EXERCISE 2.2 Write a Boolean equation in sum-of-products canonical form for each of the truth tables in Figure 2.81. In this document, see Figure 1.

(a)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(b)

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

(c)

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

(d)

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

(e)

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Figure 1: Truth tables for Exercises 2.2 and 2.4

SOLUTION In general, since we are using sum-of-products form, create the minterms for each row of the truth table. Each minterm is a product (AND) of the inputs which is TRUE for that row. The Boolean equation is the sum (OR) of each minterm.

SOLUTION (a) Create the minterms for each row of the truth table in Figure 1 (a).

A	B	Y	minterm	name
0	0	0	$\overline{A}\overline{B}$	m_0
0	1	1	$\overline{A}B$	m_1
1	0	1	$A\overline{B}$	m_2
1	1	1	AB	m_3

The Boolean equation in sum-of-products form is the sum of the minterms for which the output Y is $TRUE$.

This yields:

$$Y = m_1 + m_2 + m_3$$
$$Y = \overline{A}B + A\overline{B} + AB$$

SOLUTION (b) Create minterms for each row of the truth table in Figure 1 (b).

A	B	C	Y	minterm	name
0	0	0	0	$\overline{A}\,\overline{B}\,\overline{C}$	m_0
0	0	1	1	$\overline{A}\,\overline{B}\,C$	m_1
0	1	0	1	$\overline{A}\,B\,\overline{C}$	m_2
0	1	1	1	$\overline{A}\,B\,C$	m_3
1	0	0	1	$A\,\overline{B}\,\overline{C}$	m_4
1	0	1	0	$A\,\overline{B}\,C$	m_5
1	1	0	1	$A\,B\,\overline{C}$	m_6
1	1	1	0	$A\,B\,C$	m_7

The Boolean equation is the sum of each minterm for which Y is TRUE.

$$Y = m_1 + m_2 + m_3 + m_4 + m_6$$
$$Y = \overline{A}\,\overline{B}\,C + \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$

SOLUTION (c) Create minterms for each row of the truth table in Figure 1 (c).

A	B	C	Y	minterm	name
0	0	0	0	$\overline{A}\,\overline{B}\,\overline{C}$	m_0
0	0	1	1	$\overline{A}\,\overline{B}\,C$	m_1
0	1	0	0	$\overline{A}\,B\,\overline{C}$	m_2
0	1	1	0	$\overline{A}\,B\,C$	m_3
1	0	0	0	$A\,\overline{B}\,\overline{C}$	m_4
1	0	1	0	$A\,\overline{B}\,C$	m_5
1	1	0	1	$A\,B\,\overline{C}$	m_6
1	1	1	1	$A\,B\,C$	m_7

The Boolean equation is the sum of each minterm for which Y is TRUE.

$$Y = m_1 + m_6 + m_7$$
$$Y = \overline{A}\,\overline{B}\,C + A\,B\,\overline{C} + A\,B\,C$$

SOLUTION (d) Create minterms for each row of the truth table in Figure 1 (d).

A	B	C	D	Y	minterm	name
0	0	0	0	1	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	m_0
0	0	0	1	0	$\overline{A}\,\overline{B}\,\overline{C}\,D$	m_1
0	0	1	0	1	$\overline{A}\,\overline{B}\,C\,\overline{D}$	m_2
0	0	1	1	1	$\overline{A}\,\overline{B}\,C\,D$	m_3
0	1	0	0	0	$\overline{A}\,B\,\overline{C}\,\overline{D}$	m_4
0	1	0	1	0	$\overline{A}\,B\,\overline{C}\,D$	m_5
0	1	1	0	1	$\overline{A}\,B\,C\,\overline{D}$	m_6
0	1	1	1	1	$\overline{A}\,B\,C\,D$	m_7
1	0	0	0	1	$A\,\overline{B}\,\overline{C}\,\overline{D}$	m_8
1	0	0	1	0	$A\,\overline{B}\,\overline{C}\,D$	m_9
1	0	1	0	1	$A\,\overline{B}\,C\,\overline{D}$	m_{10}
1	0	1	1	0	$A\,\overline{B}\,C\,D$	m_{11}
1	1	0	0	0	$A\,B\,\overline{C}\,\overline{D}$	m_{12}
1	1	0	1	0	$A\,B\,\overline{C}\,D$	m_{13}
1	1	1	0	0	$A\,B\,C\,\overline{D}$	m_{14}
1	1	1	1	0	$A\,B\,C\,D$	m_{15}

The Boolean equation is the sum of each minterm for which Y is TRUE.

$$Y = m_0 + m_2 + m_3 + m_6 + m_7 + m_8 + m_{10}$$
$$Y = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,C\overline{D} + \overline{A}\,\overline{B}\,C\,D + \overline{A}\,B\,C\,\overline{D}$$
$$+ \overline{A}\,B\,C\,D + A\,\overline{B}\,\overline{C}\,\overline{D} + A\,\overline{B}\,C\,\overline{D}$$

SOLUTION (e) Create minterms for each row of the truth table in Figure 1 (e).

A	B	C	D	Y	minterm	name
0	0	0	0	0	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	m_0
0	0	0	1	0	$\overline{A}\,\overline{B}\,\overline{C}\,D$	m_1
0	0	1	0	0	$\overline{A}\,\overline{B}\,C\,\overline{D}$	m_2
0	0	1	1	1	$\overline{A}\,\overline{B}\,C\,D$	m_3
0	1	0	0	0	$\overline{A}\,B\,\overline{C}\,\overline{D}$	m_4
0	1	0	1	0	$\overline{A}\,B\,\overline{C}\,D$	m_5
0	1	1	0	1	$\overline{A}\,B\,C\,\overline{D}$	m_6
0	1	1	1	1	$\overline{A}\,B\,C\,D$	m_7
1	0	0	0	1	$A\,\overline{B}\,\overline{C}\,\overline{D}$	m_8
1	0	0	1	1	$A\,\overline{B}\,\overline{C}\,D$	m_9
1	0	1	0	1	$A\,\overline{B}\,C\,\overline{D}$	m_{10}
1	0	1	1	1	$A\,\overline{B}\,C\,D$	m_{11}
1	1	0	0	0	$A\,B\,\overline{C}\,\overline{D}$	m_{12}
1	1	0	1	0	$A\,B\,\overline{C}\,D$	m_{13}
1	1	1	0	0	$A\,B\,C\,\overline{D}$	m_{14}
1	1	1	1	0	$A\,B\,C\,D$	m_{15}

The Boolean equation is the sum of each minterm for which Y is TRUE.

$$
\begin{aligned}
Y &= m_3 + m_6 + m_7 + m_8 + m_9 + m_{10} + m_{11} \\
Y &= \overline{A}\,\overline{B}\,C\,D + \overline{A}\,B\,C\overline{D} + \overline{A}\,B\,C\,D + A\,\overline{B}\,\overline{C}\,\overline{D} \\
&\quad + A\,\overline{B}\,\overline{C}\,D + A\,\overline{B}\,C\,\overline{D} + A\,\overline{B}\,C\,D
\end{aligned}
$$

EXERCISE 2.4 Write a Boolean equation in product-of-sums canonical form for each of the truth tables in Figure 2.81. In this document, see Figure 1.

SOLUTION In general, since we are using product-of-sums form, create the maxterms for each row of the truth table. Each maxterm is a sum (OR) of the inputs which is FALSE for that row. The Boolean equation is the product (AND) of each maxterm.

SOLUTION (a) Create the maxterms for each row of the truth table in Figure 1 (a).

A	B	Y	maxterm	name
0	0	0	$A + B$	M_0
0	1	1	$A + \overline{B}$	M_1
1	0	1	$\overline{A} + B$	M_2
1	1	1	$\overline{A} + \overline{B}$	M_3

The Boolean equation in product-of-sums form is the product of the maxterms for which the output Y is FALSE.

This yields:

$$Y = M_0$$
$$Y = A + B$$

SOLUTION (b) Create maxterms for each row of the truth table in Figure 1 (b).

A	B	C	Y	minterm	name
0	0	0	0	$A+B+C$	M_0
0	0	1	1	$A+B+\overline{C}$	M_1
0	1	0	0	$A+\overline{B}+C$	M_2
0	1	1	1	$A+\overline{B}+\overline{C}$	M_3
1	0	0	1	$\overline{A}+B+C$	M_4
1	0	1	1	$\overline{A}+B+\overline{C}$	M_5
1	1	0	1	$\overline{A}+\overline{B}+C$	M_6
1	1	1	0	$\overline{A}+\overline{B}+\overline{C}$	M_7

The Boolean equation is the product of each maxterm for which Y is FALSE.

$$Y = M_0 \bullet M_5 \bullet M_7$$
$$Y = (A+B+C) \bullet (\overline{A}+B+\overline{C}) \bullet (\overline{A}+\overline{B}+\overline{C})$$

SOLUTION (c) Create maxterms for each row of the truth table in Figure 1 (c).

A	B	C	Y	minterm	name
0	0	0	0	$A+B+C$	M_0
0	0	1	1	$A+B+\overline{C}$	M_1
0	1	0	0	$A+\overline{B}+C$	M_2
0	1	1	0	$A+\overline{B}+\overline{C}$	M_3
1	0	0	0	$\overline{A}+B+C$	M_4
1	0	1	0	$\overline{A}+B+\overline{C}$	M_5
1	1	0	1	$\overline{A}+\overline{B}+C$	M_6
1	1	1	1	$\overline{A}+\overline{B}+\overline{C}$	M_7

The Boolean equation is the product of each maxterm for which Y is FALSE.

$$Y = M_0 \bullet M_2 \bullet M_3 \bullet M_4 \bullet M_5$$
$$Y = (A+B+C) \bullet (A+\overline{B}+C) \bullet (A+\overline{B}+\overline{C}) \bullet (\overline{A}+B+C) \bullet (\overline{A}+B+\overline{C})$$

SOLUTION (e) Create maxterms for each row of the truth table in Figure 1 (d).

A	B	C	D	Y	minterm	name
0	0	0	0	1	$A+B+C+D$	M_0
0	0	0	1	0	$A+B+C+\overline{D}$	M_1
0	0	1	0	1	$A+B+\overline{C}+D$	M_2
0	0	1	1	1	$A+B+\overline{C}+\overline{D}$	M_3
0	1	0	0	0	$A+\overline{B}+C+D$	M_4
0	1	0	1	0	$A+\overline{B}+C+\overline{D}$	M_5
0	1	1	0	1	$A+\overline{B}+\overline{C}+D$	M_6
0	1	1	1	1	$A+\overline{B}+\overline{C}+\overline{D}$	M_7
1	0	0	0	1	$\overline{A}+B+C+D$	M_8
1	0	0	1	0	$\overline{A}+B+C+\overline{D}$	M_9
1	0	1	0	1	$\overline{A}+B+\overline{C}+D$	M_{10}
1	0	1	1	0	$\overline{A}+B+\overline{C}+\overline{D}$	M_{11}
1	1	0	0	0	$\overline{A}+\overline{B}+C+D$	M_{12}
1	1	0	1	0	$\overline{A}+\overline{B}+C+\overline{D}$	M_{13}
1	1	1	0	0	$\overline{A}+\overline{B}+\overline{C}+D$	M_{14}
1	1	1	1	0	$\overline{A}+\overline{B}+\overline{C}+\overline{D}$	M_{15}

The Boolean equation is the product of each maxterm for which Y is FALSE.

$$Y = M_1 \bullet M_4 \bullet M_5 \bullet M_9 \bullet M_{11} \bullet M_{12} \bullet M_{13} \bullet M_{14} \bullet M_{15}$$

$$\begin{aligned} Y = {} & (A+B+C+\overline{D}) \bullet (A+\overline{B}+C+D) \bullet (A+\overline{B}+C+\overline{D}) \bullet (\overline{A}+B+C+\overline{D}) \bullet \\ & (\overline{A}+B+\overline{C}+\overline{D}) \bullet (\overline{A}+\overline{B}+C+D) \bullet (\overline{A}+\overline{B}+C+\overline{D}) \bullet (\overline{A}+\overline{B}+\overline{C}+D) \bullet \\ & (\overline{A}+\overline{B}+\overline{C}+\overline{D}) \end{aligned}$$

SOLUTION (e) Create maxterms for each row of the truth table in Figure 1 (e).

A	B	C	D	Y	minterm	name
0	0	0	0	0	$A+B+C+D$	M_0
0	0	0	1	0	$A+B+C+\overline{D}$	M_1
0	0	1	0	0	$A+B+\overline{C}+D$	M_2
0	0	1	1	1	$A+B+\overline{C}+\overline{D}$	M_3
0	1	0	0	0	$A+\overline{B}+C+D$	M_4
0	1	0	1	0	$A+\overline{B}+C+\overline{D}$	M_5
0	1	1	0	1	$A+\overline{B}+\overline{C}+D$	M_6
0	1	1	1	1	$A+\overline{B}+\overline{C}+\overline{D}$	M_7
1	0	0	0	1	$\overline{A}+B+C+D$	M_8
1	0	0	1	1	$\overline{A}+B+C+\overline{D}$	M_9
1	0	1	0	1	$\overline{A}+B+\overline{C}+D$	M_{10}
1	0	1	1	1	$\overline{A}+B+\overline{C}+\overline{D}$	M_{11}
1	1	0	0	0	$\overline{A}+\overline{B}+C+D$	M_{12}
1	1	0	1	0	$\overline{A}+\overline{B}+C+\overline{D}$	M_{13}
1	1	1	0	0	$\overline{A}+\overline{B}+\overline{C}+D$	M_{14}
1	1	1	1	0	$\overline{A}+\overline{B}+\overline{C}+\overline{D}$	M_{15}

The Boolean equation is the product of each maxterm for which Y is FALSE.

$$Y = M_0 \bullet M_1 \bullet M_2 \bullet M_4 \bullet M_5 \bullet M_{12} \bullet M_{13} \bullet M_{14} \bullet M_{15}$$

$$\begin{aligned} Y = {} & (A + B + C + D) \bullet (A + B + C + \overline{D}) \bullet (A + B + \overline{C} + D) \bullet (A + \overline{B} + C + D) \bullet \\ & (A + \overline{B} + C + \overline{D}) \bullet (\overline{A} + \overline{B} + C + D) \bullet (\overline{A} + \overline{B} + C + \overline{D}) \bullet (\overline{A} + \overline{B} + \overline{C} + D) \bullet \\ & (\overline{A} + \overline{B} + \overline{C} + \overline{D}) \end{aligned}$$

EXERCISE 2.6 Minimize each of the Boolean equations from Exercise 2.2.

SOLUTION In general, cleverly apply the Axioms of Boolean Algebra and the Boolean theorems of one and several variables to simplify the canonical sum-of-product.Boolean equations created in Exercise 2.2.

SOLUTION (a) Minimize the equation from Exercise 2.2 (a):

$$Y = \overline{A}\,B + A\,\overline{B} + A\,B$$

We can see from the truth table in Figure 1 (a) that the Boolean function is an OR so we know that the result must be

$$Y = A + B$$

Restate the equation:

$$\begin{aligned} Y &= \overline{A}\,B + A\,\overline{B} + A\,B \\ &= \overline{A}\,B + (A\,B + A\,\overline{B}) \end{aligned}$$

Here we can apply T10:

$$Y = \overline{A}\,B + A$$

But, since T10 is merely an application of theorems T8 and T5', we can also do this:

$$\begin{aligned} Y &= \overline{A}\,B + (A\,B + A\,\overline{B}) \\ &= \overline{A}\,B + A \bullet (\overline{B} + B) \\ &= \overline{A}\,B + A \bullet 1 \\ Y &= \overline{A}\,B + A \end{aligned}$$

In either approach we find the same result. Applying theorem T6':

$$Y = A + \overline{A}\,B$$

But we know that the Boolean function is just $A + B$ so this is not minimized yet. We can use the expanding trick and apply theorems T2', T1, and T10, Null Element, Identity, and Combining:

$$\begin{aligned} Y &= A \bullet (B + 1) + \overline{A}\,B \\ &= A\,B + A + \overline{A}\,B \\ &= A + A\,B + \overline{A}\,B \\ &= A + (B\,A + B\,\overline{A}) \\ Y &= A + B \end{aligned}$$

Now the Boolean equation is fully minimized.

Note this exercise. We will refer to is in later exercises that contain expressions of the form $\overline{A}\,B + A\,\overline{B} + A\,B$. This can be expressed as our own personal theorem:

$$A\,B + A\,\overline{B} + \overline{A}\,B = A + B$$

A variation on this may also be useful to reference later:

$$A + \overline{A}\,B = A + B$$

SOLUTION (b) Minimize the equation from Exercise 2.2 (b):

$$Y = \overline{A}\,\overline{B}\,C + \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$

Begin by rewriting the equation. Factor some common literals:

$$\begin{aligned} Y &= \overline{A} \bullet (\overline{B}\,C + B\,\overline{C} + B\,C) + A \bullet (\overline{B}\,\overline{C} + B\,\overline{C}) \\ &= \overline{A} \bullet (B\,C + B\,\overline{C} + \overline{B}\,C) + A \bullet (\overline{C}\,B + \overline{C}\,\overline{B}) \end{aligned}$$

Recognize that the two terms in parenthesis can be minimized using the expression derived in Exercise 2.6 (a) and theorem T10.

$$\begin{aligned} B\,C + B\,\overline{C} + \overline{B}\,C &= B + C \\ \overline{C}\,B + \overline{C}\,\overline{B} &= \overline{C} \end{aligned}$$

So we can continue by substituting these minimized expressions:

$$\begin{aligned} Y &= \overline{A} \bullet (B + C) + A \bullet (\overline{C}) \\ Y &= \overline{A}\,B + \overline{A}\,C + A\,\overline{C} \end{aligned}$$

There is an alternate solution. We can approach the minimization just a bit differently and reach a different solution that is logically equivalent.

$$\begin{aligned} Y &= \overline{A}\,\overline{B}\,C + \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C} \\ &= \overline{A} \bullet (\overline{B}\,C + B\,C) + A\,\overline{B}\,\overline{C} + B\,\overline{C} \bullet (A + \overline{A}) \end{aligned}$$

We can apply theorems T10 and T5' to the expressions in parenthesis:

$$\begin{aligned} B\,C + \overline{B}\,C &= C \\ A + \overline{A} &= 1 \end{aligned}$$

Continuing. . .

$$\begin{aligned} Y &= \overline{A} \bullet (C) + A\,\overline{B}\,\overline{C} + B\,\overline{C} \bullet (1) \\ &= \overline{A}\,C + A\,\overline{B}\,\overline{C} + B\,\overline{C} \end{aligned}$$

The equation needs an additional term so we can minimize further. Using the Null Element theorem, T2', we can expand the expression:

$$\begin{aligned} Y &= \overline{A}\,C + A\,\overline{B}\,\overline{C} + B\,\overline{C} \bullet (A+1) \\ &= \overline{A}\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C} + B\,\overline{C} \\ &= \overline{A}\,C + A \bullet (\overline{B}\,\overline{C} + B\,\overline{C}) + B\,\overline{C} \end{aligned}$$

Since the Combining theorem T10 tells us that $\overline{B}\,\overline{C} + B\,\overline{C} = \overline{C}$, we can further simplify:

$$\begin{aligned} Y &= \overline{A}\,C + A \bullet (\overline{C}) + B\,\overline{C} \\ Y &= \overline{A}\,C + A\,\overline{C} + B\,\overline{C} \end{aligned}$$

That is the alternate and logically equivalent result.

SOLUTION (c) Minimize the equation from Exercise 2.2 (c):

$$Y = \overline{A}\,\overline{B}\,C + A\,B\,\overline{C} + A\,B\,C$$

Apply theorems T5' and T1, Complements and Identity:

$$\begin{aligned} Y &= \overline{A}\,\overline{B}\,C + A\,B\,\overline{C} + A\,B\,C \\ &= A\,B\,C + A\,B\,\overline{C} + \overline{A}\,\overline{B}\,C \\ &= A\,B \bullet (C + \overline{C}) + \overline{A}\,\overline{B}\,C \\ &= A\,B \bullet (1) + \overline{A}\,\overline{B}\,C \\ Y &= A\,B + \overline{A}\,\overline{B}\,C \end{aligned}$$

SOLUTION (d) Minimize the equation from Exercise 2.2 (d):

$$\begin{aligned} Y = {} & \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,C\,\overline{D} + \overline{A}\,\overline{B}\,C\,D + \overline{A}\,B\,C\,\overline{D} + \overline{A}\,B\,C\,D + A\,\overline{B}\,\overline{C}\,\overline{D} \\ & + A\,\overline{B}\,C\,\overline{D} \end{aligned}$$

Use theorems T6–T8, Commutativity, Associativity, and Distributivity to rewrite the equation:

$$Y = \overline{B}\,\overline{C}\,\overline{D} \bullet (A + \overline{A}) + C\,\overline{D} \bullet (\overline{A}\,B + A\,\overline{B} + \overline{A}\,\overline{B}) + C\,D \bullet (\overline{A}\,\overline{B} + \overline{A}\,B)$$

Use theorems T5', Complements, T10, Combining, and the relationship established in Exercise 2.6 (a):

$$A + \overline{A} = 1$$

$$A\,B + A\,\overline{B} + \overline{A}\,B = A + B$$

$$B\,C + B\overline{C} = B$$

Now simplify the equation:

$$\begin{aligned} Y &= \overline{B}\,\overline{C}\,\overline{D} \bullet (1) + C\,\overline{D}(\overline{A} + \overline{B}) + C\,D \bullet (\overline{A}) \\ &= \overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,C\,\overline{D} + \overline{B}\,C\,\overline{D} + \overline{A}\,C\,D \\ &= \overline{B}\,\overline{C}\,\overline{D} + \overline{B}\,C\,\overline{D} + \overline{A}\,C\,\overline{D} + \overline{A}\,C\,D \end{aligned}$$

Again, use theorems T6–T8, and T5' to rewrite the equation:

$$\begin{aligned} Y &= \overline{B}\,\overline{D} \bullet (C + \overline{C}) + \overline{A}\,C \bullet (D + \overline{D}) \\ &= \overline{B}\,\overline{D} \bullet (1) + \overline{A}\,C \bullet (1) \\ &= \overline{B}\,\overline{D} + \overline{A}\,C \\ Y &= \overline{A}\,C + \overline{B}\,\overline{D} \end{aligned}$$

Undoubtedly there are many ways to approach this. Some may be more difficult that others because of the expressions encountered along the way. Some are more difficult than others to minimize.

Check the result using *perfect induction* against the truth table in Figure 1 (d). Note that this does not necessarily indicate that the result is fully minimized, only that there have been no algebraic erros in arriving at the result.

SOLUTION (e) Minimize the equation from Exercise 2.2 (e):

$$\begin{aligned} Y = {} & \overline{A}\,\overline{B}\,C\,D + \overline{A}\,B\,C\overline{D} + \overline{A}\,B\,C\,D + A\,\overline{B}\,\overline{C}\overline{D} \\ & + A\,\overline{B}\,\overline{C}\,D + A\,\overline{B}\,C\,\overline{D} + A\,\overline{B}\,C\,D \end{aligned}$$

Use theorems T6–T8, Commutativity, Associativity, and Distributivity to rewrite the equation:

$$Y = C\,D \bullet (\overline{A}\,B + \overline{A}\,\overline{B}) + A\,\overline{B} \bullet (\overline{C}\,D + \overline{C}\,\overline{D}) + A\,\overline{B} \bullet (C\,D + C\,\overline{D}) + \overline{A}\,B\,C\,\overline{D}$$

Apply theorems T6–T8 and T10 to rewrite:

$$\begin{aligned} Y &= C\,D \bullet (\overline{A}) + A\,\overline{B} \bullet (\overline{C}) + A\,\overline{B} \bullet (C) + \overline{A}\,B\,C\,\overline{D} \\ &= \overline{A}\,C\,D + A\,\overline{B}\,\overline{C} + A\,\overline{B}\,C + \overline{A}\,B\,C\,\overline{D} \\ &= \overline{A}\,C\,D + \overline{B} \bullet (A\,C + A\,\overline{C}) + \overline{A}\,B\,C\,\overline{D} \\ &= \overline{A}\,C\,D + \overline{B} \bullet (A) + \overline{A}\,B\,C\,\overline{D} \\ &= \overline{A}\,C\,D + A\,\overline{B} + \overline{A}\,B\,C\,\overline{D} \end{aligned}$$

At this point, none of our theorems will further minimize this without expanding an implicant. Use the Null Element theorem (T2') to expand an implicant that will be easy to factor with an additional B literal.

$$\begin{aligned} Y &= \overline{A}\,C\,D \bullet (B + 1) + A\,\overline{B} + \overline{A}\,B\,C\,\overline{D} \\ &= \overline{A}\,B\,C\,D + \overline{A}\,B\,C\,\overline{D} + A\,\overline{B} + \overline{A}\,C\,D \\ &= \overline{A}\,B \bullet (C\,D + C\,\overline{D}) + A\,\overline{B} + \overline{A}\,C\,D \\ &= \overline{A}\,B\,C + A\,\overline{B} + \overline{A}\,C\,D \\ Y &= A\,\overline{B} + \overline{A}\,B\,C + \overline{A}\,C\,D \end{aligned}$$

EXERCISE 2.8 Sketch a reasonably simple combinational circuit implementing each of the functions from Exercise 2.6.

SOLUTION Sketch the combinational logic functions using the rules for arranging inputs and outputs from the text.

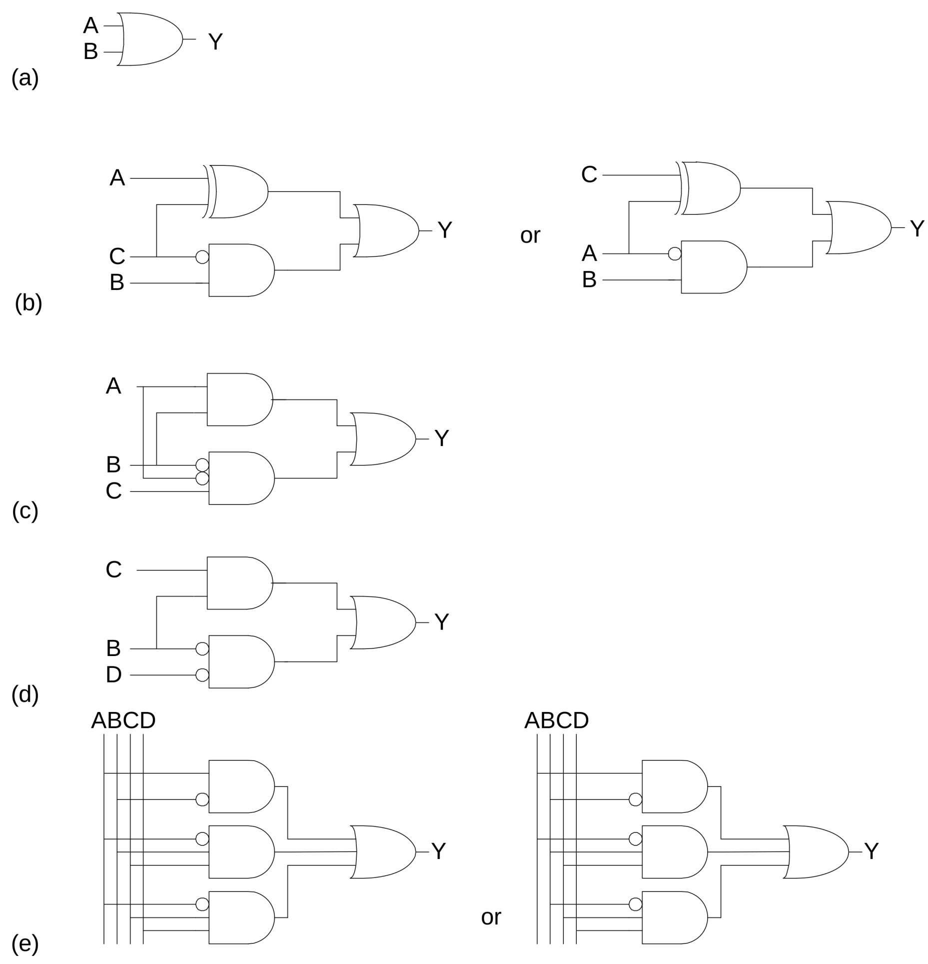

Figure 2: Circuits for functions from Exercise 2.6

EXERCISE 2.14 Simplify the following Boolean equations using Boolean theorems. Check for correctness using a truth table or K-map.

a. $Y = \overline{A}\,B\,C + \overline{A}\,B\,\overline{C}$

b. $Y = \overline{ABC} + A\,\overline{B}$

c. $Y = A\,B\,C\,\overline{D} + A\,\overline{BCD} + (\overline{A + B + C + D})$

SOLUTION (a) Using theorems T6–T8 and T5', rewrite and simpllify the equation.

$$
\begin{aligned}
Y &= \overline{A}\,B\,C + \overline{A}\,B\overline{C} \\
&= \overline{A}\,B \bullet (C + \overline{C}) \\
&= \overline{A}\,B \bullet (1) \\
Y &= \overline{A}\,B
\end{aligned}
$$

The truth table for the original and simplified expression demonstrates that both forms produce the same logical result.

A	B	C	$\overline{A}\,B\,C + \overline{A}\,B\,\overline{C}$	$\overline{A}\,B$
0	0	0	0	0
0	0	1	0	0
0	1	0	1	1
0	1	1	1	1
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	0	0

SOLUTION (b) Using theorems T2', T5', T6–T8, and T12, rewrite and simpllify the equation.

$$
\begin{aligned}
Y &= \overline{A\,B\,C} + A\,\overline{B} \\
&= \overline{A} + \overline{B} + \overline{C} + A\,\overline{B} \\
&= \overline{A} + \overline{C} + (\overline{B} + A\,\overline{B}) \\
&= \overline{A} + \overline{C} + \overline{B} \bullet (1 + A) \\
&= \overline{A} + \overline{C} + \overline{B} \bullet (1) \\
Y &= \overline{A\,B\,C}
\end{aligned}
$$

The truth table below demonstrates correctness of the simplification.

A	B	C	$\overline{A\,B\,C}$	$A\,\overline{B}$	$\overline{A\,B\,C} + A\,\overline{B}$
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	1	0	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	0	0	0

SOLUTION (c) Using theorems T2', T5', T6–T8, and T12, rewrite and simpllify the equation.

$$
\begin{aligned}
Y &= A\,B\,C\,\overline{D} + A\,\overline{BCD} + (\overline{A + B + C + D}) \\
&= A\,B\,C\,\overline{D} + A \bullet (\overline{B} + \overline{C} + \overline{D}) + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} \\
&= A\,B\,C\,\overline{D} + A\,\overline{B} + A\,\overline{C} + A\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} \\
&= A \bullet (B\,C\,\overline{D} + \overline{B} + \overline{C} + \overline{D}) + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} \\
&= A \bullet (\overline{B} + \overline{C} + \overline{D} \bullet (1 + B\,C)) + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} \\
&= A \bullet (\overline{B} + \overline{C} + \overline{D} \bullet (1)) + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} \\
&= A \bullet (\overline{B} + \overline{C} + \overline{D}) + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} \\
&= A\,\overline{B} + A\,\overline{C} + A\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}
\end{aligned}
$$

At this point, we need to simplify the 4th term. This can be accomplished by factoring and expanding before continuing to simplify.

$$
\begin{aligned}
Y &= A\,\overline{B} + A\,\overline{C} + A\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} \\
&= A\,\overline{B} + A\,\overline{C} + \overline{D} \bullet (A + \overline{A}\,\overline{B}\,\overline{C}) \\
&= A\,\overline{B} + A\,\overline{C} + \overline{D} \bullet (A \bullet (\overline{B}\,\overline{C} + 1) + \overline{A}\,\overline{B}\,\overline{C}) \\
&= A\,\overline{B} + A\,\overline{C} + \overline{D} \bullet (A\,\overline{B}\,\overline{C} + A + \overline{A}\,\overline{B}\,\overline{C}) \\
&= A\,\overline{B} + A\,\overline{C} + \overline{D} \bullet (A + \overline{B}\,\overline{C} \bullet (A + \overline{A})) \\
&= A\,\overline{B} + A\,\overline{C} + \overline{D} \bullet (A + \overline{B}\,\overline{C} \bullet (1)) \\
&= A\,\overline{B} + A\,\overline{C} + A\,\overline{D} + \overline{B}\,\overline{C}\,\overline{D} \\
&= A \bullet (\overline{B} + \overline{C} + \overline{D}) + \overline{B}\,\overline{C}\,\overline{D} \\
&= A \bullet (\overline{B\,C\,D}) + \overline{B}\,\overline{C}\,\overline{D} \\
Y &= A\,\overline{B\,C\,D} + \overline{B}\,\overline{C}\,\overline{D}
\end{aligned}
$$

This truth table is simply too large to consider including in this document. I may change my mind. Problem is that `texinfo` tables are so complex to work with—it is just too horrible to contemplate. I may change my mind after completing the rest of the exercises.

EXERCISE 2.16 Sketch a reasonably simple combinational circuit implementing each of the functions from Exercise 2.14.

SOLUTION Sketch the combinational logic functions using the rules for the minimized functions.

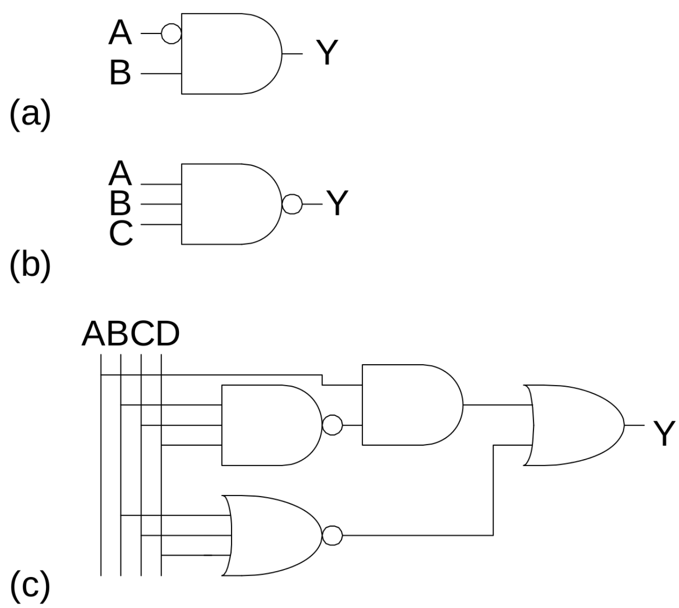

Figure 3: Circuits for functions from Exercise 2.14

EXERCISE 2.22 Prove that the following theorems are true using perfect induction. You need not prove their duals.

a. The idempotency theorem (T3)
b. The distributivity theorem (T8)
c. The combining theorem (T10)

SOLUTION Construct a truth table for each theorem using as many inputs as needed as stated in the text. Fill in the inputs in counting order. Compute the value of each term and then compute the result for each side of the theorem. Verify that they are equivalent.

SOLUTION (a) Theorem T3 Idempotency states:

$$B \bullet B = B$$

The truth table is simple and contains only two rows. This is because there are only two discrete states.

B	B	$B \bullet B$
0	0	0
1	1	1

SOLUTION (b) Theorem T8, Distributivity states:

$$(B \bullet C) + (B \bullet D) = B \bullet (C + D)$$

B	C	D	$B\,C + B\,D$	$B \bullet (C + D)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

SOLUTION (c) Theorem T10, Combining states:

$$(B \bullet C) + (B \bullet \overline{C}) = B$$

The truth table is below.

B	C	$(B \bullet C) + (B \bullet \overline{C})$
0	0	0
0	1	0
1	0	1
1	1	1