Let $\psi(x,y)$ be the antisymmetrized wave function for two electrons in a box of width L.

$$\psi(x,y) = \frac{1}{\sqrt{2}}\big(\phi_1(x)\phi_2(y) - \phi_1(y)\phi_2(x)\big)$$

$$\phi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

For $L = 10^{-9}$ meter the expected potential energy is

$$V = \frac{e^2}{4\pi\epsilon_0}\int_0^L\int_0^L \frac{\psi^*(x,y)\psi(x,y)}{|x-y|}\,dx\,dy = 4.67\,\mathrm{eV}$$

Next calculate the potential energy for a wave function that is not antisymmetrized.

$$V_0 = \frac{e^2}{4\pi\epsilon_0}\int_0^L\int_0^L \frac{\phi_1^*(x)\phi_2^*(y)\phi_1(x)\phi_2(y)}{|x-y|}\,dx\,dy = 12.80\,\mathrm{eV}$$

The difference is the exchange energy.

$$V_{ex} = V - V_0 = -8.13\,\mathrm{eV}$$

Note that the formula for V_0 has a singularity at $x = y$. The computed value shown above is the result of an arbitrary cutoff in numerical integration. The real value of V_0 goes to infinity.

Note also that there is a singularity at $x = y$ in the formula for V. However, due to antisymmetry we have $\psi(x,x) = 0$ and hence the integral converges.

We are left to ponder the reality of exchange energy since it cannot really be computed.