COMPUTER ARCHITECTURE
CMPT 328

MIDTERM EXAM

STUDENT: AUDIT COPY

▸ **QUESTION 1**

An RSA public key of 2,048 bits is the minimum length considered secure today. How many decimal digits does that represent?

Hint: The straightforward approach to this problem will have you writing a *very* large number b^N. That is not practical. Consider the logarithmic relationship

$$log_k(b^N) = N \bullet log_k(b)$$

Estimated time to complete this question is 10 minutes.

SOLUTION

In general, the number of discrete states is $M_b = b^N$ where b is the base and N is the number of digits. The number of decimal digits, N_{10} is to be determined. The number of discrete states represented in decimal is, obviously, the same as the binary representation. Therefore, we can represent this relationship as follows:

$$M_{10} = M_2$$

We can express the number of discrete states in decimal as $M_{10} = 10^{N_{10}}$ where N_{10} is the number of decimal digits represented.

We are given $b = 2$, where b is the base—i.e. binary—and $N_2 = 2,048$, where N_2 is the number of binary digits, The number of discrete states, M_2 represented by 2,048 binary digits is $M_2 = 2^{2,048}$.

$$\begin{aligned}
M_{10} &= M_2 \\
log_{10}(10^{N_{10}}) &= \frac{log_2(2^{2,048})}{log_2(10)} \\
N_{10} \bullet log_{10}(10) &= \frac{2,048 \bullet log_2(2)}{log_2(10)} \\
N_{10} \bullet (1) &= \frac{2,048 \bullet (1)}{3.32} \\
N_{10} &= 616.87 \qquad (1)
\end{aligned}$$

▸ **QUESTION 2**

In elliptic curve encryption, a public key of 224 bits is considered secure today. How many decimal digits does that represent?

Hint: The straightforward approach to this problem will have you writing a *very* large number b^N. That is not practical. Consider the logarithmic relationship

$$log_k(b^N) = N \bullet log_k(b)$$

Estimated time to complete this question is 10 minutes.

SOLUTION

In general, the number of discrete states is $M_b = b^N$ where b is the base and N is the number of digits. The number of decimal digits, N_{10} is to be determined. The number of discrete states represented in decimal is, obviously, the same as the binary representation. Therefore, we can represent this relationship as follows:

$$M_{10} = M_2$$

We can express the number of discrete states in decimal as $M_{10} = 10^{N_{10}}$ where N_{10} is the number of decimal digits represented.

We are given $b = 2$, where b is the base—i.e. binary—and $N_2 = 224$, where N_2 is the number of binary digits, The number of discrete states, M_2 represented by 224 binary digits is $M_2 = 2^{224}$.

$$
\begin{aligned}
M_{10} &= M_2 \\
log_{10}(10^{N_{10}}) &= \frac{log_2(2^{224})}{log_2(10)} \\
N_{10} \bullet log_{10}(10) &= \frac{224 \bullet log_2(2)}{log_2(10)} \\
N_{10} \bullet (1) &= \frac{224 \bullet (1)}{3.32} \\
N_{10} &= 67.47
\end{aligned} \tag{2}
$$

▸ **QUESTION 3**

In AES symmetric key encryption, a symmetric key of 112 bits is considered secure today. How many decimal digits does that represent?

Hint: The straightforward approach to this problem will have you writing a *very* large number b^N. That is not practical. Consider the logarithmic relationship

$$log_k(b^N) = N \bullet log_k(b)$$

Estimated time to complete this question is 10 minutes.

SOLUTION

In general, the number of discrete states is $M_b = b^N$ where b is the base and N is the number of digits. The number of decimal digits, N_{10} is to be determined. The number of discrete states represented in decimal is, obviously, the same as the binary representation. Therefore, we can represent this relationship as follows:

$$M_{10} = M_2$$

We can express the number of discrete states in decimal as $M_{10} = 10^{N_{10}}$ where N_{10} is the number of decimal digits represented.

We are given $b = 2$, where b is the base—i.e. binary—and $N_2 = 112$, where N_2 is the number of binary digits, The number of discrete states, M_2 represented by 112 binary digits is $M_2 = 2^{112}$.

$$
\begin{aligned}
M_{10} &= M_2 \\
log_{10}(10^{N_{10}}) &= \frac{log_2(2^{112})}{log_2(10)} \\
N_{10} \bullet log_{10}(10) &= \frac{112 \bullet log_2(2)}{log_2(10)} \\
N_{10} \bullet (1) &= \frac{112 \bullet (1)}{3.32} \\
N_{10} &= 33.73
\end{aligned} \tag{3}
$$

▸ **QUESTION 4**

Perform longhand conversion of the following decimal numbers to 6–bit two's complement binary numbers. Perform indicated addition or subtraction. Indicate whether or not the sum overflows a 6-bit result. In each case, specify why or why not.

(a) $-16 - 15$

(b) $-31 - 20$

(c) $9 - 26$

Estimated time to complete this question is 10 minutes.

SOLUTION

$(a) - 16 - 15$

First, perform longhand conversion of -16 and -15 to 6–bit two's complement binary:

$$\begin{aligned} \frac{16}{2} &= 8r0 \\ \frac{8}{2} &= 4r0 \\ \frac{4}{2} &= 2r0 \\ \frac{2}{2} &= 1r0 \\ \frac{1}{2} &= 0r1 \end{aligned} \tag{4}$$

The magnitude of -16 is 10000. The magnitude is positive so the sign bit is 0. Perform sign extension by copying the sign bit into the most significant bit positions. This yields 010000.

Now perform two's complement by inverting the digits and adding 1. Inverting the bits yields 101111.

$$\begin{array}{r} 101111 \\ \underline{+\,000001} \\ 110000 \end{array}$$

So, -16 is equal to 110000.

$$\begin{aligned} \frac{15}{2} &= 7r1 \\ \frac{7}{2} &= 3r1 \\ \frac{3}{2} &= 1r1 \\ \frac{1}{2} &= 0r1 \end{aligned} \tag{5}$$

The magnitude of -15 is 1111. Since this is the magnitude, it is positive and the sign bit is 0. Perform sign extension by copying the sign bit in to the most significant positions to make a 6–bit binary number. This yields 001111.

Now perform two's complement by inverting the digits and adding 1. Inverting the bits yields 110000.

$$\begin{array}{r} 110000 \\ +\underline{100001} \\ 110001 \end{array}$$

So, -15 is equal to 110001.
Now add the two numbers:

$$\begin{array}{r} 110000 \\ +\underline{110001} \\ 1100001 \end{array}$$

There is a carry out of the msb. Both sign bits are the same and the sign bit of the result is the same. The operation does not overflow.
$(b) - 31 - 20$
First, perform longhand conversion of -31 and -20 to 6–bit two's complement binary:

$$\begin{aligned} \frac{31}{16} &= 1r15 \\ \frac{15}{8} &= 1r7 \\ \frac{7}{4} &= 1r3 \\ \frac{3}{2} &= 1r1 \\ \frac{1}{2} &= 1r0 \end{aligned} \tag{6}$$

The magnitude of -31 is 11111. The magnitude is positive so the sign bit is 0. Perform sign extension by copying the sign bit into the most significant bit positions. This yields 011111.
Now perform two's complement by inverting the digits and adding 1. Inverting the bits yields 100000.

$$\begin{array}{r} 100000 \\ +\underline{000001} \\ 100001 \end{array}$$

So, -31 is equal to 100001.

$$\begin{aligned} \frac{20}{16} &= 1r4 \\ \frac{4}{8} &= 0r4 \\ \frac{4}{4} &= 1r0 \\ \frac{0}{2} &= 0r0 \\ \frac{0}{1} &= 0r0 \end{aligned} \tag{7}$$

The magnitude of -20 is 10100. Since this is the magnitude, it is positive and the sign bit is 0.
Perform sign extension by copying the sign bit in to the most significant positions to make a 6–bit binary number. This yields 010100.
Now perform two's complement by inverting the digits and adding 1. Inverting the bits yields 101011.

$$\begin{array}{r} 101011 \\ +\underline{000001} \\ 101100 \end{array}$$

So, -20 is equal to 101100.
Now add the two numbers:

$$\begin{array}{r} 100001 \\ \underline{+\,101100} \\ 1001101 \end{array}$$

The sign bits of the numbers being added are negative but the sign bit of the result is positive. The operation does overflow. There is also a carry out of the msb.

$(c) 9 - 26$
First, perform longhand conversion of 9 and -26 to 6–bit two's complement binary:

$$\begin{aligned} \frac{9}{8} &= 1r1 \\ \frac{1}{4} &= 0r1 \\ \frac{1}{2} &= 0r1 \\ \frac{1}{1} &= 1r0 \end{aligned} \tag{8}$$

The magnitude of 9 is 1001. The magnitude is positive so the sign bit is 0. Perform sign extension by copying the sign bit into the most significant bit positions. This yields **001001**. This number is positive and so already a valid two's complement binary number.

$$\begin{aligned} \frac{26}{16} &= 1r10 \\ \frac{10}{8} &= 1r2 \\ \frac{2}{4} &= 0r2 \\ \frac{2}{2} &= 1r0 \\ \frac{0}{1} &= 0r0 \end{aligned} \tag{9}$$

The magnitude of -26 is 11010. Since this is the magnitude, it is positive and the sign bit is 0. Perform sign extension by copying the sign bit in to the most significant positions to make a 6–bit binary number. This yields 011010.
Now perform two's complement by inverting the digits and adding 1. Inverting the bits yields 100101.

$$\begin{array}{r} 011010 \\ \underline{+\,000001} \\ 100110 \end{array}$$

So, -26 is equal to 100110.
Now add the two numbers:

$$\begin{array}{r} 100110 \\ \underline{+\,001001} \\ 101111 \end{array}$$

Adding a positive and a negative two's complement number cannot overflow. The operation does not overflow.

▸ **QUESTION 5**

Perform longhand conversion of the following decimal numbers to 6–bit two's complement binary numbers. Perform indicated addition or subtraction. Indicate whether or not the sum overflows a 6-bit result. In each case, specify why or why not.

(a) $-25 + 30$

(b) $-17 - 16$

(c) $5 - 10$

Estimated time to complete this question is 10 minutes.

SOLUTION

$(a) - 25 + 30$

First, perform longhand conversion of -25 and 30 to 6–bit two's complement binary:

$$\begin{aligned} \frac{25}{16} &= 1r9 \\ \frac{9}{8} &= 1r1 \\ \frac{1}{4} &= 0r1 \\ \frac{1}{2} &= 0r1 \\ \frac{1}{1} &= 1r0 \end{aligned} \tag{10}$$

The magnitude of -25 is 11001. The magnitude is positive so the sign bit is 0. Perform sign extension by copying the sign bit into the most significant bit positions. This yields 011001.

Now perform two's complement by inverting the digits and adding 1. Inverting the bits yields 100110.

$$\begin{array}{r} 100110 \\ \underline{+\,000001} \\ 100111 \end{array}$$

So, -25 is equal to 110111.

$$\begin{aligned} \frac{30}{2} &= 15r0 \\ \frac{15}{2} &= 7r1 \\ \frac{7}{2} &= 3r1 \\ \frac{3}{2} &= 1r1 \\ \frac{1}{2} &= 0r1 \end{aligned} \tag{11}$$

The magnitude of 30 is 11110. Since this is a positive number, the sign bit is 0. Perform sign extension by copying the sign bit in to the most significant positions to make a 6–bit binary number. This yields 011110.

Now add the two numbers:

$$\begin{array}{r} 110111 \\ \underline{+\,011110} \\ 1000101 \end{array}$$

Adding a positive and a negative two's complement number cannot overflow. The operation does not overflow. Note there is a carry out of the msb.
$(b) - 17 - 16$
First, perform longhand conversion of -17 and -16 to 6–bit two's complement binary:

$$\begin{aligned} \frac{17}{2} &= 8r1 \\ \frac{8}{2} &= 4r0 \\ \frac{4}{2} &= 2r0 \\ \frac{2}{2} &= 1r0 \\ \frac{1}{2} &= 0r1 \end{aligned} \tag{12}$$

The magnitude of -17 is 10001. The magnitude is positive so the sign bit is 0. Perform sign extension by copying the sign bit into the most significant bit positions. This yields 010001.
Now perform two's complement by inverting the digits and adding 1. Inverting the bits yields 101110.

$$\begin{array}{r} 101110 \\ \underline{+\,000001} \\ 101111 \end{array}$$

So, -17 is equal to 101111 in two's complement representation.

$$\begin{aligned} \frac{16}{2} &= 8r0 \\ \frac{8}{2} &= 4r0 \\ \frac{4}{2} &= 2r0 \\ \frac{2}{2} &= 1r0 \\ \frac{1}{2} &= 0r1 \end{aligned} \tag{13}$$

The magnitude of -16 is 10000. Since this is the magnitude, it is positive and the sign bit is 0. Perform sign extension by copying the sign bit in to the most significant positions to make a 6–bit binary number. This yields 010000.
Now perform two's complement by inverting the digits and adding 1. Inverting the bits yields 101111.

$$\begin{array}{r} 101111 \\ \underline{+\,000001} \\ 110000 \end{array}$$

So, -16 is equal to 110000.
Now add the two numbers:

$$\begin{array}{r} 101111 \\ \underline{+\,110000} \\ 1011111 \end{array}$$

The sign bits of the numbers being added are negative but the sign bit of the result is positive. The operation does overflow. There is also a carry out of the msb.

$(c) 5 - 10$
First, perform longhand conversion of 5 and -10 to 6–bit two's complement binary:

$$\begin{aligned} \frac{5}{2} &= 2r1 \\ \frac{2}{2} &= 1r0 \\ \frac{1}{2} &= 0r1 \end{aligned} \tag{14}$$

The magnitude of 5 is 101. The magnitude is positive so the sign bit is 0. Perform sign extension by copying the sign bit into the most significant bit positions. This yields **000101**. This number is positive and so already a valid two's complement binary number.

$$\begin{aligned} \frac{10}{2} &= 5r0 \\ \frac{5}{2} &= 2r1 \\ \frac{2}{2} &= 1r0 \\ \frac{1}{2} &= 0r1 \end{aligned} \tag{15}$$

The magnitude of -10 is 1010. Since this is the magnitude, it is positive and the sign bit is 0. Perform sign extension by copying the sign bit in to the most significant positions to make a 6–bit binary number. This yields 001010.
Now perform two's complement by inverting the digits and adding 1. Inverting the bits yields 110101.

$$\begin{array}{r} 110101 \\ \underline{+\,000001} \\ 110110 \end{array}$$

So, -10 is equal to 110110.
Now add the two numbers:

$$\begin{array}{r} 000101 \\ \underline{+\,110110} \\ 111011 \end{array}$$

Adding a positive and a negative two's complement number cannot overflow. The operation does not overflow. There is no carry.

▸ QUESTION 6

Prove theorem T9 from Table 2.3 of *Digital Design and Computer Architecture.* Do not prove the dual.

$$B \bullet (B + C) = B$$

Hint: Try expanding a term with $(C + 1)$.

Estimated time to complete this question is 10 minutes.

SOLUTION

$$
\begin{aligned}
B \bullet (B+C) &= B \\
BB + BC &= B \\
B \bullet (C+1) + BC &= B \ (T2') \\
BC + B + BC &= B \\
B + BC + BC &= B \ (T3') \\
B &= B
\end{aligned} \tag{16}
$$

▸ **QUESTION 7**

Prove the following Boolean expression:

$$B + \overline{B}\,C = B + C$$

Hint: Begin by expanding a term with $(C+1)$.

Estimated time to complete this question is 10 minutes.

SOLUTION

$$
\begin{aligned}
B + \overline{B}\,C &= B + C \\
B \bullet (C+1) + \overline{B}\,C &= B + C \quad (T2') \\
B\,C + B + \overline{B}\,C &= B + C \\
B + (B\,C + \overline{B}\,C) &= B + C \\
B + (C\,B + C\,\overline{B}) &= B + C \quad (T10) \\
B + C &= B + C
\end{aligned} \tag{17}
$$

▸ **QUESTION 8**

Simplify the following Boolean equation:

$$Y = A\,C + \overline{A}\,\overline{B}\,C$$

Hint: Begin by expanding a term with $(\overline{B}+1)$.

Estimated time to complete this question is 10 minutes.

SOLUTION

$$
\begin{aligned}
Y &= A\,C + \overline{A}\,\overline{B}\,C \\
&= A\,C \bullet (\overline{B}+1) + \overline{A}\,\overline{B}\,C \\
&= A\,\overline{B}\,C + A\,C + \overline{A}\,\overline{B}\,C \\
&= \overline{B}\,C \bullet (A + \overline{A}) + A\,C \\
&= \overline{B}\,C \bullet (1) + A\,C \\
&= A\,C + \overline{B}\,C
\end{aligned} \tag{18}
$$

▸ **QUESTION 9**
Consider the following Boolean expression:

$$\overline{B}\,\overline{C} + \overline{B\,C}$$

Is it possible to apply theorem T3'? Why?

Estimated time to complete this question is 5 minutes.

SOLUTION

No, it is not possible to apply theorem T3'. For theorem T3' to apply, each term must be the same. In this equation, one term is the product of the inverts while the other is the invert of the product.

▸ **QUESTION 10**
Consider the following Boolean expression:

$$B\,C + \overline{B\,C}$$

Is it possible to apply theorem T5'? Why?

Estimated time to complete this question is 5 minutes.

SOLUTION

Yes, it is possible to apply theorem T5'. For theorem T5' to apply, each term must be inverse of each other. In this equation, one term is the product while the other is the inverse of the product.

▸ **QUESTION 11**
Consider the following Boolean expression:

$$B\,C + \overline{B}\,\overline{C}$$

Is it possible to apply theorem T5'? Why?

Estimated time to complete this question is 5 minutes.

SOLUTION

No, it is not possible to apply theorem T5'. For theorem T5' to apply, each term must be inverse of each other. In this equation, one term is the product of the inverts while the other is the invert of the product.

▸ **QUESTION 12**
Consider the following Boolean expression:

$$A \bullet B\,C + A \bullet \overline{B}\,\overline{C}$$

Is it possible to apply theorem T10? Why?

Estimated time to complete this question is 5 minutes.

SOLUTION

No, it is not possible to apply theorem T10. For theorem T10 to apply, each term $B\,C$ and $\overline{B}\,\overline{C}$ must be inverse of each other. In this equation, one term is the product of the inverts while the other is the invert of the product.

▸ QUESTION 13

Consider the following logic circuit:

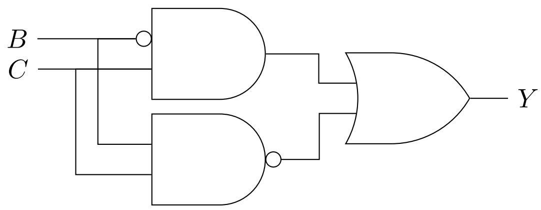

Figure 1: Logic circuit.

(a) Draw a complete truth table with minterms and minterm names. Write a Boolean equation in sum-of-products canonical form.

(b) Draw a complete truth table with maxterms and maxterm names. Write a Boolean equation in product-of-sums canonical form.

(c) Simplify the sum-of-products Boolean equation.

Estimated time to complete this question is 10 minutes.

SOLUTION

The Boolean function is $\overline{B}\,C + \overline{B\,C}$.

B	C	$\overline{B}\,C$	$\overline{B\,C}$	Y	minterm	name
0	0	0	1	1	$\overline{B}\,\overline{C}$	m_0
0	1	1	1	1	$\overline{B}\,C$	m_1
1	0	0	1	1	$B\,\overline{C}$	m_2
1	1	0	0	0	$B\,C$	m_3

Figure 2: Truth table for $\overline{B}\,C + \overline{B\,C}$—sum-of-products.

The Boolean equation in sum-of-products canonical form is the sum of the minterms for which Y is 1.

$$\begin{aligned} Y &= m_0 + m_1 + m_2 \\ Y &= \overline{B}\,\overline{C} + \overline{B}\,C + B\,\overline{C} \end{aligned} \tag{19}$$

B	C	$\overline{B}\,C$	$\overline{B\,C}$	Y	maxterm	name
0	0	0	1	1	$B + C$	M_0
0	1	1	1	1	$B + \overline{C}$	M_1
1	0	0	1	1	$\overline{B} + C$	M_2
1	1	0	0	0	$\overline{B} + \overline{C}$	M_3

Figure 3: Truth table for $\overline{B}\,C + \overline{B\,C}$—product-of-sums.

The Boolean equation in product-of-sums canonical form is the product of the maxterms for which Y is 0.

$$
\begin{aligned}
Y &= M_3 \\
Y &= \overline{B} + \overline{C} \qquad (20)
\end{aligned}
$$

Simplify the sum-of-products Boolean equation.

$$
\begin{aligned}
Y &= m_0 + m_1 + m_2 \\
&= \overline{B}\,\overline{C} + \overline{B}\,C + B\,\overline{C} \\
&= \overline{B} \bullet (C + \overline{C}) + B\,\overline{C} \\
&= \overline{B} \bullet (1) + B\,\overline{C} \\
&= \overline{B} + B\,\overline{C} \\
Y &= \overline{B} + \overline{C} \qquad (21)
\end{aligned}
$$

▸ **QUESTION 14**

Consider the following logic circuit:

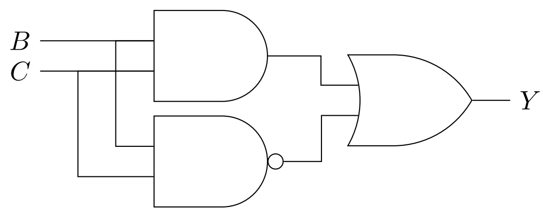

Figure 4: Logic circuit.

(a) Draw a complete truth table with minterms and minterm names. Write a Boolean equation in sum-of-products canonical form.

(b) Draw a complete truth table with maxterms and maxterm names. Write a Boolean equation in product-of-sums canonical form.

(c) Simplify the sum-of-products Boolean equation.

Estimated time to complete this question is 10 minutes.

SOLUTION

The Boolean function is $B\,C + \overline{B\,C}$.

B	C	$B\,C$	$\overline{B\,C}$	Y	minterm	name
0	0	0	1	1	$\overline{B}\,\overline{C}$	m_0
0	1	0	1	1	$\overline{B}\,C$	m_1
1	0	0	1	1	$B\,\overline{C}$	m_2
1	1	1	0	1	$B\,C$	m_3

Figure 5: Truth table for $B\,C + \overline{B\,C}$—sum-of-products.

The Boolean equation in sum-of-products canonical form is the sum of the minterms for which Y is 1.

$$\begin{aligned} Y &= m_0 + m_1 + m_2 + m_3 \\ Y &= \overline{B}\,\overline{C} + \overline{B}\,C + B\,\overline{C} + B\,C \end{aligned} \tag{22}$$

B	C	$\overline{B}\,C$	$\overline{B\,C}$	Y	maxterm	name
0	0	0	1	1	$B + C$	M_0
0	1	0	1	1	$B + \overline{C}$	M_1
1	0	0	1	1	$\overline{B} + C$	M_2
1	1	1	0	1	$\overline{B} + \overline{C}$	M_3

Figure 6: Truth table for $B\,C + \overline{B\,C}$—product-of-sums.

The Boolean equation in product-of-sums canonical form is the product of the maxterms for which Y is 0. There are no terms for which Y is 0. Therefore,

$$Y = 1 \tag{23}$$

Simplify the sum-of-products Boolean equation.

$$\begin{aligned} Y &= m_0 + m_1 + m_2 + m_3 \\ &= \overline{B}\,\overline{C} + \overline{B}\,C + B\,\overline{C} + B\,C \\ &= (\overline{B}\,\overline{C}) + (\overline{B}\,C + B\,\overline{C} + B\,C) \\ &= (\overline{B}) + (B) \\ &= (\overline{B} + B) \\ Y &= 1 \end{aligned} \tag{24}$$

▸ **QUESTION 15**

Consider the following logic circuit:

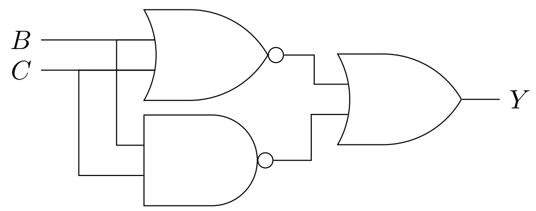

Figure 7: Logic circuit.

(a) Draw a complete truth table with minterms and minterm names. Write a Boolean equation in sum-of-products canonical form.

(b) Draw a complete truth table with maxterms and maxterm names. Write a Boolean equation in product-of-sums canonical form.

(c) Simplify the sum-of-products Boolean equation.

Estimated time to complete this question is 10 minutes.

SOLUTION

The Boolean function is $\overline{B}\,\overline{C} + \overline{B\,C}$.

B	C	$\overline{B}\,\overline{C}$	$\overline{B\,C}$	Y	minterm	name
0	0	1	1	1	$\overline{B}\,\overline{C}$	m_0
0	1	0	1	1	$\overline{B}\,C$	m_1
1	0	0	1	1	$B\,\overline{C}$	m_2
1	1	0	0	0	$B\,C$	m_3

Figure 8: Truth table for $\overline{B}\,\overline{C} + \overline{B\,C}$—sum-of-products.

The Boolean equation in sum-of-products canonical form is the sum of the minterms for which Y is 1.

$$
\begin{aligned}
Y &= m_0 + m_1 + m_2 \\
Y &= \overline{B}\,\overline{C} + \overline{B}\,C + B\,\overline{C}
\end{aligned}
\tag{25}
$$

B	C	$\overline{B}\,C$	$\overline{B\,C}$	Y	maxterm	name
0	0	1	1	1	$B + C$	M_0
0	1	0	1	1	$B + \overline{C}$	M_1
1	0	0	1	1	$\overline{B} + C$	M_2
1	1	0	0	0	$\overline{B} + \overline{C}$	M_3

Figure 9: Truth table for $\overline{B}\,\overline{C} + \overline{B\,C}$—product-of-sums.

The Boolean equation in product-of-sums canonical form is the product of the maxterms for which Y is 0.

$$
\begin{aligned}
Y &= M_3 \\
Y &= \overline{B} + \overline{C}
\end{aligned}
\tag{26}
$$

Simplify the sum-of-products Boolean equation.

$$
\begin{aligned}
Y &= m_0 + m_1 + m_2 \\
&= \overline{B}\,\overline{C} + \overline{B}\,C + B\,\overline{C} \\
&= \overline{B} \bullet (\overline{C} + C) + B\,\overline{C} \\
&= \overline{B} \bullet (1) + B\,\overline{C} \\
&= \overline{B} + B\,\overline{C} \\
Y &= \overline{B} + \overline{C}
\end{aligned}
\tag{27}
$$

▸ **QUESTION 16**

Consider the following logic circuit:

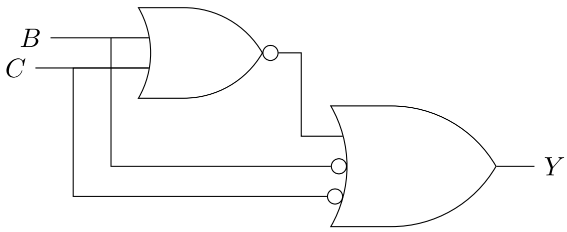

Figure 10: Logic circuit.

(a) Draw a complete truth table with minterms and minterm names. Write a Boolean equation in sum-of-products canonical form.

(b) Draw a complete truth table with maxterms and maxterm names. Write a Boolean equation in product-of-sums canonical form.

(c) Simplify the sum-of-products Boolean equation.

Estimated time to complete this question is 10 minutes.

SOLUTION

The Boolean function is $\overline{B}\,\overline{C} + \overline{B\,C}$ but expressed in the circuit after applying DeMorgans theorem.

B	C	$\overline{B}\,\overline{C}$	$\overline{B}+\overline{C}$	Y	minterm	name
0	0	1	1	1	$\overline{B}\,\overline{C}$	m_0
0	1	0	1	1	$\overline{B}\,C$	m_1
1	0	0	1	1	$B\,\overline{C}$	m_2
1	1	0	0	0	$B\,C$	m_3

Figure 11: Truth table for $\overline{B}\,\overline{C} + \overline{B\,C}$—sum-of-products.

The Boolean equation in sum-of-products canonical form is the sum of the minterms for which Y is 1.

$$\begin{aligned} Y &= m_0 + m_1 + m_2 \\ Y &= \overline{B}\,\overline{C} + \overline{B}\,C + B\,\overline{C} \end{aligned} \tag{28}$$

B	C	$\overline{B}\,C$	$\overline{B\,C}$	Y	maxterm	name
0	0	1	1	1	$B+C$	M_0
0	1	0	1	1	$B+\overline{C}$	M_1
1	0	0	1	1	$\overline{B}+C$	M_2
1	1	0	0	0	$\overline{B}+\overline{C}$	M_3

Figure 12: Truth table for $\overline{B}\,\overline{C} + \overline{B\,C}$—product-of-sums.

The Boolean equation in product-of-sums canonical form is the product of the maxterms for which Y is 0.

$$\begin{aligned} Y &= M_3 \\ Y &= \overline{B} + \overline{C} \end{aligned} \tag{29}$$

Simplify the sum-of-products Boolean equation.

$$
\begin{aligned}
Y &= m_0 + m_1 + m_2 \\
&= \overline{B}\,\overline{C} + \overline{B}\,C + B\,\overline{C} \\
&= \overline{B} \bullet (\overline{C} + C) + B\,\overline{C} \\
&= \overline{B} \bullet (1) + B\,\overline{C} \\
&= \overline{B} + B\,\overline{C} \\
Y &= \overline{B} + \overline{C} \qquad (30)
\end{aligned}
$$

▸ **QUESTION 17**

The binary full adder takes the sum of two binary literals A and B. It also takes a carry bit from an optional *lsb* stage adder. The two outputs are S, which is the sum of A, B, and C_{in}, and C_{out} which is the carry.

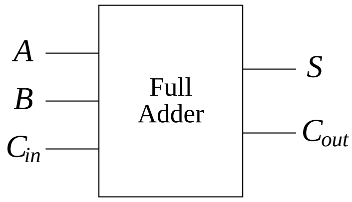

Figure 13: Binary Full Adder

(a) Draw a complete truth table for the binary full adder with minterms and minterm names. For the output S only, write a Boolean equation in sum-of-products canonical form. Do not simplify.

(b) Draw a complete truth table for the binary full adder with maxterms and maxterm names. For the output S only, write a Boolean equation in product-of-sums canonical form. Do not simplify.

Estimated time to complete this question is 10 minutes.

SOLUTION

A	B	C_{in}	S	C_{out}	minterm	name
0	0	0	0	0	$\overline{A}\,\overline{B}\,\overline{C}$	m_0
0	0	1	1	0	$\overline{A}\,\overline{B}\,C$	m_1
0	1	0	1	0	$\overline{A}\,B\,\overline{C}$	m_2
0	1	1	0	1	$\overline{A}\,B\,C$	m_3
1	0	0	1	0	$A\,\overline{B}\,\overline{C}$	m_4
1	0	1	0	1	$A\,\overline{B}\,C$	m_5
1	1	0	0	1	$A\,B\,\overline{C}$	m_6
1	1	1	1	1	$A\,B\,C$	m_7

Figure 14: Truth table for binary full adder—sum-of-products.

The boolean equation is the sum of the minterms for which the result S is 1.

$$S = m_1 + m_2 + m_4 + m_7$$
$$S = \overline{A}\,\overline{B}\,C_{in} + \overline{A}\,B\,\overline{C_{in}} + A\,\overline{B}\,\overline{C_{in}} + A\,B\,C_{in} \tag{31}$$

A	B	C_{in}	S	C_{out}	maxterm	name
0	0	0	0	0	$A + B + C_{in}$	M_0
0	0	1	1	0	$A + B + \overline{C_{in}}$	M_1
0	1	0	1	0	$A + \overline{B} + C_{in}$	M_2
0	1	1	0	1	$A + \overline{B} + \overline{C_{in}}$	M_3
1	0	0	1	0	$\overline{A} + B + C_{in}$	M_4
1	0	1	0	1	$\overline{A} + B + \overline{C_{in}}$	M_5
1	1	0	0	1	$\overline{A} + \overline{B} + C_{in}$	M_6
1	1	1	1	1	$\overline{A} + \overline{B} + \overline{C_{in}}$	M_7

Figure 15: Truth table for binary full adder—product-of-sums.

The boolean equation is the product of the maxterms for which the result S is 0.

$$S = M_0 \bullet M_3 \bullet M_5 \bullet M_6$$
$$S = (A + B + C_{in}) \bullet (A + \overline{B} + \overline{C_{in}}) \bullet (\overline{A} + B + \overline{C_{in}}) \bullet (\overline{A} + \overline{B} + C_{in}) \tag{32}$$

▸ **QUESTION 18**

This combinational logic circuit is built from two multiplexers. The multiplexer data lines are hardwired to V_{DD} or GND. The three input selectors, A, B, and C act as inputs to the function.

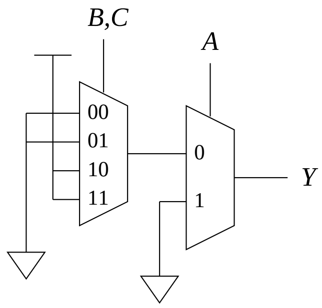

Figure 16: Multiplexer Logic

(a) Draw a complete truth table for the combinational multiplexer logic circuit with minterms and minterm names. Write a Boolean equation in sum-of-products canonical form. Do not simplify.

(b) Draw a complete truth table for the combinational multiplexer logic circuit with maxterms and maxterm names. Write a Boolean equation in product-of-sums canonical form. Do not simplify.

Estimated time to complete this question is 10 minutes.

SOLUTION

A	B	C	Y	minterm	name
0	0	0	0	$\overline{A}\,\overline{B}\,\overline{C}$	m_0
0	0	1	0	$\overline{A}\,\overline{B}\,C$	m_1
0	1	0	1	$\overline{A}\,B\,\overline{C}$	m_2
0	1	1	1	$\overline{A}\,B\,C$	m_3
1	0	0	0	$A\,\overline{B}\,\overline{C}$	m_4
1	0	1	0	$A\,\overline{B}\,C$	m_5
1	1	0	0	$A\,B\,\overline{C}$	m_6
1	1	1	0	$A\,B\,C$	m_7

Figure 17: Truth table for multiplexer logic circuit—sum-of-products.

The boolean equation is the sum of the minterms for which the result Y is 1.

$$\begin{aligned} Y &= m_2 + m_3 \\ Y &= \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C \end{aligned} \tag{33}$$

A	B	C	Y	maxterm	name
0	0	0	0	$A+B+C$	M_0
0	0	1	0	$A+B+\overline{C}$	M_1
0	1	0	1	$A+\overline{B}+C$	M_2
0	1	1	1	$A+\overline{B}+\overline{C}$	M_3
1	0	0	0	$\overline{A}+B+C$	M_4
1	0	1	0	$\overline{A}+B+\overline{C}$	M_5
1	1	0	0	$\overline{A}+\overline{B}+C$	M_6
1	1	1	0	$\overline{A}+\overline{B}+\overline{C}$	M_7

Figure 18: Truth table for multiplexer logic circuit—product-of-sums.

The boolean equation is the product of the maxterms for which the result Y is 0.

$$\begin{aligned} Y &= M_0 \bullet M_1 \bullet M_4 \bullet M_5 \bullet M_6 \bullet M_7 \\ Y &= (A+B+C) \bullet (A+B+\overline{C}) \bullet (\overline{A}+B+C) \bullet (\overline{A}+B+\overline{C}) \bullet (\overline{A}+\overline{B}+C) \bullet (\overline{A}+\overline{B}+\overline{C}) \end{aligned} \tag{34}$$

▸ **QUESTION 19**

Perform longhand conversion of the following decimal numbers to hexadecimal. Show all your work.

(a) 7911

(b) 4541

Estimated time to complete this question is 5 minutes.

SOLUTION

(a)

$$
\begin{aligned}
\frac{7911}{4096} &= 1r3815 \\
\frac{3815}{256} &= 14r231 \\
\frac{231}{16} &= 14r7 \\
\frac{7}{1} &= 7r0 \qquad (35)
\end{aligned}
$$

0x1EE7
(b)

$$
\begin{aligned}
\frac{4541}{16} &= 283r13 \\
\frac{283}{16} &= 17r11 \\
\frac{17}{16} &= 1r1 \\
\frac{1}{16} &= 0r1 \qquad (36)
\end{aligned}
$$

0x11BD

▸ **QUESTION 20**
Perform longhand conversion of the following decimal numbers to hexadecimal. Show all your work.

(a) 6672

(b) 4505

Estimated time to complete this question is 5 minutes.

SOLUTION

(a)

$$
\begin{aligned}
\frac{6672}{4096} &= 1r2576 \\
\frac{3815}{256} &= 10r16 \\
\frac{231}{16} &= 1r0 \\
\frac{7}{1} &= 0r0 \qquad (37)
\end{aligned}
$$

0x1A10
(b)

$$
\begin{aligned}
\frac{4505}{16} &= 281r9 \\
\frac{281}{16} &= 17r9 \\
\frac{17}{16} &= 1r1 \\
\frac{1}{16} &= 0r1
\end{aligned} \tag{38}
$$

0x1199

▸ **QUESTION 21**

Perform longhand conversion of the following decimal numbers to hexadecimal. Show all your work.

(a) 4387

(b) 5723

Estimated time to complete this question is 5 minutes.

SOLUTION

(*a*)

$$
\begin{aligned}
\frac{4387}{4096} &= 1r291 \\
\frac{291}{256} &= 1r35 \\
\frac{35}{16} &= 2r3 \\
\frac{3}{1} &= 3r0
\end{aligned} \tag{39}
$$

0x1123

(*b*)

$$
\begin{aligned}
\frac{5723}{16} &= 357r11 \\
\frac{357}{16} &= 22r5 \\
\frac{22}{16} &= 1r6 \\
\frac{1}{16} &= 0r1
\end{aligned} \tag{40}
$$

0x165B

▸ **QUESTION 22**
Perform longhand conversion of the following decimal numbers to hexadecimal. Show all your work.

(a) 5455

(b) 9963

Estimated time to complete this question is 5 minutes.

SOLUTION

(a)

$$
\begin{aligned}
\frac{5455}{4096} &= 1r1359 \\
\frac{1359}{256} &= 5r79 \\
\frac{79}{16} &= 4r15 \\
\frac{15}{1} &= 15r0
\end{aligned}
\tag{41}
$$

0x154F
(b)

$$
\begin{aligned}
\frac{9963}{16} &= 622r11 \\
\frac{622}{16} &= 38r14 \\
\frac{38}{16} &= 2r6 \\
\frac{2}{16} &= 0r2
\end{aligned}
\tag{42}
$$

0x26EB

▸ **QUESTION 23**
You hold 10 coins in your hand. You toss them into the air and they fall to the ground. Each coin comes to rest face up or face down.

(a) How many discrete states are there in this system?

(b) What is the base of this system?

(c) In decimal, how many numbers can be represented—i.e. what is the range?

Estimated time to complete this question is 5 minutes.

SOLUTION

(a) There are 20 discrete states—2 per coin with 10 coins.

(b) Each penny represents 2 states. Therefore the base is 2.

(c) In general, the range is b^N where N is the number of devices. In this case, $N = 10$ so the number of states is $2^{10} = 1,024$.

▸ QUESTION 24

You hold 3 six-sided dice in your hand. You toss them on a table where they come to rest squarely.

(a) How many discrete states are there in this system?

(b) What is the base of this system?

(c) In decimal, how many numbers can be represented—i.e. what is the range?

Estimated time to complete this question is 5 minutes.

SOLUTION

(a) There are 18 discrete states—6 per die with 3 die.

(b) Each die represents 6 states. Therefore the base is 6.

(c) In general, the range is b^N where N is the number of devices. In this case, $N = 3$ so the number of states is $6^3 = 216$.

▸ QUESTION 25

Convert the following hexadecimal numbers to decimal. You may convert each hexadecimal digit to it's corresponding decimal value 'by inspection.'

(a) 0x3840

(b) 0xFA8E

Estimated time to complete this question is 5 minutes.

SOLUTION

(a) 0x3840

$$\begin{aligned} VAL_D &= (3 * 4096) + (8 * 256) + (4 * 16) + (0 * 1) \\ &= 12,288 + 2,048 + 64 + 0 \\ &= 14,400 \end{aligned} \tag{43}$$

(b) 0xFA8E

$$\begin{aligned} VAL_D &= (15 * 4096) + (10 * 256) + (8 * 16) + (14 * 1) \\ &= 61,440 + 3,840 + 128 + 14 \\ &= 64,142 \end{aligned} \tag{44}$$

▸ **QUESTION 26**

Convert the following hexadecimal numbers to decimal. You may convert each hexadecimal digit to it's corresponding decimal value 'by inspection.'

(a) 0xACDC

(b) 0x4F91

Estimated time to complete this question is 5 minutes.

SOLUTION

(a) 0xACDC

$$\begin{aligned} VAL_D &= (10 * 4096) + (12 * 256) + (13 * 16) + (12 * 1) \\ &= 40,960 + 3,072 + 208 + 12 \\ &= 44,252 \end{aligned} \tag{45}$$

(b) 0x4F91

$$\begin{aligned} VAL_D &= (4 * 4096) + (15 * 256) + (9 * 16) + (1 * 1) \\ &= 16,384 + 3,840 + 144 + 1 \\ &= 20,369 \end{aligned} \tag{46}$$

▸ **QUESTION 27**

Convert the following hexadecimal numbers to decimal. You may convert each hexadecimal digit to it's corresponding decimal value 'by inspection.'

(a) 0xE9F5

(b) 0xDD7B

Estimated time to complete this question is 5 minutes.

SOLUTION

(a) 0xE9F5

$$\begin{aligned} VAL_D &= (14 * 4096) + (9 * 256) + (15 * 16) + (5 * 1) \\ &= 57,344 + 2,304 + 240 + 5 \\ &= 59,893 \end{aligned} \tag{47}$$

(b) 0xDD7B

$$\begin{aligned} VAL_D &= (13 * 4096) + (13 * 256) + (7 * 16) + (11 * 1) \\ &= 53,248 + 3,328 + 112 + 11 \\ &= 56,699 \end{aligned} \tag{48}$$

▸ **QUESTION 28**
Convert the following hexadecimal numbers to decimal. You may convert each hexadecimal digit to it's corresponding decimal value 'by inspection.'

(a) 0x987F

(b) 0x4005

Estimated time to complete this question is 5 minutes.

SOLUTION

(a) 0x997F

$$\begin{aligned} VAL_D &= (9*4096)+(8*256)+(7*16)+(15*1) \\ &= 36,864+2,048+112+15 \\ &= 39,039 \end{aligned} \tag{49}$$

(b) 0x4005

$$\begin{aligned} VAL_D &= (4*4096)+(0*256)+(0*16)+(5*1) \\ &= 16,384+0+0+5 \\ &= 16,389 \end{aligned} \tag{50}$$

▸ **QUESTION 29**
Consider the input logic levels in the following table:

Parameter	Parameter Name	Min	Max
V_{CC}	High-level input voltage	-	$5.0V$
V_{IH}	High-level input voltage	$2.0V$	-
V_{IL}	Low-level input voltage	-	$0.8V$

Figure 19: Input logic levels for $5V$ device.

(a) Compute the noise margins between the LaunchPad output logic levels and this device's input logic levels.

(b) Sketch a logic level diagram with the LaunchPad driving the input of this device.

(c) If the output of the LaunchPad is driving these inputs, are the logic levels compatible?

Estimated time to complete this question is 10 minutes.

SOLUTION
The answer to (c) is YES—the LaunchPad output logic levels are compatible with the device input logic levels.

$$
\begin{aligned}
NM_H &= V_{OH} - V_{IH} \\
NM_L &= V_{IL} - V_{OL} \\
NM_H &= 2.4V - 2.0V \\
NM_H &= 0.4V \\
NM_L &= 0.8V - 0.4V \\
NM_L &= 0.4V
\end{aligned} \tag{51}
$$

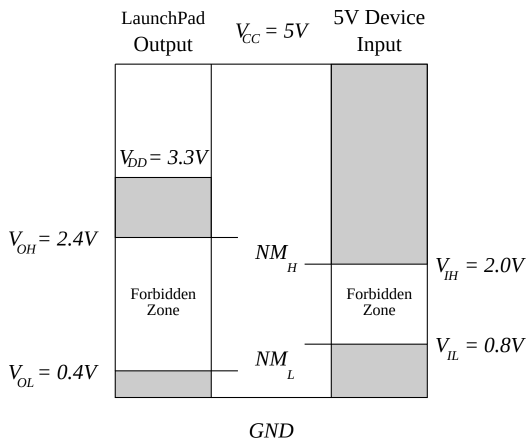

Figure 20: Logic level diagram for LP to 5V.

▸ **QUESTION 30**

Consider the input logic levels in the following table:

Parameter	Parameter Name	Min	Max
V_{CC}	High-level input voltage	-	$3.3V$
V_{IH}	High-level input voltage	$2.0V$	-
V_{IL}	Low-level input voltage	-	$0.8V$

Figure 21: Input logic levels for $3.3V$ device.

(a) Compute the noise margins between the LaunchPad output logic levels and this device's input logic levels.

(b) Sketch a logic level diagram with the LaunchPad driving the input of this device.

(c) If the output of the LaunchPad is driving these inputs, are the logic levels compatible?

Estimated time to complete this question is 10 minutes.

SOLUTION

The answer to (c) is YES—the LaunchPad output logic levels are compatible with the device input logic levels.

$$\begin{aligned} NM_H &= V_{OH} - V_{IH} \\ NM_L &= V_{IL} - V_{OL} \\ NM_H &= 2.4V - 2.0V \\ NM_H &= 0.4V \\ NM_L &= 0.8V - 0.4V \\ NM_L &= 0.4V \end{aligned} \tag{52}$$

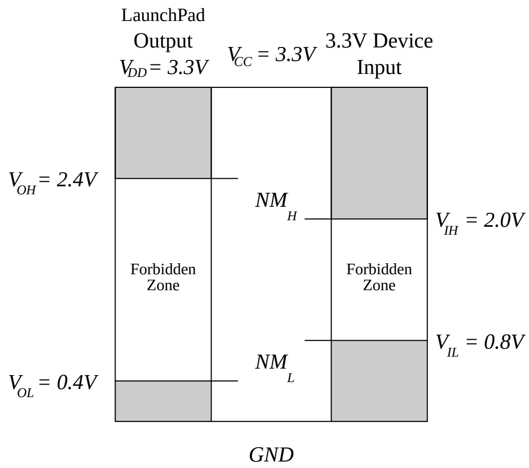

Figure 22: Logic level diagram for LP to 3.3V.

▸ **QUESTION 31**

Consider the input logic levels in the following table:

Parameter	Parameter Name	Min	Max
V_{CC}	High-level input voltage	-	$2.5V$
V_{IH}	High-level input voltage	$1.7V$	-
V_{IL}	Low-level input voltage	-	$0.7V$

Figure 23: Input logic levels for $2.5V$ device.

(a) Compute the noise margins between the LaunchPad output logic levels and this device's input logic levels.

(b) Sketch a logic level diagram with the LaunchPad driving the input of this device.

(c) If the output of the LaunchPad is driving these inputs, are the logic levels compatible? Assume the inputs are at least $3.3V$ tolerant.

Estimated time to complete this question is 10 minutes.

SOLUTION

The answer to (c) is YES—the LaunchPad output logic levels are compatible with the device input logic levels.

$$\begin{aligned} NM_H &= V_{OH} - V_{IH} \\ NM_L &= V_{IL} - V_{OL} \\ NM_H &= 2.4V - 1.7V \\ NM_H &= 0.7V \\ NM_L &= 0.7V - 0.4V \\ NM_L &= 0.3V \end{aligned} \tag{53}$$

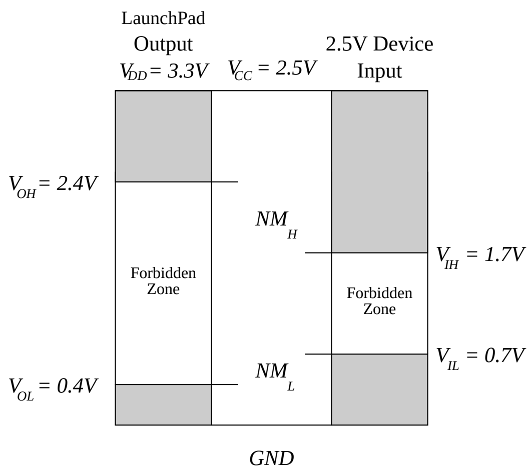

Figure 24: Logic level diagram for LP to 2.5V.

▸ **QUESTION 32**

Consider the input logic levels in the following table:

Parameter	Parameter Name	Min	Max
V_{CC}	High-level input voltage	-	$1.8V$
V_{IH}	High-level input voltage	$1.17V$	-
V_{IL}	Low-level input voltage	-	$0.63V$

Figure 25: Input logic levels for $1.8V$ device.

(a) Compute the noise margins between the LaunchPad output logic levels and this device's input logic levels.

(b) Sketch a logic level diagram with the LaunchPad driving the input of this device.

(c) If the output of the LaunchPad is driving these inputs, are the logic levels compatible?
Assume the inputs are at least $3.3V$ tolerant.

Estimated time to complete this question is 10 minutes.

SOLUTION
The answer to (c) is YES—the LaunchPad output logic levels are compatible with the device input logic levels.

$$
\begin{aligned}
NM_H &= V_{OH} - V_{IH} \\
NM_L &= V_{IL} - V_{OL} \\
NM_H &= 2.4V - 1.17V \\
NM_H &= 1.23V \\
NM_L &= 0.63V - 0.4V \\
NM_L &= 0.23V
\end{aligned}
\tag{54}
$$

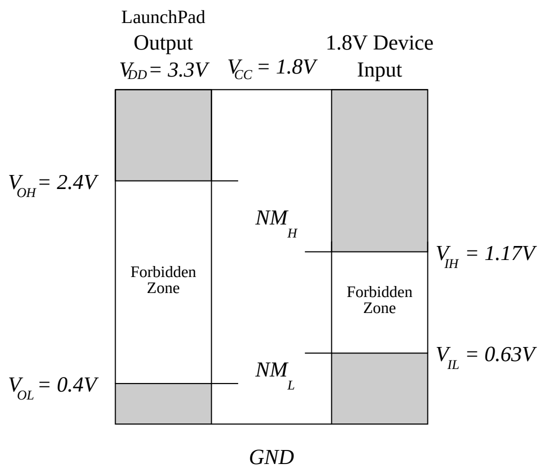

Figure 26: Logic level diagram for LP to 1.8V.

QUESTION 33
Consider the input logic levels in the following table:

Parameter	Parameter Name	Min	Max
V_{CC}	High-level input voltage	-	$1.5V$
V_{IH}	High-level input voltage	$0.98V$	-
V_{IL}	Low-level input voltage	$0V$	$0.53V$

Figure 27: Input logic levels for $1.5V$ device.

(a) Compute the noise margins between the LaunchPad output logic levels and this device's input logic levels.

(b) Sketch a logic level diagram with the LaunchPad driving the input of this device.

(c) If the output of the LaunchPad is driving these inputs, are the logic levels compatible?

Assume the inputs are at least $3.3V$ tolerant.

Estimated time to complete this question is 10 minutes.

SOLUTION

The answer to (c) is YES—the LaunchPad output logic levels are compatible with the device input logic levels.

$$
\begin{aligned}
NM_H &= V_{OH} - V_{IH} \\
NM_L &= V_{IL} - V_{OL} \\
NM_H &= 2.4V - 0.98V \\
NM_H &= 1.42V \\
NM_L &= 0.53V - 0.4V \\
NM_L &= 0.13V
\end{aligned}
\tag{55}
$$

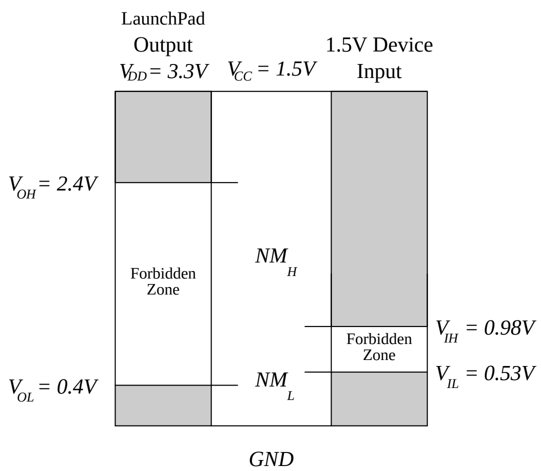

Figure 28: Logic level diagram for LP to 1.5V.

▸ QUESTION 34

Convert the following octal numbers to hexadecimal. Use the fact that each octal or hexadecimal digit corresponds to 3–bit or 4–bit, respectively, binary numbers.

(a) 0354

(b) 0177

Estimated time to complete this question is 5 minutes.

SOLUTION

(a) 0354
Convert each octal digit by inspection to 3–bit binary:

$$0354 = 011.101.100$$

Now rearrange the bits into 4–bit groups that represent each hexadecimal digit.

$$\begin{aligned} 0354 &= 0.1110.1100 \\ &= 0\text{xE} \quad 0\text{xC} \end{aligned} \tag{56}$$

Combine the hexadecimal digits: 0xEC.
(b) 0177
Convert each octal digit by inspection to 3–bit binary:

$$0354 = 001.111.111$$

Now rearrange the bits into 4–bit groups that represent each hexadecimal digit.

$$\begin{aligned} 0354 &= 0.0111.1111 \\ &= 0\text{x}7 \quad 0\text{xF} \end{aligned} \tag{57}$$

Combine the hexadecimal digits: 0x7F.

▸ **QUESTION 35**
Convert the following octal numbers to hexadecimal. Use the fact that each octal or hexadecimal digit corresponds to 3–bit or 4–bit, respectively, binary numbers.

(a) 0673

(b) 0172

Estimated time to complete this question is 5 minutes.

SOLUTION

(a) 0673
Convert each octal digit by inspection to 3–bit binary:

$$0673 = 110.111.011$$

Now rearrange the bits into 4–bit groups that represent each hexadecimal digit.

$$\begin{aligned} 0673 &= 0001.1011.1011 \\ &= 0\text{x}1 \quad 0\text{xB} \quad 0\text{xB} \end{aligned} \tag{58}$$

Combine the hexadecimal digits: 0x1BB.
(b) 0172
Convert each octal digit by inspection to 3–bit binary:

$$0172 = 001.111.010$$

Now rearrange the bits into 4–bit groups that represent each hexadecimal digit.

$$\begin{aligned} 0172 &= 0.0111.1010 \\ &= 0\text{x}7 \quad 0\text{xA} \end{aligned} \tag{59}$$

Combine the hexadecimal digits: 0x7A.

▸ **QUESTION 36**

Convert the following octal numbers to hexadecimal. Use the fact that each octal or hexadecimal digit corresponds to 3–bit or 4–bit, respectively, binary numbers.

(a) 0162

(b) 0426

Estimated time to complete this question is 5 minutes.

SOLUTION

(*a*) 0162
Convert each octal digit by inspection to 3–bit binary:

$$0162 = 001.110.010$$

Now rearrange the bits into 4–bit groups that represent each hexadecimal digit.

$$\begin{aligned} 0162 &= 0.0111.0010 \\ &= 0x7 \quad 0x2 \end{aligned} \tag{60}$$

Combine the hexadecimal digits: 0x72.
(*b*) 0426
Convert each octal digit by inspection to 3–bit binary:

$$0426 = 100.010.110$$

Now rearrange the bits into 4–bit groups that represent each hexadecimal digit.

$$\begin{aligned} 0426 &= 0001.0001.0110 \\ &= 0x1 \quad 0x1 \quad 0x6 \end{aligned} \tag{61}$$

Combine the hexadecimal digits: 0x116

▸ **QUESTION 37**

Convert the following octal numbers to hexadecimal. Use the fact that each octal or hexadecimal digit corresponds to 3–bit or 4–bit, respectively, binary numbers.

(a) 0272

(b) 0735

Estimated time to complete this question is 5 minutes.

SOLUTION

(*a*) 0272
Convert each octal digit by inspection to 3–bit binary:

$$0272 = 010.111.010$$

Now rearrange the bits into 4–bit groups that represent each hexadecimal digit.

$$\begin{aligned} 0272 &= 0.1011.1010 \\ &= 0xB \quad 0xA \end{aligned} \tag{62}$$

Combine the hexadecimal digits: 0xBA.
(b) 0735
Convert each octal digit by inspection to 3–bit binary:

$$0735 = 111.011.101$$

Now rearrange the bits into 4–bit groups that represent each hexadecimal digit.

$$\begin{aligned} 0735 &= 00001.1101.1101 \\ &= 0x1 \quad 0xD \quad 0xD \end{aligned} \tag{63}$$

Combine the hexadecimal digits: 0x1DD.

▸ QUESTION 38

Use perfect induction to prove:

$$(B \bullet C) + (B \bullet \overline{C}) = B$$

Estimated time to complete this question is 5 minutes.

SOLUTION

B	C	$B\,C$	$B\,\overline{C}$	$B\,C + B\,\overline{C}$
0	0	0	0	0
0	1	0	0	0
1	0	0	1	1
1	1	1	0	1

Figure 29: Truth table for $(B \bullet C) + (B \bullet \overline{C}) = B$.

▸ QUESTION 39

Use perfect induction to prove:

$$B \bullet (B + C) = B$$

Estimated time to complete this question is 5 minutes.

B	C	$B + C$	$B \bullet (B + C)$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

Figure 30: Truth table for $B \bullet (B + C) = B$.

▸ **QUESTION 40**

Use perfect induction to prove:

$$B + \overline{B}\,C = B + C$$

Estimated time to complete this question is 5 minutes.

B	C	$\overline{B}\,C$	$B + \overline{B}\,C$	$B + C$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

Figure 31: Truth table for $B + \overline{B}\,C = B + C$.

▸ **QUESTION 41**

Use perfect induction to prove:

$$\overline{B\,C} = \overline{B} + \overline{C}$$

Estimated time to complete this question is 5 minutes.

B	C	$\overline{B\,C}$	$\overline{B} + \overline{C}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Figure 32: Truth table for $\overline{B\,C} = \overline{B} + \overline{C}$.

▸ **QUESTION 42**

Use perfect induction to prove:

$$\overline{B + C} = \overline{B} \bullet \overline{C}$$

Estimated time to complete this question is 5 minutes.

B	C	$\overline{B + C}$	$\overline{B} \bullet \overline{C}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Figure 33: Truth table for $\overline{B\,C} = \overline{B} + \overline{C}$.