Let $\hat{\psi}$ be a wavefunction operator such that

$$\hat{\psi}|a\rangle = \psi(x,y)|0\rangle$$

Note that

$$\langle 0|\hat{\psi}|a\rangle = \psi(x,y)$$

and consequently

$$\langle a|\hat{\psi}^\dagger|0\rangle = \psi^*(x,y)$$

It follows that

$$\langle E\rangle = \int \langle a|\hat{\psi}^\dagger|0\rangle \hat{H} \langle 0|\hat{\psi}|a\rangle \, dx \, dy$$

Let $\hat{E}$ be the operator

$$\hat{E} = \int \hat{\psi}^\dagger|0\rangle \hat{H} \langle 0|\hat{\psi} \, dx \, dy$$

Then

$$\langle E\rangle = \langle a|\hat{E}|a\rangle$$

Let $|\xi\rangle$ be a linear combination of state vectors.

$$|\xi\rangle = \sum_k c_k|k\rangle, \qquad \langle\xi|\xi\rangle = 1$$

The expected energy can be determined by matrix multiplication.

$$\langle E\rangle = \langle\xi|\hat{E}|\xi\rangle = \sum_k |c_k|^2 \langle E_k\rangle$$