



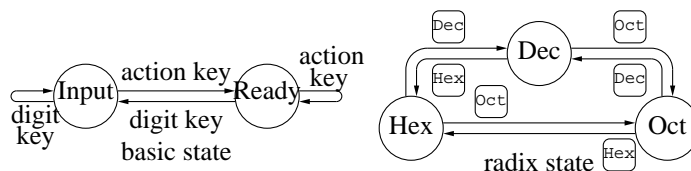
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## Happy Hacking Calculator

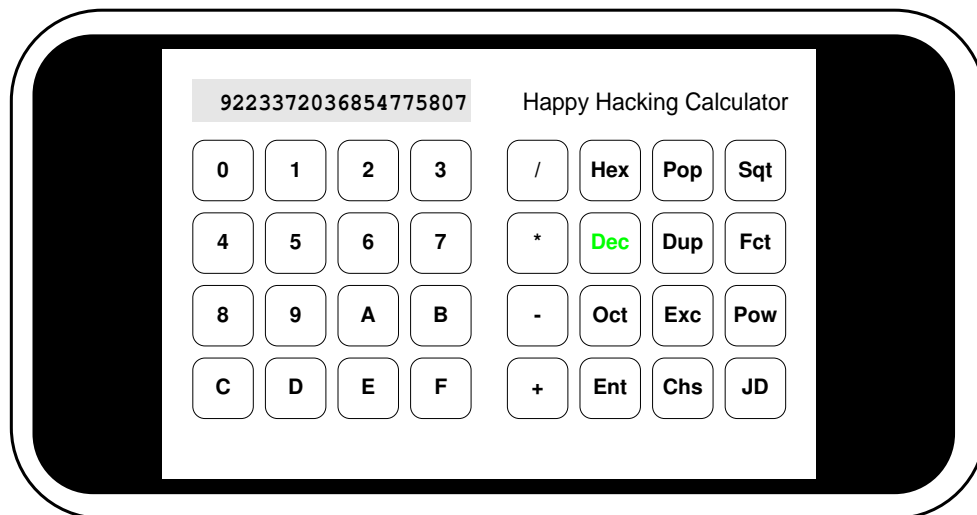
Happy Hacking Calculator (HappyCalc) is a pocket calculator for integer arithmetics in the reverse Polish notations (RPN), implemented for both iPod touch and iPhone. The elements of the stack for processing the expressions are *2's complement numbers of 64 bits*. The *depth of the stack is 16*; when the numbers are pushed in more than 16 times, the lowest element of the stack will be lost. 0's are popped up from the bottom of the stack.

The HappyCalc has the *basic states* with 'Input' and 'Ready', and the *radix states* with 'Hex', 'Dec' and 'Oct'.



**Figure 0** State transitions of the HappyCalc

The HappyCalc has 16 *digit keys* of 0, 1, ..., F on the left and 16 *action keys* of  $\square$ ,  $\square$ , ...,  $\square$  on the right. A small window above the digit keys is for displaying the results.



**Figure 1** The interface of the HappyCalc

The radix states are used for the radix conversion of input/output numbers and binary numbers inside of HappyCalc. Radix state can be changed with the action keys  $\square$ ,  $\square$  and  $\square$ , which set the state to 16, 10 and 8 respectively. The state is remembered in *ir*.

The present radix state is shown with the **green** letter. In the figure above,  $\boxed{\text{Dec}}$  is shown in green. When the HappyCalc is switched on, the radix is set to 'Dec'.

When any digit key is pressed in the 'Ready' state, the basic state goes to 'Input', the stack is pushed in and a sequence of digits until next action key is placed on the new stack top. Digit keys 0 to 9 input integers of 0 to 9, and A to F input 10 to 15 regardless the radix state.

Action keys perform operation on the operands on the top of the stack and replace the stack top with the result, which is displayed in the window. The basic state moves to 'Ready'.

Here,  $s[ix]$  indicates the stack top;  $s[ix - 1]$  is the content in the stack immediately below the top.  $ix$  is the index of the stack top, and  $ir$  is the radix register. ( $ix$  is increased or decreased in modulo 16.)

Note: No backspace key is provided. Incorrect input value must be discarded with  $\boxed{\text{Pop}}$ .

Note: Almost all operations end instantly. Factorization of primes larger than  $2^{45}$  will take time. Factorization of  $2^{47} - 115$  needs 7 seconds. During the operation,  $\boxed{\text{Fct}}$  key remains in **blue**.

## Operations

- $\boxed{+}$  Addition.  $s[ix - 1] \leftarrow s[ix - 1] + s[ix]$ ,  $s[ix] \leftarrow 0$ ,  $ix \leftarrow ix - 1$ .
- $\boxed{-}$  Subtraction.  $s[ix - 1] \leftarrow s[ix - 1] - s[ix]$ ,  $s[ix] \leftarrow 0$ ,  $ix \leftarrow ix - 1$ .
- $\boxed{*}$  Multiplication.  $s[ix - 1] \leftarrow s[ix - 1] \times s[ix]$ ,  $s[ix] \leftarrow 0$ ,  $ix \leftarrow ix - 1$ .
- $\boxed{/}$  Division.  $n \leftarrow s[ix - 1]$ ,  $d \leftarrow s[ix]$ . If  $d \neq 0$ ,  $r \leftarrow n \% d$ ,  $q \leftarrow (n - r)/d$ , (When  $0 < d$ ,  $0 \leq r < d$ ; when  $d < 0$ ,  $d \leq r < 0$ .)  $s[ix] \leftarrow q$ ,  $s[ix - 1] \leftarrow r$ . If  $d = 0$ , exception\*.
- $\boxed{\text{Hex}}$  Hexadecimal.  $ir \leftarrow 16$ .
- $\boxed{\text{Dec}}$  Decimal.  $ir \leftarrow 10$ .
- $\boxed{\text{Oct}}$  Octal.  $ir \leftarrow 8$ .
- $\boxed{\text{Ent}}$  No operation.
- $\boxed{\text{Pop}}$  Delete.  $s[ix] \leftarrow 0$ ,  $ix \leftarrow ix - 1$ .
- $\boxed{\text{Dup}}$  Duplicate.  $s[ix + 1] \leftarrow s[ix]$ ,  $ix \leftarrow ix + 1$ .
- $\boxed{\text{Exc}}$  Exchange.  $t \leftarrow s[ix]$ ,  $s[ix] \leftarrow s[ix - 1]$ ,  $s[ix - 1] \leftarrow t$ .
- $\boxed{\text{Chs}}$  Change Sign.  $s[ix] \leftarrow -s[ix]$ .
- $\boxed{\text{Sqt}}$  Square Root.  $n \leftarrow s[ix]$ . If  $n \geq 0$ ,  $q \leftarrow \lfloor \sqrt{n} \rfloor$ ,  $r \leftarrow n - q \times q$ ,  $s[ix + 1] \leftarrow q$ ,  $s[ix] \leftarrow r$ ,  $ix \leftarrow ix + 1$ . If  $n < 0$ , exception\*.
- $\boxed{\text{Fct}}$  Factorization.  $n \leftarrow s[ix]$ . If  $n \geq 2$ ,  $r \leftarrow n/p^k$ ,  $k \geq 1$  (where  $p \geq 2$  is the smallest factor of  $n$ ;  $p$  is not a factor of  $r$ .)  $s[ix + 2] \leftarrow p$ ,  $s[ix + 1] \leftarrow k$ ,  $s[ix] \leftarrow r$ ,  $ix \leftarrow ix + 2$ . If  $n < 2$ , exception\*.
- $\boxed{\text{Pow}}$  Power.  $b \leftarrow s[ix - 1]$ ,  $p \leftarrow s[ix]$ . If  $p > 0$  or ( $p = 0$  and  $b \neq 0$ ),  $s[ix - 1] \leftarrow b^p$ ,  $s[ix] \leftarrow 0$ ,  $ix \leftarrow ix - 1$ . If  $p < 0$  or ( $p = 0$  and  $b = 0$ ), exception\*.
- $\boxed{\text{JD}}$  Julian Day.  $s[ix] \leftarrow$  Julian day of noon of year  $y$ , month  $m$ , day  $d$ , represented as  $y \times 10000 + m \times 100 + d$  in  $s[ix]$ .

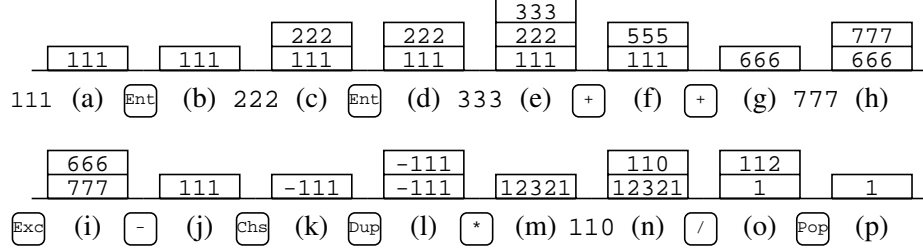
\* In case of exception, the name of action key turns to **red**, and the stack is unchanged..

## Examples

### Stack

Relations of input and stack are shown in Figure 2. (a) Input 111. Stack top is 111. (b) When two numbers are input without operation in between, press  $\boxed{\text{Ent}}$ . (c) Since it becomes ready for input next

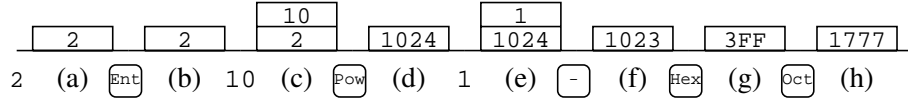
number, input 222, which is placed above 111. (d) Press  $\boxed{\text{Ent}}$  for the next number. (e) Input 333 to be placed above 222. (f) Press  $\boxed{+}$ . 2 operands on the stack top are added. (g) Press  $\boxed{+}$ . 666 is placed on the stack top. (h) Input 777. (i) Press  $\boxed{\text{Exc}}$ . 2 numbers on the stack top are exchanged. (j) Press  $\boxed{-}$ . The result of subtraction remains on the stack. (k) Press  $\boxed{\text{Chs}}$ . Sign is reversed. (l) Press  $\boxed{\text{Dup}}$ . The number of the stack top is copied. (m) Press  $\boxed{*}$ . Multiplication. (n) Input 110. (o) Press  $\boxed{/}$ . Quotient 112, remainder 1 are kept on the stack. (p) The quotient is seen in the window, and  $\boxed{\text{Pop}}$  will show the remainder. The result of these operations are the same in any radix.



**Figure 2** Stack transition (Arithmetic)

### Radix change

2  $\boxed{\text{Ent}}$  10  $\boxed{\text{Pow}}$  1  $\boxed{-}$  ( $\rightarrow 1023$ )  $\boxed{\text{Hex}}$  ( $\rightarrow 3FF$ )  $\boxed{\text{Oct}}$  ( $\rightarrow 1777$ ). During number input, radix may not be changed.



**Figure 3** Stack transition (Radix)

### Quotient and remainder

Sign of remainder is that of divisor. Be careful when the divisor is negative.

100  $\boxed{\text{Ent}}$  8  $\boxed{/}$  ( $\rightarrow$  quotient 12 remainder 4),      200  $\boxed{\text{Ent}}$  8  $\boxed{/}$  ( $\rightarrow$  quotient 25 remainder 0),  
100  $\boxed{\text{Chs}}$  8  $\boxed{/}$  ( $\rightarrow$  quotient  $-13$  remainder 4),      200  $\boxed{\text{Chs}}$  8  $\boxed{/}$  ( $\rightarrow$  quotient  $-25$  remainder 0),  
100  $\boxed{\text{Ent}}$  8  $\boxed{\text{Chs}}$   $\boxed{/}$  ( $\rightarrow$  quotient  $-13$  remainder  $-4$ ),      200  $\boxed{\text{Ent}}$  8  $\boxed{\text{Chs}}$   $\boxed{/}$  ( $\rightarrow$  quotient  $-26$  remainder  $-8$ ),  
100  $\boxed{\text{Chs}}$  8  $\boxed{\text{Chs}}$   $\boxed{/}$  ( $\rightarrow$  quotient 12 remainder  $-4$ ),      200  $\boxed{\text{Chs}}$  8  $\boxed{\text{Chs}}$   $\boxed{/}$  ( $\rightarrow$  quotient 24 remainder  $-8$ ).

### Square root of $2^{63} - 1$

$\boxed{\text{Hex}}$  7FFFFFFFFFFFFFFF (15 F's)  $\boxed{\text{Sqt}}$  ( $\rightarrow$  B504F333, remainder 1615E23D6)  $\boxed{\text{Dup}}$   $\boxed{*}$  ( $\rightarrow$  7FFFFFFE9EA1DC29)  $\boxed{+}$  ( $\rightarrow$  7FFFFFFFFFFFFFFF, the original number).

### Square root of 2

$\boxed{\text{Dec}}$  20000000000000000 (16 0's)  $\boxed{\text{Sqt}}$  ( $\rightarrow 141421356 = \lfloor \sqrt{2} \times 10^8 \rfloor$ )  $\boxed{\text{Dup}}$   $\boxed{*}$  ( $\rightarrow 19999999932878736$ )  $\boxed{+}$  ( $\rightarrow 20000000000000000$ ).

### Factorization of $2^{63} - 1$

$2^{63} - 1 = 7 \times 7 \times 73 \times 127 \times 337 \times 92737 \times 649657$ .

$\boxed{\text{Dec}}$  2  $\boxed{\text{Ent}}$  63  $\boxed{\text{Pow}}$  1  $\boxed{-}$  ( $\rightarrow 9223372036854775807$ ) (  $\boxed{\text{Fct}}$   $\boxed{\text{Pop}}$   $\boxed{\text{Pop}}$  )<sup>6</sup> ( $\rightarrow 7, 2, 188232082384791343, 73, 1, 2578521676503991, 127, 1, 20303320287433, 337, 1, 60247241209, 92737, 1, 649657, 649657, 1, 1$ ).

## Factorization of $20!$

Dec 2 Ent 3 \* 4 \* ... \* 9 \* A \* B \* ... \* F \* 16 \* 17 \* 18 \* 19 \* 1A \* ( $\rightarrow$  2432902008176640000 = 20!).

$$20! = 2^1 \times 3^1 \times 2^2 \times 5^1 \times (2^1 \times 3^1) \times 7^1 \times 2^3 \times 3^2 \times (2^1 \times 5^1) \times 11^1 \times (2^2 \times 3^1) \times 13^1 \times (2^1 \times 7^1) \times (3^1 \times 5^1) \times 2^4 \times 17^1 \times (2^1 \times 3^2) \times 19^1 \times (2^2 \times 5^1) = 2^{18} \times 3^8 \times 5^4 \times 7^2 \times 11^1 \times 13^1 \times 17^1 \times 19^1$$

Here,  $\boxed{\text{Fct}} \rightarrow 2 = \text{smallest factor}$   $\boxed{\text{Pop}} \rightarrow 18 = \text{power}$   $\boxed{\text{Pop}} \rightarrow 9280784638125$   $\boxed{\text{Fct}} \rightarrow 3 = \text{smallest factor}$   $\boxed{\text{Pop}} \rightarrow 8 = \text{power}$   $\boxed{\text{Pop}} \dots$

## Julian astronomical day

Julian day of October 20, 2008:

20081020  $\boxed{\text{JD}}$  ( $\rightarrow 2454760$ )1  $\boxed{+}$  7  $\boxed{/}$   $\boxed{\text{Pop}}$  ( $\rightarrow 1 = \text{Monday}$ . (Day of week is given by  $(\text{JD} + 1) \% 7$ ).

Julian day of January 1,  $-4712$ :

4712  $\boxed{\text{Chs}}$  10000  $\boxed{*}$  101  $\boxed{+}$   $\boxed{\text{JD}}$  ( $\rightarrow 0$ . Note:  $-47119899 \boxed{\text{Ent}}$  10000  $\boxed{/}$  ( $\rightarrow$  quotient  $-4712$  remainder 101)).

## The life of the Bard

William Shakespeare was born on April 23, 1564 and died on April 23, 1616. How many days did he live?  
 16160424  15640423   ( $\rightarrow$  18984 days).

## Repeated digits

Dec 12345679012345679 (omitting 8's) Ent 81 \* (→ 999999999999999999) Sqrt (→ 999999999)

Hex 123456789ABCDEF (omitting E)'s Ent E1 \* (→ FFFFFFFFFFFFFFFF)

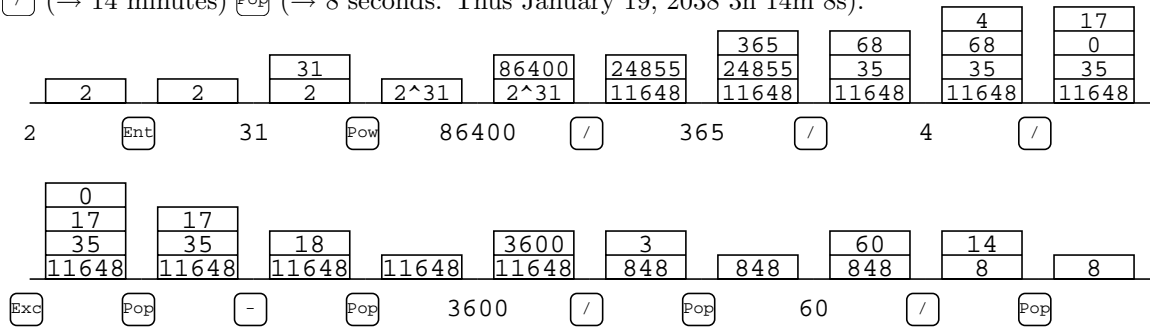
**Oct** 12345701234570123457 (omitting 6's) **Ent** 61 **\*** ( $\rightarrow$  77777777777777777777)

The case of twice 1...7  1234570123457  61  ( $\rightarrow$  77777777777777 even 7's)  ( $\rightarrow$  7777777).

## Year 2038

UNIX time clock will overflow on  $2^{31}$  seconds after midnight (0h) of January 1, 1970. When does the overflow occur?

2 **Ent** 31 **Pow** ( $\rightarrow$  2147483648) 86400 **/** ( $\rightarrow$  24855 days later) 365 **/** ( $\rightarrow$  68 years later = 2038) 4 **/** ( $\rightarrow$  17 leap years) **Exc** **Pop** ( $\rightarrow$  17, subtract 17 from 35 in the stack) **-** ( $\rightarrow$  18. (January) 18 will count 24855 days. Calculate time of January 19 ) **Pop** ( $\rightarrow$  11648 seconds) 3600 **/** ( $\rightarrow$  3 o'clock) **Pop** ( $\rightarrow$  848) 60 **/** ( $\rightarrow$  14 minutes) **Pop** ( $\rightarrow$  8 seconds. Thus January 19, 2038 3h 14m 8s).



**Figure 4** Unix time overflow

### Continued fraction of $\pi$

3141592654 Ent 10 Ent 9 Pow / ( $\rightarrow$  3) Pop ( $\rightarrow$  141592654) 10 Ent 18 Pow Exc / 10 Ent 9 Pow / ( $\rightarrow$  7) Pop ( $\rightarrow$  62513285) 10 Ent 18 Pow Exc / 10 Ent 9 Pow / ( $\rightarrow$  15) Pop ( $\rightarrow$  99699762) 10 Ent 18 Pow Exc / 10 Ent 9 Pow / ( $\rightarrow$  1) Pop ( $\rightarrow$  3411839) 10 Ent 18 Pow Exc / 10 Ent 9 Pow / ( $\rightarrow$  293) Pop ( $\rightarrow$  97065834).  $\pi = 3 + //7, 15, 1, 293, \dots// \approx 355/113$ . The correct value of the 4th term is 292.

end