StatInf Course Project Part 1

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Distribution of averages of exponentials

In this paper we explore the properties of the distribution of the mean of $n = 40$ exponentionals. The rate parameter λ is 0.2, and that means that mean of our exponentional distribution is $\mu = 1/\lambda$, i.e. 5 and the standard deviation is also $\sigma = 1/\lambda$, i.e. 5.

First, let's simulate our data. We will have 10000×40 matrix filled with exponentionally randomly generated values with rate parameter 0.2.

```
n<-40
nosim<-10000
lambda<-0.2
sim<-matrix(rexp(nosim*n,lambda),nosim)
```

Now, let's create a vector with means of all the rows. It will be our sample from the distribution of the means.

```
means<-apply(sim,1,mean)
```

Central Limit Theorem tells us that this distribution should be normal with mean $\mu = 1/\lambda = 5$ and standard deviation $\sigma/\sqrt{n} = 5/\sqrt{40} = 0.79$. Since we were asked about variance, square that: $\sigma^2/n = 25/40 = 0.625$. Let's see what we've really got.

Mean of means

First, let's look on mean of our distribution:

```
mean(means)
```

```
## [1] 5.000502
```

Seems pretty close to 5.

Variance

Now, variance:

```
sd(means)^2
```

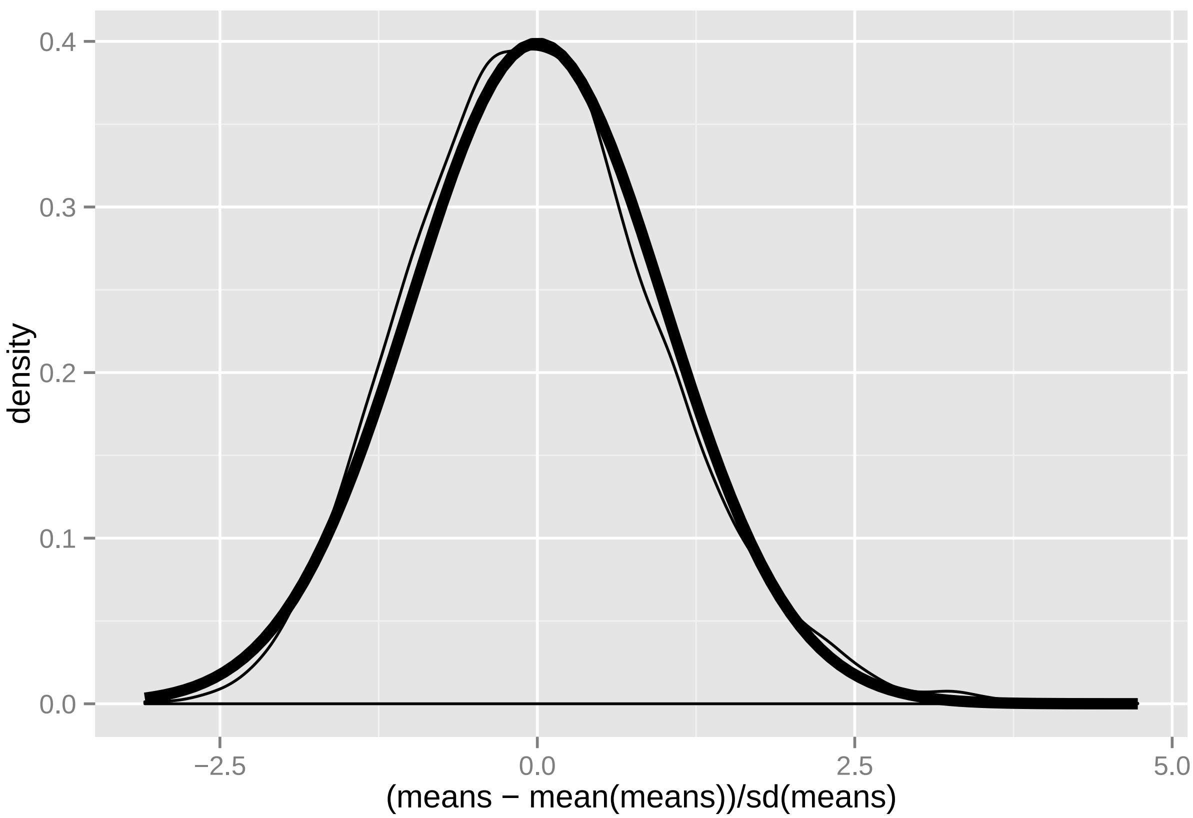
```
## [1] 0.6245041
```

Not so far from 0.625.

Compare with normal distribution

Let's compare the looks of our sample and normal distribution.

```
library(ggplot2)
g<-qplot((means-mean(means))/sd(means),geom="density")
g<-g+stat_function(fun = dnorm, size = 2)
g
```



Again, quite close.

Coverage of the confidence interval

We were also asked to evaluate the coverage of the confidence interval for $1/\lambda : \bar{X} \pm 1.96S/\sqrt{n}$

```
left<-mean(means)-1.96/(lambda*sqrt(n))
right<-mean(means)+1.96/(lambda*sqrt(n))
sum(means>left & means < right)/length(means)
```

```
## [1] 0.953
```

And we almost hit 95% confidence interval, as we should have! Thank you very much.