Numerical Optimization

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1 Introduction

The scope of this assignment was to solve two optimization problems using various solvers hosted in the GAMS platform, general algebraic modeling system. For the first problem, the task included building the GAMS model, while the second problem's model was provided.

2 Mechanical Spring

2.1 Model definition

The first problem was about minimizing the weight of a mechanical spring. As can be seen in the problem statement, this problem can be formulated as follows.
Minimize

$\text{Weight} = 1.92 \times 10^{-5} \cdot d^2 \cdot D_2 \cdot (n+2)$

subject to

$-350 + 163 \cdot \frac{D_2^{0.84}}{d^{2.84}} \leq 0,$

$-2500 + \frac{d^4}{D_2^3 \cdot n} \leq 0,$

$375 - \frac{3.56 \cdot 10^5 \cdot d}{D_2^2 \cdot n} \leq 0,$

$4.5 - \frac{D_2}{d} \leq 0,$

$\frac{D_2}{d} - 30 \leq 0,$

$10 - d - D_2 \leq 0,$

$d + D_2 - 30 \leq 0$

where, d, D_2 and n are bounded as follows,

$1 \leq d \leq 4$,

$4.5 \leq n \leq 50$

In order to set this problem in GAMS, one first has to define the variables, d, D_2, n, and the weight and the equations, containing the objective function, the constraints it is subject to, and the bounds of the variables. Then the problem is solved using one specified solver at a time. Lastly, it is essential to define the initial values for d, D_2, and n, set to 100000, 150000, and 200000 respectively.

2.2 Results

The optimization solvers' outcomes for the given problem showed a mix of efficiency and problem feasibility. IPOPT completed its run in 39 iterations, achieving an objective value of $2.2203729 \cdot 10^{-3}$ in 0.092 seconds. MINOS, however, determined the problem to be infeasible, thus not providing any objective value or time efficiency data.

SNOPT required a substantial number of iterations, 2028 in total, to reach an objective value very close to that of IPOPT, specifically $2.2203724017 \cdot 10^{-3}$, and did so in a time of 0.11 seconds. CONOPT showed a remarkable iteration efficiency, arriving at an objective of $2.2203724045 \cdot 10^{-3}$ in only 7 iterations, albeit with a slightly longer execution time of 0.109 seconds compared to SNOPT.

In summary, while IPOPT and SNOPT both identified solutions with similar objective values and demonstrated swift execution times, CONOPT achieved comparable precision with fewer iterations but took marginally longer. MINOS did not find an optimal value for the objective function.

3 Rocket

3.1 Problem description

This problem involves maximizing a rocket's reached height using thrust as a control variable. For the model, the initial mass, the fuel, and the drag characteristics are given. Then the interval of each assessment step is defined, along with the equations, the constraints, the bounds, and the initial values. The model is then set to run using the respective solver each time. As can be seen in the model, the objective function that is attempted to be maximized is the final velocity of the rocket. To enable the solution of this problem using the educational licence, the steps are limited to 150, instead of the 1000 originally specified.

3.2 Results

The optimization solvers presented varied outcomes in both performance and feasibility for the assigned problem. CONOPT successfully found a solution with an objective value of approximately 1.0128346337 and was notably efficient with an execution time of 0.438 seconds. In contrast, MINOS could not find a feasible solution, declaring the problem infeasible after 6 iterations and an objective estimate of 1.04910.

SNOPT, completed nearly 8000 iterations, reaching an objective value very close to that achieved by CONOPT, at 1.0128316046, with a relatively swift execution time of 0.84 seconds. This indicates that while SNOPT took many iterations to converge, it did so quickly in terms of computation time.

IPOPT found a solution with an objective value of 1.0128347281110957 when considering the absolute value, which seems consistent with the solutions found by CONOPT and SNOPT, but it required 1.334 seconds, suggesting it was less time-efficient than the SNOPT or the CONOPT approach for this problem.

These results highlight a divergence in solver approaches: while CONOPT and SNOPT found similar solutions with high efficiency, MINOS, similarly to the mechanical spring problem, reported infeasibility, and IPOPT, despite finding a consistent solution, was less efficient in terms of time.

4 Conclusions

The key takeaways from this assignment are two. This assignment showcases the ease with which a model is set up and optimized using GAMS. What could have been hours of writing a script in matlab or python, setting up the right parameters etc, is dealt with in minutes using the solvers in GAMS. The MINOS solver in both cases did not find the optimal value for the problem, which along with the fact that the rest of the solvers convergence indicates that the problem was not correctly set for this solver. The greater picture here is that for all the user friendliness of the GAMS platform, approaching a problem with a specific solver is not as straight forward as it might seem.