Numerical Optimization

Dimitrios Nentidis, 6821

December 2023

1 Introduction

This report is focused on optimizing a truss structure for a specific case of static loading. First, the static problem is solved using Matlab and then using GAMS, using Matlab generated code for the equation part of the GAMS problem. The two solutions are compared and then a rudimentary sensitivity analysis is performed on the GAMS model.

The main part of the problem is optimizing the truss to minimize the total mass of all the rods while maintaining the structural integrity.

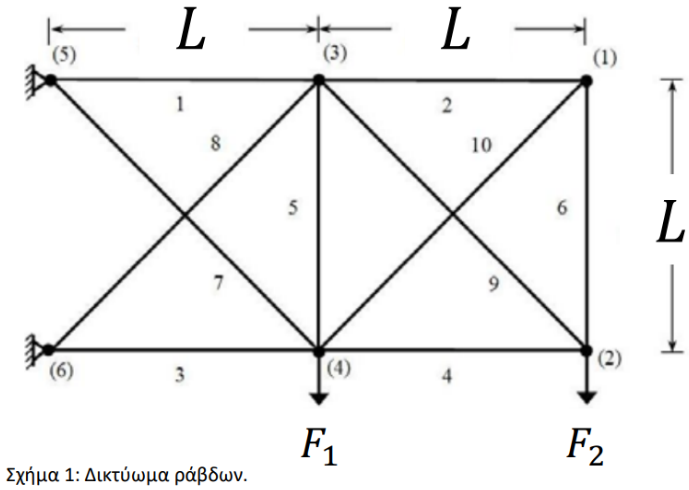

Figure 1: Truss geometry.

2 Static problem in Matlab

The primary reason the static problem is first tackled in Matlab is to get intuition on how the structure behaves under loading in a familiar environment. At this stage, the solution is approached numerically using the relevant Matlab script found separately in the assignment folder. The results of the Matlab analysis are summed in the two figures below, illustrating the deformation of the truss in terms of displacement of each node and the axial stresses of each element.

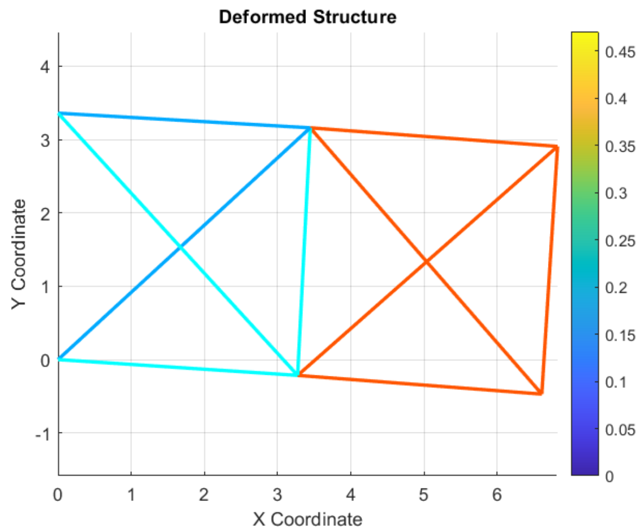

Figure 2: Deformed structure, deformation in meters.

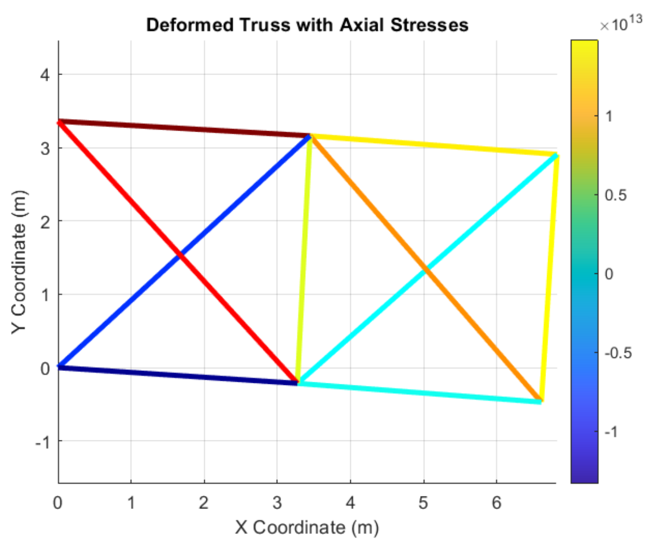

Figure 3: Stresses, in N/m^m.

More specifically the displacement of each node and degree of freedom can be seen in the table below.

Node	X (m)	X - disp (m)	Y (m)	Y - disp (m)
1	6.72	0.1013	3.36	-0.4533
2	6.72	-0.1137	0	-0.4705
3	3.36	0.0840	3.36	-0.2000
4	3.36	-0.0880	0	-0.2152
5	0	0	3.36	0
6	0	0	0	0

Table 1: Node coordinates and displacements

When it comes to the axial stresses experienced it seems that the numbers do not add up. In particular, the magnitude of the stresses is way above the material's yield strength. This is most likely down to the definition of the parameters but even after rechecking the problem did not go away. As it stands it is not true. That being said, it does not harm the scope of this assignment, as these are just numbers, so it turn, for the rest of the assignment the focus will only be in displacements.

3 Static problem in GAMS

For this part essentially the static truss problem must be formulated as an optimization problem in GAMS. The tricky part of this, or so it seemed in the beginning, was developing an objective function. It turns out that the objective function could be anything, here it was chosen to be the sum of all node displacements. Interestingly since the equations governing the problem are particularly robust, they are equations not relationships, choosing to maximize or minimize the objective function did not make any difference. The GAMS code found the exact same solutions every time, regardless of the starting point. All the available solvers converged to the same solution in about 1 second, regardless of initial conditions.

```
                          LOWER          LEVEL          UPPER         MARGINAL

---- EQU r1                0.1013         0.1013         0.1013         1.0000
---- EQU r2               -0.4533        -0.4533        -0.4533         1.0000
---- EQU r3               -0.1137        -0.1137        -0.1137         1.0000
---- EQU r4               -0.4705        -0.4705        -0.4705         1.0000
---- EQU r5                0.0840         0.0840         0.0840         1.0000
---- EQU r6               -0.2000        -0.2000        -0.2000         1.0000
---- EQU r7               -0.0880        -0.0880        -0.0880         1.0000
---- EQU r8               -0.2152        -0.2152        -0.2152         1.0000
---- EQU r9                 .              .              .             1.0000
---- EQU r10                .              .              .             1.0000
---- EQU r11                .              .              .             1.0000
---- EQU r12                .              .              .             1.0000
---- EQU object             .              .              .             1.0000

                          LOWER          LEVEL          UPPER         MARGINAL

---- VAR u1               -INF            0.1013        +INF             .
---- VAR u2               -INF           -0.4533        +INF             .
---- VAR u3               -INF           -0.1137        +INF             .
---- VAR u4               -INF           -0.4705        +INF             .
---- VAR u5               -INF            0.0840        +INF             .
---- VAR u6               -INF           -0.2000        +INF             .
---- VAR u7               -INF           -0.0880        +INF             .
---- VAR u8               -INF           -0.2152        +INF             .
---- VAR u9               -INF             .            +INF             .
---- VAR u10              -INF             .            +INF             .
---- VAR u11              -INF             .            +INF             .
---- VAR u12              -INF             .            +INF             .
---- VAR obj              -INF           -1.3555        +INF             .


**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED
                             0     ERRORS


EXECUTION TIME       =        0.078 SECONDS      3 MB  45.2.0 e4d2ee31 WEX-WEI
```

Figure 4: Static problem, GAMS output.

4 Optimizing the structure's mass

To optimize the structure's mass it is necessary to reformulate the above static model to a proper optimization problem. Facilitating this is an alteration to the Matlab GAMS generation script. Up to this point, the cross section was the same for all the rods, however, to optimize the structure, the cross section needs to be a variable. This essentially means that in Matlab not one but ten different symbolic variables are defined from A1 to A10, each being part of the K matrix in its respective rod element. Now the objective function can take a more appropriate form.

Interestingly, the optimization problem could only be solved by the IPOPT solver option. The rest of the solvers presented errors due to division with almost zero numbers ($< 1e - 150$).

As seen below the mass has gone down from 115.5412 kg to 76.1134 kg. This has been accomplished by essentially removing some of the rods, particularly A1, A2, A3, and A6 are almost zero, while at the same time thickening the remaining rods. As a consequence of this, the displacement of the nodes and, therefore, the stresses experienced by the structure are also lowered.

```
                          LOWER          LEVEL          UPPER         MARGINAL

---- EQU r1                 .        -2.77556E-17         .           -0.0002
---- EQU r2                 .        2.220446E-16         .        4.390591E-11
---- EQU r3                 .        -8.32667E-17         .          -34.5486
---- EQU r4                 .        -3.33067E-16         .         -120.5137
---- EQU r5                 .        -6.93889E-18         .        3.379065E-10
---- EQU r6                 .              .              .        -4.36536E-10
---- EQU r7                 .        1.387779E-17         .        -3.07191E-10
---- EQU r8                 .        -1.66533E-16         .          -71.9493
---- EQU r9                 .              .              .             EPS
---- EQU r10                .              .              .             EPS
---- EQU r11                .              .              .             EPS
---- EQU r12                .              .              .             EPS
---- EQU object             .        1.776357E-15         .            1.0000

                          LOWER          LEVEL          UPPER         MARGINAL

---- VAR u1               -0.1013        -0.1013         0.1013         0.0002
---- VAR u2               -0.4533         0.2795         0.4533   -4.39059E-11
---- VAR u3               -0.1137        -0.1137         0.1137        34.5486
---- VAR u4               -0.4705        -0.4705         0.4705       120.5137
---- VAR u5               -0.0840         0.0595         0.0840   -3.37906E-10
---- VAR u6               -0.2000        -0.1784         0.2000   4.365356E-10
---- VAR u7               -0.0880        -0.0613         0.0880   3.071907E-10
---- VAR u8               -0.2152        -0.2152         0.2152        71.9493
---- VAR u9               -INF              .            +INF             EPS
---- VAR u10              -INF              .            +INF             EPS
---- VAR u11              -INF              .            +INF             EPS
---- VAR u12              -INF              .            +INF             EPS
---- VAR mass               .            76.1134        +INF       1.313830E-13
---- VAR A1                 .        2.143435E-11       +INF           0.0823
---- VAR A2                 .        -1.00000E-10       +INF       285048.4636
---- VAR A3                 .        -9.99999E-11       +INF        34219.2347
---- VAR A4                 .             0.0004        +INF       2.6289797E-8
---- VAR A5                 .             0.0003        +INF       3.2776854E-8
---- VAR A6                 .        -9.99997E-11       +INF        34628.1496
---- VAR A7                 .             0.0005        +INF       1.8589697E-8
---- VAR A8                 .             0.0004        +INF       2.6289797E-8
---- VAR A9                 .             0.0003        +INF       3.4024512E-8
---- VAR A10                .             0.0005        +INF       1.9154956E-8


**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED
                             0     ERRORS
```

Figure 5: Optimization problem, GAMS output.

5 Constraint relaxation

```
COIN-OR Ipopt    45.2.0 e4d2ee31 Oct 30, 2023          WEI x86 64bit/MS Window

                           LOWER          LEVEL          UPPER         MARGINAL

---- EQU r1                  .        -2.77556E-17         .            -0.0001
---- EQU r2                  .        1.110223E-16         .        1.008672E-11
---- EQU r3                  .        -8.32667E-17         .           -12.8186
---- EQU r4                  .        -3.33067E-16         .            -9.1124
---- EQU r5                  .              .              .        3.3643540E-9
---- EQU r6                  .        1.110223E-16         .          -111.4286
---- EQU r7                  .              .              .           -73.5936
---- EQU r8                  .        1.110223E-16         .           -14.5405
---- EQU r9                  .              .              .              EPS
---- EQU r10                 .              .              .              EPS
---- EQU r11                 .              .              .              EPS
---- EQU r12                 .              .              .              EPS
---- EQU object              .              .              .             1.0000

                           LOWER          LEVEL          UPPER         MARGINAL

---- VAR u1               -0.1013        -0.1013         0.1013          0.0001
---- VAR u2               -0.4533         0.0987         0.4533    -1.00867E-11
---- VAR u3               -0.2274        -0.2274         0.2274         12.8186
---- VAR u4               -0.9410        -0.9410         0.9410          9.1124
---- VAR u5               -0.0840         0.0811         0.0840    -3.364354E-9
---- VAR u6               -0.2000        -0.2000         0.2000        111.4286
---- VAR u7               -0.0880        -0.0880         0.0880         73.5936
---- VAR u8               -0.4304        -0.4304         0.4304         14.5405
---- VAR u9               -INF              .            +INF             EPS
---- VAR u10              -INF              .            +INF             EPS
---- VAR u11              -INF              .            +INF             EPS
---- VAR u12              -INF              .            +INF             EPS
---- VAR mass                .           46.5099         +INF     2.150082E-13
---- VAR A1                  .        1.754132E-11       +INF           0.0851
---- VAR A2                  .        -1.00000E-10       +INF      215488.1300
---- VAR A3                  .        -1.00006E-10       +INF       35362.8834
---- VAR A4                  .            0.0001         +INF     9.5606935E-8
---- VAR A5                  .            0.0001         +INF     8.7154786E-8
---- VAR A6                  .        -9.99999E-11       +INF      167372.5626
---- VAR A7                  .            0.0004         +INF     2.5346255E-8
---- VAR A8                  .            0.0004         +INF     2.6286842E-8
---- VAR A9                  .            0.0001         +INF     7.5686019E-8
---- VAR A10                 .            0.0004         +INF     2.7509351E-8


**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED
                             0     ERRORS


EXECUTION TIME       =        4.454 SECONDS      3 MB  45.2.0 e4d2ee31 WEX-WEI
```

Figure 6: Relaxed constraints optimization.

One way to improve on the optimization value found is to relax some of the constraints. Choosing which constraints shall be relaxed is simply a matter of looking at the Lagrange multipliers, or the Marginal, in GAMS notation. The greater the value of these, the greater the "pressure" applied. Relaxing the constraints with the highest LM values will improve the results the most. Here, relaxing the allowed displacements of node 2, both degrees of freedom, and node 4, only Y. This brought the mass down to just above 45 kg.

6 Half the original displacements

The opposite procedure is required here, although there is not much of a choice of which constraints to make stricter. What can be observed though is that the Lagrangian multipliers are getting greater, meaning that the constraints are indeed applying more pressure to the model, further restricting the solver. The outcome now is a mass exceeding 150 kg.

```
COIN-OR Ipopt    45.2.0 e4d2ee31 Oct 30, 2023         WEI x86 64bit/MS Window

                             LOWER          LEVEL         UPPER         MARGINAL

---- EQU r1                    .        -3.46945E-17        .             -0.0003
---- EQU r2                    .        -5.55112E-17        .        -2.21947E-10
---- EQU r3                    .        -6.93889E-18        .           -138.1945
---- EQU r4                    .        -1.11022E-16        .           -482.0552
---- EQU r5                    .        1.734723E-17        .        6.758125E-10
---- EQU r6                    .              .             .        -8.73072E-10
---- EQU r7                    .        -1.04083E-17        .        -6.14381E-10
---- EQU r8                    .        6.938894E-17        .           -287.7967
---- EQU r9                    .              .             .              EPS
---- EQU r10                   .              .             .              EPS
---- EQU r11                   .              .             .              EPS
---- EQU r12                   .              .             .              EPS
---- EQU object                .        -1.42109E-14        .              1.0000

                             LOWER          LEVEL         UPPER         MARGINAL

---- VAR u1                 -0.0507        -0.0506        0.0507          0.0003
---- VAR u2                 -0.2266        -0.1860        0.2266    2.219471E-10
---- VAR u3                 -0.0568        -0.0569        0.0568        138.1945
---- VAR u4                 -0.2352        -0.2353        0.2352        482.0552
---- VAR u5                 -0.0420         0.0297        0.0420    -6.75812E-10
---- VAR u6                 -0.1000        -0.0892        0.1000    8.730715E-10
---- VAR u7                 -0.0440        -0.0306        0.0440    6.143811E-10
---- VAR u8                 -0.1076        -0.1076        0.1076        287.7967
---- VAR u9                 -INF              .           +INF             EPS
---- VAR u10                -INF              .           +INF             EPS
---- VAR u11                -INF              .           +INF             EPS
---- VAR u12                -INF              .           +INF             EPS
---- VAR mass                  .          152.2268        +INF      6.569147E-14
---- VAR A1                    .        1.633114E-10      +INF            0.0380
---- VAR A2                    .        -9.99999E-11      +INF       156755.3132
---- VAR A3                    .        2.309573E-10      +INF            0.0302
---- VAR A4                    .            0.0008        +INF      1.3144894E-8
---- VAR A5                    .            0.0006        +INF      1.6388424E-8
---- VAR A6                    .        -9.99997E-11      +INF        34628.1458
---- VAR A7                    .            0.0011        +INF      9.2948441E-9
---- VAR A8                    .            0.0008        +INF      1.3144895E-8
---- VAR A9                    .            0.0006        +INF      1.7012270E-8
---- VAR A10                   .            0.0010        +INF      9.5774805E-9


**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED
                             0     ERRORS


EXECUTION TIME       =        0.672 SECONDS      3 MB  45.2.0 e4d2ee31 WEX-WEI
```

Figure 7: Half the original Displacements.

7 Deliverables Guide

The following deliverables resulted from this assignment:

- The report in PDF format.
- `original_truss.m`, a MATLAB script solving the static problem numerically.
- `GamsModelEquations.gms`, a GAMS script modeling the static problem as an optimization problem.
- `opti.gms`, a GAMS script modeling the optimization problem for the truss geometry.
- `gams_translator.m` and `optimal_As.m`, the symbolic equation creating scripts.