Digital Control

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1 1st and 2nd order approximation

This is the original function:

$$G(s) = \frac{250e^{-1.5s}}{(s+1.2)(s+0.75)(s+0.28)(s^2+0.5s+160)}$$

To approximate the original transfer function process, one has only to look at its step response and the information acquired by the pole placement.

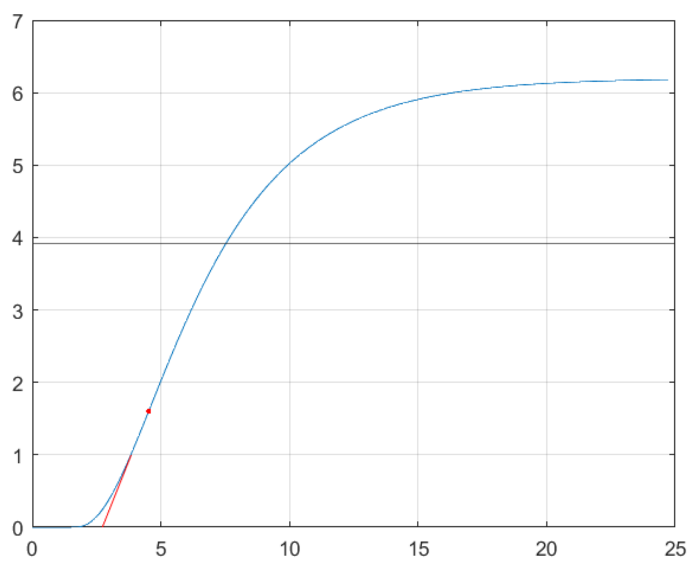

Figure 1: Step response of original transfer function.

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-2.80e-01	1.00e+00	2.80e-01	3.57e+00
-7.50e-01	1.00e+00	7.50e-01	1.33e+00
-1.20e+00	1.00e+00	1.20e+00	8.33e-01
-2.50e-01 + 1.26e+01i	1.98e-02	1.26e+01	4.00e+00
-2.50e-01 - 1.26e+01i	1.98e-02	1.26e+01	4.00e+00

Figure 2: Poles of original transfer function.

Using the information provided by the above plot, it can be determined that for the first order system, the time delay can be thought of as equal to 2.5 seconds, whereas the time constant would be $tau = 7.52 - 2.5 = 5.02$ seconds, rounded to 5 seconds, and the gain is 6.2.

All in all the first order approximation is the following.

$$G_{1st-order} = (e^{-2.5 \cdot s}) \cdot \frac{6.2}{5s+1}$$

For the second order approximation, the dominant pole method shall be used. The dominant poles are complex, thus to avoid any unwanted oscillatory behaviour, the poles at 0.28 and 0.75 shall be used.

$$G_{2nd-order} = (e^{-2.5 \cdot s}) \cdot \frac{1.302}{(s+0.28)(s+0.75)}$$

To assess the fit of the approximation both the step response as well as the impulse response of the approximate functions are plotted together with the original. These approximations fare quite well in approximating the process behaviour as can be seen in the graph below. Note that, as expected, the second order comes closer to replicating the original function.

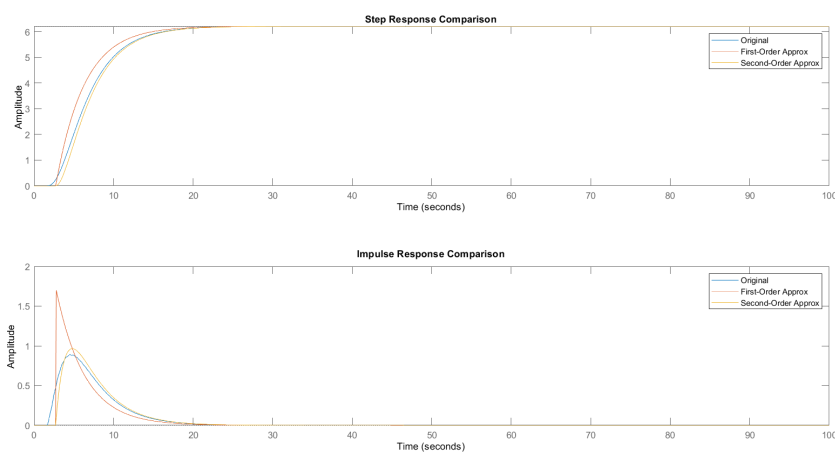

Figure 3: Approximations assessment.

2 Digital Dahlin controller

The assignment requires the process to get the following dynamical behaviour:

$$T(s) = \frac{e^{-theta \cdot s}}{tau \cdot s + 1}$$

where, as specified by the assignment statement, tau is equal to 3. Using the Shannon law, since the time delay is equal to 4.8 seconds, the sampling time shall be 0.48 seconds. The sampling time was taken as 0.5 seconds due to inefficient gain otherwise. Supposing n to be equal to 5:

Essentially here, the first order system from the first part of the assignment is used, along with a Dahlin controller. First we must discretize the process using the ZOH method and the first order approximation as can be seen below:

$$G_p H(z) = Z[G_{1st-order} \cdot \frac{1-e^{-Ts/q}}{s}]$$

plugging in the values above, the result is the following:

$$G_p H(z) = \frac{0.4537z^{-5} + 0.1138z^{-6}}{1 - 0.9085z^{-1}}$$

The form of the Dahlin controller is defined as:

$$D(z) = \frac{z^{-n-1}(1-e^{-Ts/q}}{1-e^{-Ts/q} \cdot z^{-1} - z^{-n-1} \cdot (1-e^{-Ts/q})} \cdot \frac{1}{G_p H(z)}$$

So the Controller's transfer function takes the following form:

$$D(z) = \frac{0.1535z^6 - 0.1389z^5}{0.59z^6 + 0.4994z^5 - 0.09697}$$

Interestingly, if the sampling time is 0.48 the controller creates a system with no overshoot, but without a gain of 1.

$$D(z) = \frac{0.1479z^6 - 0.1343z^5}{0.6708z^6 + 0.01683z^5 - 0.3866z - 0.09697}$$

Below is the step response of the closed-loop system. Note that for the rest of the assignment, the sampling time is taken as 0.5 seconds.

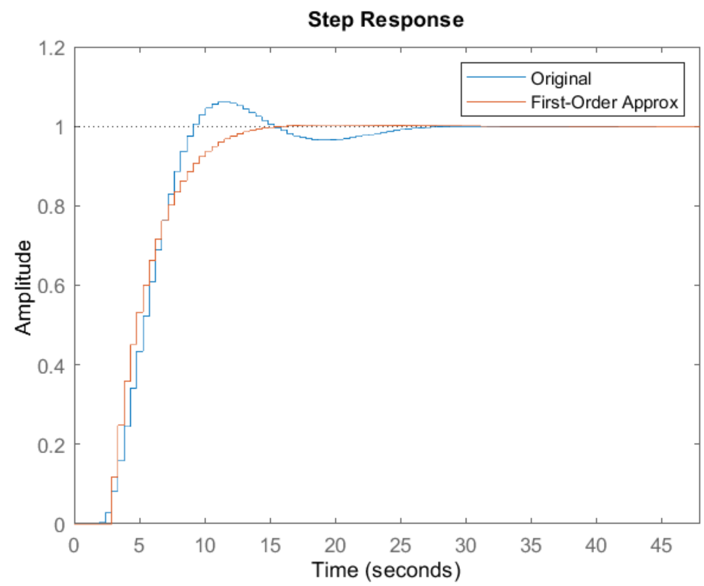

Figure 4: Response sampling time is equal to 0.5.

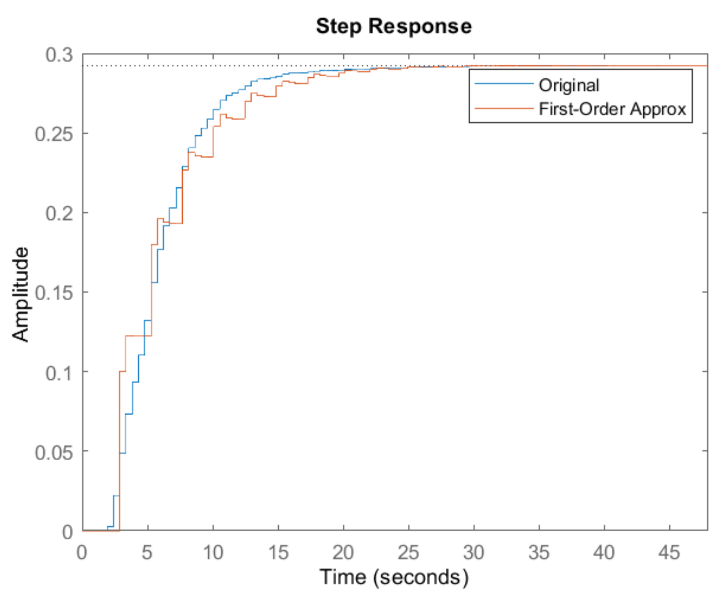

Figure 5: Response sampling time is equal to 0.48.

3 Manipulated variable behaviour

One important characteristic of a controlled system is the effort of the actuators. A great indicator of this is the behaviour of the manipulated variable. To find out more a Simulink model was created. Below the effort of the manipulated variable is shown.

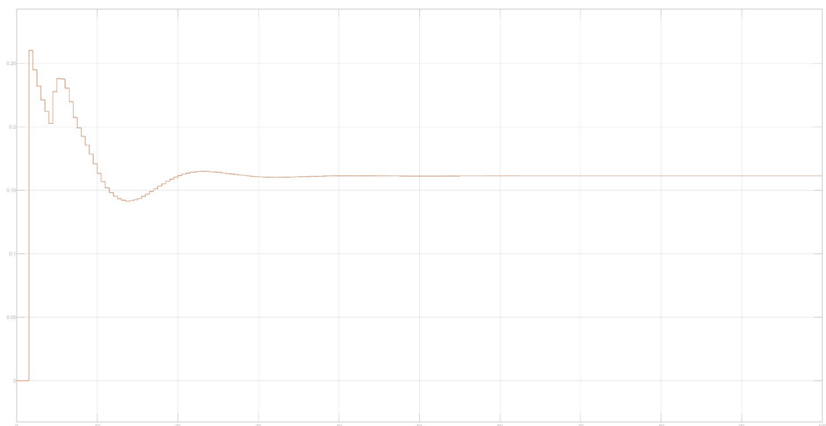

Figure 6: Manipulated variable response.

In this case here, it is evident that before it reaches its steady state, the manipulated variable experiences oscillations. To cure this the Dahlin controller is altered. Most likely, this oscillatory behaviour is traced back to the existence of complex roots in the controller's transfer function.

Pole	Magnitude	Damping	Frequency (rad/seconds)	Time Constant (seconds)
1.00e+00	1.00e+00	1.00e+00	4.00e-05	2.50e+04
5.05e-01 + 5.54e-01i	7.49e-01	3.28e-01	1.83e+00	1.66e+00
5.05e-01 - 5.54e-01i	7.49e-01	3.28e-01	1.83e+00	1.66e+00
-2.64e-01 + 6.01e-01i	6.56e-01	2.08e-01	4.23e+00	1.14e+00
-2.64e-01 - 6.01e-01i	6.56e-01	2.08e-01	4.23e+00	1.14e+00
-6.35e-01	6.35e-01	1.43e-01	6.61e+00	1.06e+00

Figure 7: Controller poles.

After some trial and error, removing some of the complex poles, and to maintain feasible control, the poles at $5.05e-01+5.54e-01i$ and $5.05e-01-5.54e-01i$ were removed along with the necessary zeros. The new controller transfer function can be seen below:

$$D_{mv} = \frac{0.2602z^3 0.2354z^2}{z^3 0.472z^2 0.0971z 0.4309}$$

Comparing the manipulated variable effort, it seems that the new controller reduces the effort required. That being said, the oscillatory behaviour persists, although reduced. There is an initial jitter which is then followed by reduced oscillations.

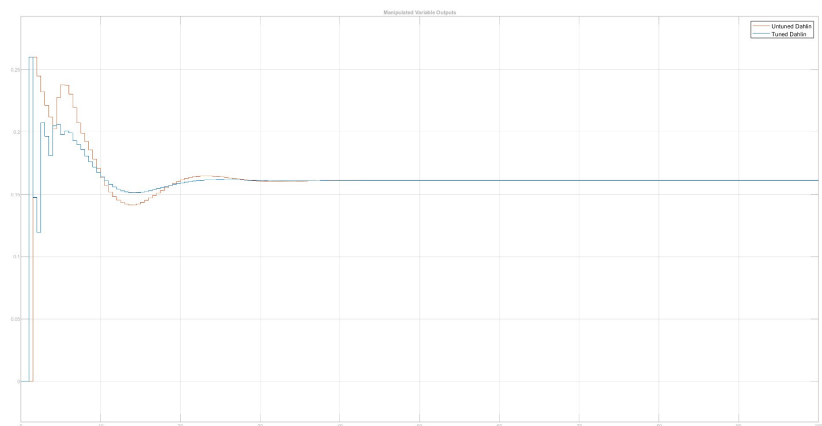

Figure 8: Tuned controller and original controller manipulated variable effort.

4 Time delay increase

In the last part of this assignment, we are asked to assess the system response if the initial time delay is increased by 1 second and 2 seconds, without changing the controller developed to reduce the effort of the manipulated variable. Following a similar process to before the following transfer functions are plugged in the Simulink model and the outputs seen in the last figure are produced.

$$G_{+1sec} = z^{-5} \cdot \frac{0.02162z^4 + 0.04281z^3 0.1142z^2 + 0.04236z + 0.009132}{z^5 3.869z^4 + 5.944z^3 4.528z^2 + 1.709z 0.2554}$$

and

$$G_{+2sec} = z^{-7} \cdot \frac{0.02162z^4 + 0.04281z^3 0.1142z^2 + 0.04236z + 0.009132}{z^5 3.869z^4 + 5.944z^3 4.528z^2 + 1.709z 0.2554}$$

The only difference between the two being the first term, the exponent is -5 when the time delay is set to 2.5 seconds and -7 when it is 3.5 seconds.

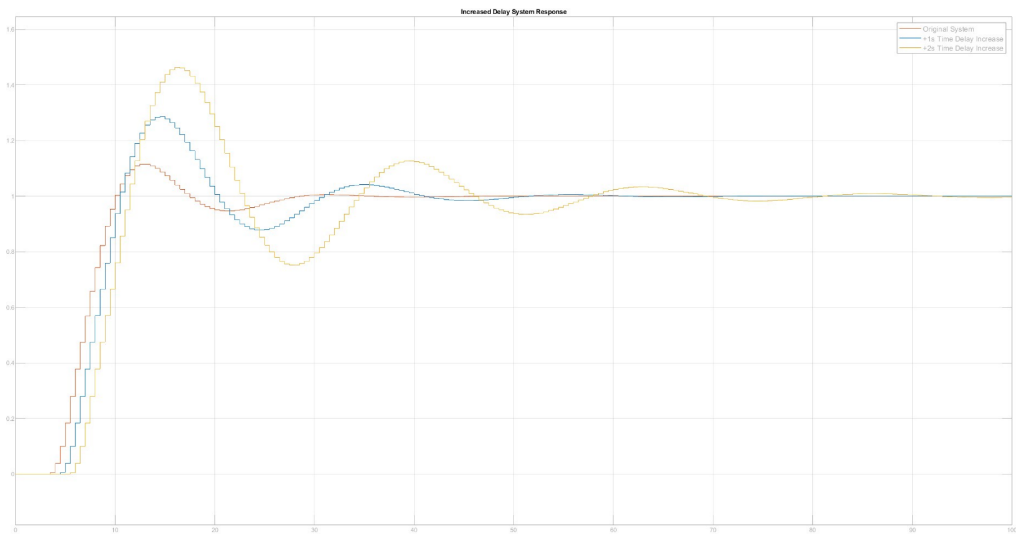

Figure 9: Added delay step responses.

The figure above illustrates that an alteration in the system dynamics by means of increased time delay does not throw off the control scheme. The step response is not ideal, there is an increased overshoot, or settling time but crucially the system remains controllable. Considering that the controller was tuned using the discrete form of the first order approximation of the original system, $G_pH(z)$, and thus it's poles and zeros are dependent on it, this behaviour is expected, since the approximation error is far greater.