Introduction to Computational Macroeconomics

Tokyo 2022

Assignment

June 29, 2022

- Due date: 12th July
- Weight: 40% of total marks

This assignment should be submitted by email as a single Jupyter notebook no later than midnight on the due date. Please note that 20% of the total mark will be deducted for late assignments. Please send your email to `john.stachurski@anu.edu.au`. Make sure that you include your NAME AND STUDENT NUMBER in the Jupyter notebook. The subject of your email should be "Comp econ assignment".

Note that proofs and other discussion should also be included in the notebook. If you are not sure how to do this please ask. No other files should be submitted. I recommend that you stop and restart Jupyter before you submit, and make sure that the notebook runs from start to finish without error.

Please note that *you may collaborate with up to two other students*. Please be generous and offer to collaborate with any student who does not have a partner. You should include the names of the students with whom you collaborated in your Jupyter notebook.

Groups of students should not collaborate with other groups.

A worker is currently employed and deciding when to retire. While employed, the worker is paid a fixed wage w in each period. When she retires, she is paid a single lump sum pension s. Both w and s are nonnegative and constant.

The worker discounts future earnings using the real interest rate $r_t = i - \pi_t$, where i is the nominal risk-free interest rate and π_t is time t inflation. In other words, a payment y_{t+1} at $t+1$ is discounted to time t present value via $\beta_t y_{t+1}$, where $\beta_t = 1/(1+r_t)$. The nominal risk-free rate is constant.

Assume that the inflation rate obeys $\pi_t = \pi(Z_t)$ for some fixed function $\pi \in \mathbb{R}^{\mathsf{Z}}$, where Z is a finite subset of $\mathbb{R}$ and $(Z)_{t \geq 0}$ is Q-Markov for some stochastic matrix Q on Z. In what follows we set $n := |\mathsf{Z}|$.

Question 1. Write down the Bellman equation for the problem, in terms of the lifetime value for an employed worker. Define the Bellman operator T corresponding to the Bellman equation as a self-map on $\mathbb{R}^{\mathsf{Z}}$.

Question 2. Prove that T is a contraction map on $\mathbb{R}^{\mathsf{Z}}$ with respect to the supremum norm whenever $\bar{r} := \min_{z \in \mathsf{Z}}(i - \pi(z))$ satisfies $\bar{r} > 0$.

Although the condition $\bar{r} > 0$ is sufficient for global stability of T, it is not necessary. As we will learn later, it sufficies that the matrix $n \times n$ matrix

$$L(z, z') := \frac{1}{1 + i - \pi(z)} Q(z, z')$$

satisfies $r(L) < 1$.

Question 3. Let $\pi(z) = p + z$, where p is a parameter indicating the long-run average inflation rate. Show that $r(L) < 1$ holds when (Z_t) is the Tauchen discretization of the AR(1) process $X_{t+1} = \rho X_t + \nu W_{t+1}$, where $(W_t)_{t \geq 0}$ is IID standard normal and the parameters are given by

$$s = 100, \; w = 5, \; i = 0.1, \; p = 0.06, \; \rho = 0.9 \text{ and } \nu = 0.01 \qquad (1)$$

The AR(1) process should be discretized to a grid of $n = 100$ points (which is the size of the state space Z). To discretize, you can use the commands `import quantecon as qe` and

```
mc = qe.tauchen(ρ, ν, m=10, n=n)
```

Question 4. Solve for the value function v^* using value function iteration using the default parameters in (1). Plot v^* as a function of z. Plot the stopping value s on the same figure.

Question 5. Plot the v^*-greedy policy as a function of z. Provide some interpretation of the figure.

Question 6. Prove that the value function v^* is increasing on Z under the parameters in (1). Provide some economic intuition if you can.

Question 7. Using the default parameters and simulation, compute the average number of years until retirement at the default parameters. In answering this question, you should regard one period in the model as one month (i.e., wages are paid monthly and interest and inflation rates are monthly). If you can, accelerate your simulation using Numba.

Question 8. Now set $w = 4.9$ and repeat the last exercise. What change do you observe? How can you explain this change?