Example 1. Compute the probability p that 23 people have different birthdays.

$$p = \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \cdots \times \frac{343}{365} = \frac{365!/(365-23)!}{365^{23}}$$

The probability that at least two people have the same birthday is $1 - p$.

```
-- www.eigenmath.org/example1.txt
"Product method"
p = product(k,1,23,(365-k+1)/365)
float(p)
"Factorial method"
p = 365! / (365 - 23)! / 365^23
float(p)
"Probability of at least one shared birthday"
1.0 - p
```

To run this script, click on the following link.

http://www.eigenmath.org/example1.txt

Then select all text, copy, and paste into the Eigenmath script window. Then click the Run button.
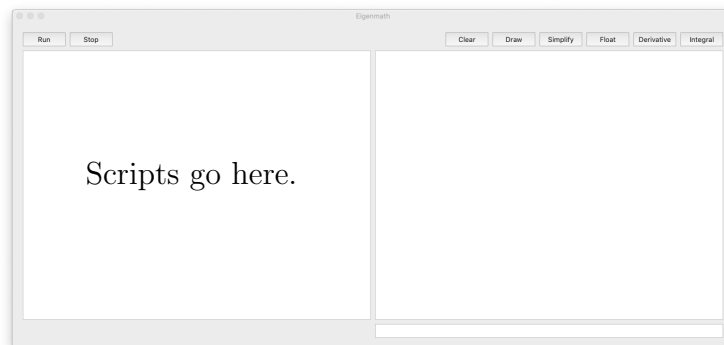
Example 2. Show that
$$\frac{E^2 + m^2 + p^2 \cos\theta}{8p^4} = \frac{1 - \beta^2 \sin^2 \theta/2}{4p^2\beta^2}$$
where $p = \sqrt{E^2 - m^2}$ and $\beta = p/E$. The exponential forms of sine and cosine are used so that trigonometric identities are not needed.

```
-- www.eigenmath.org/example2.txt
p = sqrt(E^2 - m^2)
beta = p/E
A = (E^2 + m^2 + p^2 expcos(theta)) / (8 p^4)
B = (1 - beta^2 expsin(theta/2)^2) / (4 p^2 beta^2)
-- print 1 if A equals B
A == B
```

http://www.eigenmath.org/example2.txt

Example 3. Let

$$\mathbf{E} = \begin{pmatrix} A\sin(kz - \omega t + \phi) \\ 0 \\ 0 \end{pmatrix} \qquad \text{and} \qquad \mathbf{B} = \begin{pmatrix} 0 \\ A\sin(kz - \omega t + \phi) \\ 0 \end{pmatrix}$$

where $k = \omega/c$. Verify that $\mathbf{E}$ and $\mathbf{B}$ are solutions to the free-field Maxwell equations

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} + c^{-1}\frac{\partial}{\partial t}\mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$
$$\nabla \times \mathbf{B} - c^{-1}\frac{\partial}{\partial t}\mathbf{E} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
-- www.eigenmath.org/example3.txt
k = omega/c
E = (A sin(k z - omega t + phi), 0, 0)
B = (0, A sin(k z - omega t + phi), 0)
div(E)
div(B)
curl(E) + d(B,t)/c
curl(B) - d(E,t)/c
```

http://www.eigenmath.org/example3.txt

Example 4. Let
$$\psi = \exp(ik_x x + ik_y y + ik_z z - i\omega t)$$
where
$$\omega = \sqrt{k_x^2 + k_y^2 + k_z^2 + m^2}$$

Verify that ψ is a solution to the Klein-Gordon equation

$$\frac{\partial^2}{\partial t^2}\psi - \frac{\partial^2}{\partial x^2}\psi - \frac{\partial^2}{\partial y^2}\psi - \frac{\partial^2}{\partial z^2}\psi + m^2\psi = 0$$

```
-- www.eigenmath.org/example4.txt
omega = sqrt(kx^2 + ky^2 + kz^2 + m^2)
psi = exp(i kx x + i ky y + i kz z - i omega t)
-- result should be zero
d(psi,t,t) - d(psi,x,x) - d(psi,y,y) - d(psi,z,z) + m^2 psi
```

http://www.eigenmath.org/example4.txt

Example 6. Show that
$$u_1 \bar{u}_1 + u_2 \bar{u}_2 = (E + m)(\not{p} + m)$$

where
$$E = \sqrt{p_x^2 + p_y^2 + p_z^2 + m^2}$$

The spinors are
$$u_1 = \begin{pmatrix} E + m \\ 0 \\ p_z \\ p_x + ip_y \end{pmatrix} \qquad u_2 = \begin{pmatrix} 0 \\ E + m \\ p_x - ip_y \\ -p_z \end{pmatrix}$$

The adjoint of u is $\bar{u} = u^\dagger \gamma^0$. Note that $\bar{u}$ is a row vector hence $u\bar{u}$ is an outer product. This problem demonstrates a trick regarding normalization constants. The normalization constant $E + m$ is placed on the right-hand side of the equation instead of normalizing the spinors with $1/\sqrt{E + m}$. This is because the software is better at simplifying expressions with multipliers instead of divisors.

```
-- www.eigenmath.org/example6.txt
E = sqrt(px^2 + py^2 + pz^2 + m^2)
u1 = (E + m, 0, pz, px + i py)
u2 = (0, E + m, px - i py, -pz)
p = (E, px, py, pz)
I = ((1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1))
gmunu = ((1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1))
gamma0 = ((1,0,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1))
gamma1 = ((0,0,0,1),(0,0,1,0),(0,-1,0,0),(-1,0,0,0))
gamma2 = ((0,0,0,-i),(0,0,i,0),(0,i,0,0),(-i,0,0,0))
gamma3 = ((0,0,1,0),(0,0,0,-1),(-1,0,0,0),(0,1,0,0))
gamma = (gamma0,gamma1,gamma2,gamma3)
pslash = dot(gmunu,p,gamma)
bar(u) = dot(conj(u),gamma0)
A = outer(u1,bar(u1)) + outer(u2,bar(u2))
B = (E + m) (pslash + m I)
-- print 1 if A equals B
A == B
```

http://www.eigenmath.org/example6.txt