The Lifeguard Problem through a Monte Carlo Simulation

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Abstract: A Monte Carlo simulation is a predictive tool used when numerical solutions are expensive to produce or to assist human intuition. This report uses these methods to solve The Lifeguard Problem. That is, where should the lifeguard enter the water to reach the drowning child fastest: as (s)he is faster on-land than in the water. Through ten simulations the results varied from the optimum ($178m$) by $\pm 9m$ with one simulation resulting the optimum. Future work will include stock price forecasting.

Key Contents: Monte Carlo Simulations, The Lifeguard Problem, The_Lifeguard_Problem.py, Optimisation, python3.

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1. Introduction

A Monte Carlo simulation, or applied method is a computational algorithm used to estimate the potential outcomes of randomly evolving events. Developed by John von Neumann and Stanislaw Ulam when working on the Manhattan Project to improve decision making under uncertainty **[1]**. It was named after the city of Monte Carlo, Monaco: named aptly due to the casinos in Monte Carlo which each introduce elements of chance and randomness.

Instances in which numerical solutions are unnecessarily difficult or computationally expensive to find presents the case for the Monte Carlo method to be used. Furthermore, the application in forecast modelling, risk assessment and so forth allows for a larger degree of accuracy as opposed to human intuition, or to assist with human intuition **[1]**.

A Monte Carlo simulation builds a model of possible results by leveraging random sampling for any factor with inherent uncertainty. This process is then repeated for each simulation, recalculating and each time using a different set of random samples between the set bounds.

2. Methodology

Introducing Monte Carlo methods varies from case to case but generally follows a particular pattern;

1. Define the domain. Identify the dependant variable/s, to be predicted. Identify the independent variable/s to drive the prediction.
2. Generate inputs randomly from a probability distribution over the independent variable/s. Historical data or subjective judgement can be used to define the bounds of likely values.
3. Run the simulations and aggregate the results until the data is representative of the near infinite number of results. The data set can then be analysed **[1]**.

3. The Lifeguard Problem

The Lifeguard Problem represents an optimization problem. That is, determining the optimum position for the lifeguard to enter the water (denoted 'a' in Figure 1) as the velocity of the Lifeguard on land is greater than that in the water.

Posing the question; *A lifeguard swims at a rate of 3m/s but can run at a rate of 7.5m/s. (s)He spots a drowning child 200m down the shore and 50m out to sea. How far down the shore should the lifeguard run before swimming? See Figure 1.*

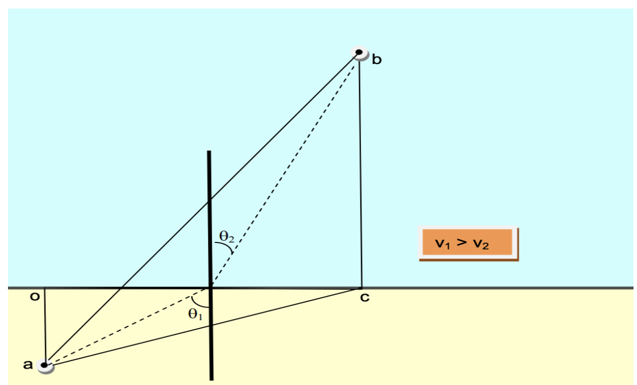

Figure 1: The Lifeguard Problem where *a* represents the starting point of the lifeguard and *b* is the point of the drowning child. **[1]**

Solving analytically:

$$Time_{Total} = Time_{beach} + Time_{water} \tag{1}$$

Using,

$$Time = \frac{Distance}{Speed} \tag{2}$$

$$Time_{beach} = \frac{x[m]}{7.5[ms^{-1}]} \tag{3}$$

$$Time_{water} = \frac{\sqrt{50^2[m] + (200 - x)^2[m]}}{3[ms^{-1}]} \tag{4}$$

Equation 1 becomes,

$$Time_{Total} = \frac{x[m]}{7.5[ms^{-1}]} + \frac{\sqrt{50^2[m] + (200 - x)^2[m]}}{3[ms^{-1}]} \tag{5}$$

Taking the derivative of Eq.5 with respect to x yields:

$$Time_{Total} = 0.1\overline{33}[ms^{-1}] - \frac{200[m] - x[m]}{3[ms^{-1}]\sqrt{(200[m] - x[m])^2 + 2500[m]}} \tag{6}$$

NB: Computed using Wolfram Alpha.

Setting the right-hand side of Eq.6 to 0 and solving for x is both complex and computationally expensive. Therefore, the Monte Carlo method is used to solve for the optimal x value.

NB: Finding the derivative of the function with respect to x (distance along the shore the lifeguard enters the water) is a measure of the rate of change $Time_{Total}$ with respect to the rate of change of x. That is, as we increase the distance we enter the water what effect does this have on the total time to reach the swimmer.

The following refers to the code file: *The_Lifeguard_Problem.py* and is written in *python3*.

The first step of building the Monte Carlo simulation introduces the domain. The dependant variable is identified as the time at which it takes the lifeguard to reach the swimmer ($Time_{Total}$). The independent variable is identified as the distance along the shore at the point the lifeguard enters the water (x).

As can be seen in Figure 2 the probability distribution of the independent variable is uniformly distributed. Therefore, a uniform random distribution will be used to drive the simulation.

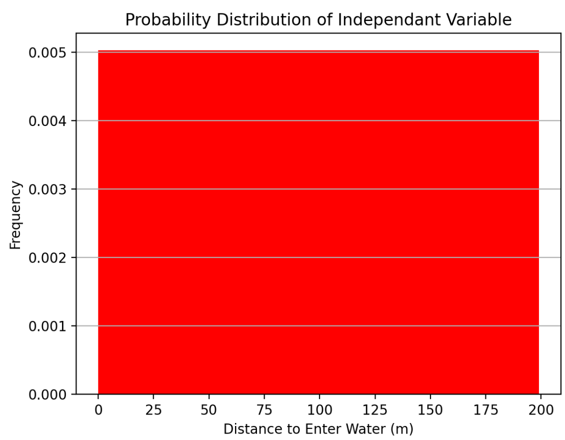

Figure 2: Uniform distribution shown in the independent variable.

Using uniform distribution a data set of random (pseudo-random) variables bound by the distance the lifeguard can travel along the shore is generated: $0m - 200m$.

NB: true randomness is only achieved with maximum entropy occurs, therefore the 'random' values generated are recognized as being pseudo-random.

The simulated dependant variables result when the randomly generated independent variables are inputted into the objective function (Eq.6). The result of five simulations can be seen in Figure 3. The distribution of the dependant variable over the Monte Carlo simulated dependant variables can be see in Figure 4.

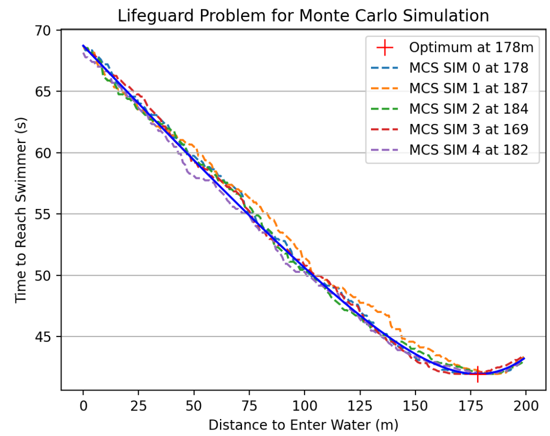

Figure 3: Uniform input used to show the optimum entry-point of the lifeguard at 178m down-shore. Five Monte Carlo simulations with a degree of accuracy of $\pm 9m$.

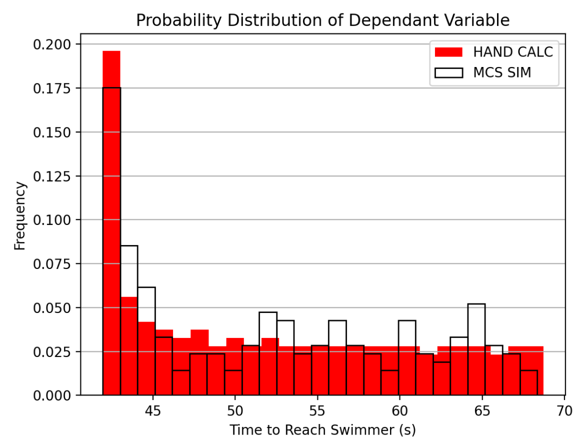

Figure 4: Frequency distribution of Monte Carlo simulated dependant variables (*MCS SIM*) over dependant variables (*HAND CALC*).

4. Future Work

The next set of work will use Monte Carlo methods for forecasting stock prices. It should also include using multiple independent variables and the effect that these have with each other and the effect on the dependant variable.

References

[1] Barreto, H.; Weppler, R. Testing Optimization: Solving the Lifeguard Problem with Discrete and Continuous Methodologies `https://www.depauw.edu/learn/econexcel/Lifeguard%5C%2520Paper%5C%2520With%5C%2520Appendix%5C%2520v5.pdf`, (accessed: 22.01.2021).