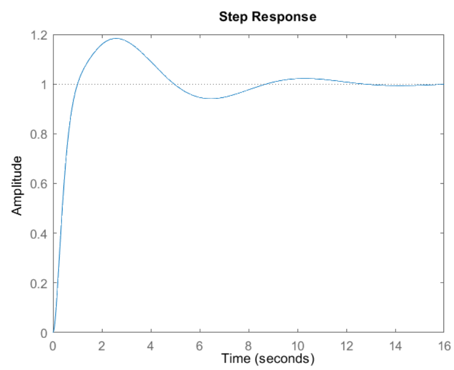

Figure 7: Step response for controlled closed loop $1.3V_1$.

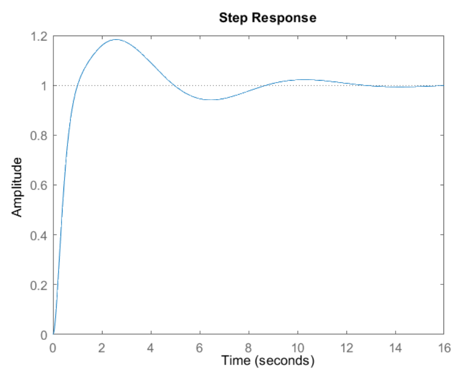

Figure 8: Step response for controlled closed loop with $1.5k_{20}$.

3 Cascade control

Another way to control this system is using cascade control. Essentially an additional isoflurane controller is implemented along with the BIS controller as seen in the figure below.

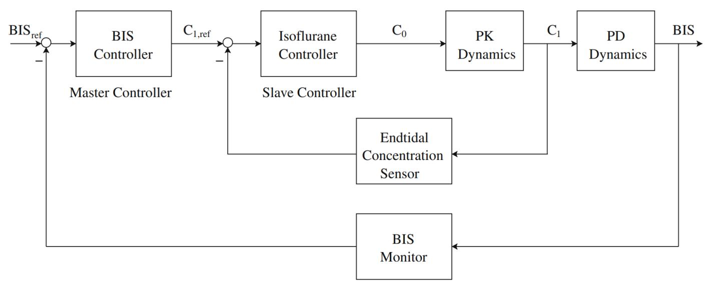

Figure 9: Cascade control schematic.

Initially, an attempt was made to implement two PID controllers, but fine-tuning proved to be challenging. As a result, a more straightforward control method was adopted. The same PID controller used previously was applied to the inner loop. Below, you can observe both the uncontrolled closed-loop step response and the controlled closed-loop response. The inner loop demonstrates instability in the absence of a controller, but its behavior becomes manageable when the controller scheme is introduced.

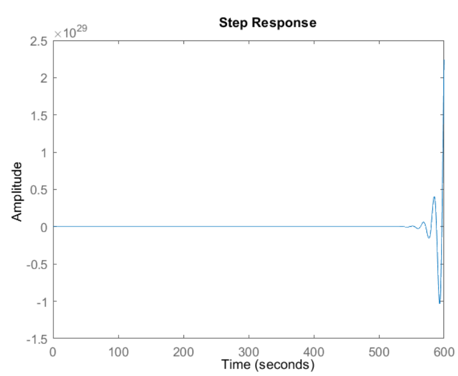

Figure 10: Step response inner uncontrolled loop.

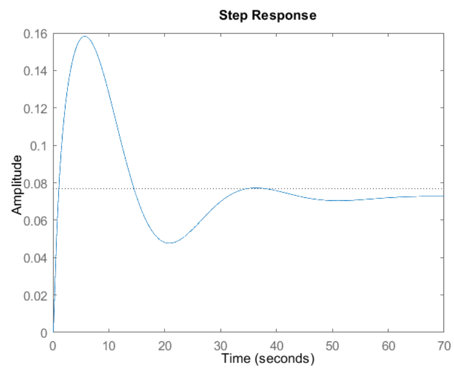

Figure 11: Step response inner closed loop.

For the outer loop a simpler controller with a transfer function of $C_t f = \frac{3}{2000s+1}$ was chosen. The results of the implementation can be seen below

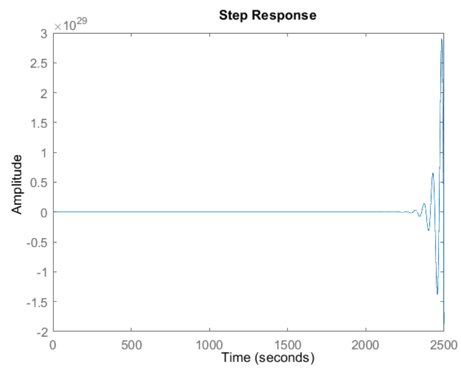

Figure 12: Step response outer open loop.

As can be seen in the step response of the controlled closed loop of cascade control scheme, the steady state gain is not one. This means that the user should set a set-point to about 1/0.9 times the desired value. The system responds in a very slow fashion but that is to be expected.

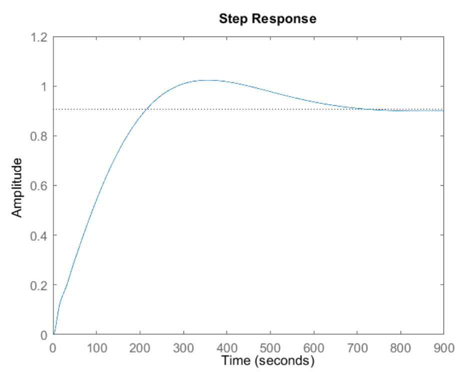

Figure 13: Step response outer closed loop.

4 Conclusion

To conclude this extensive report, it is apparent that the implementation of cascade control did not enhance the performance of the closed-loop system. This could likely be attributed to inadequate tuning of this more complex control scheme.

5 appendix

Here you can find all the transfer functions used to model the system.

PK dynamics:

$$PK(s) = \frac{5.38s^5+3.396s^4+0.5414s^3+0.01234s^2+3.782\times10^{-5}s+1.589\times10^{-8}}{s^7+9.89s^6+13.86s^5+5.257s^4+0.5356s^3+0.00924s^2+2.359\times10^{-5}s+9.137\times10^{-9}} \tag{8}$$

PD dynamics:

$$\frac{106.3s^{11}+1078s^{10}+1733s^9+927.4s^8+203.1s^7+18.31s^6+0.5904s^5+0.007459s^4+3.338\times10^{-5}s^3+5.896\times10^{-8}s^2+3.693\times10^{-11}s+7.447\times10^{-15}}{5.38s^{12}+56.6s^{11}+108.7s^{10}+80.72s^9+28.36s^8+4.886s^7+0.3869s^6+0.01189s^5+0.0001471s^4+6.539\times10^{-7}s^3+1.152\times10^{-9}s^2+7.204\times10^{-13}s+1.452\times10^{-16}} \tag{9}$$

For the BIS monitor the transfer function was found in the paper by Bibian S, Dumont GA, Zikov T., "Dynamic behavior of BIS, M-Entropy and NeuroSENSE brain function monitors":

$$\frac{0.049s^4 - 0.0856s^3 + 1.002s^2 - 0.4464s + 0.0224}{s^5 + 2.67s^4 + 7.261s^3 + 7.411s^2 + 2.003s + 0.022} \tag{10}$$

For the other sensor a simple time delay function was utilized:

$$\frac{-10s + 4}{10s^2 + 5s + 0.4} \tag{11}$$