Calculus A II One-to-One Tutoring

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Question 2.1 (Some Basic Derivatives).

Find the derivative with respect to x for the following functions:

1. $y = x^n$;
2. $y = e^x$;
3. $y = a^x$;
4. $y = \ln x$;
5. $y = \log_a x$;
6. $y = \sin x$;
7. $y = \cos x$; and
8. $y = \tan x$;

Question 2.2 (Utilizing the Chain Rule).

Find the derivative of the following functions:

1. $y = \sqrt[3]{e^x + 1}$;
2. $y = e^{\tan\theta}$;
3. $y = \sin\left(\dfrac{e^x}{1+e^x}\right)$;
4. $y = t\sin(\pi t)$;
5. $y = \sin(\ln x)$;
6. $y = \ln\left(\dfrac{x^a}{b^x}\right)$;
7. $y = \dfrac{1}{\ln x}$;
8. $y = \ln\Big((\sin x)^2\Big)$; and
9. $y = \dfrac{\ln x}{1+\ln x}$.

Question 2.3 (Applying the Chain Rule).

Use the fact that $|x| = \sqrt{x^2}$ to find $\dfrac{\mathrm{d}}{\mathrm{d}x}(|x|)$.

Question 2.4 (Applying the Chain Rule).

Find the derivative of $y = x^x$. (Hint: take log to both sides.)

Question 2.5 (Comprehensive Applications).

Find the derivative of the following functions:

1. $y = \sqrt[4]{x\sqrt[3]{x\sqrt{x}}}$;
2. $y = (x-3)\sqrt{x^2+2x+3}$;
3. $y = x^{(\ln x)^{111}}$;
4. $y = \cos(\sin 3x)$;
5. $y = e^t(1+te^t)$;
6. $y = x^3e^x$;
7. $y = \dfrac{x}{e^x}$; and
8. $y = \dfrac{e^x}{1-e^x}$.

Example 2.6.

Compute $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sin\left(\cos 5x\right)\right)$.

Example 2.7.

Compute $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sin\left(3\cos x\right)\right)$.

Example 2.8.

Compute $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sin\left(\cos bx\right)\right)$ for $b \in \mathbb{R}^{+}$.

Example 2.9.

Compute $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sin\left(a\cos x\right)\right)$ for $a \in \mathbb{R}^{+}$.

Example 2.10.

Compute $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sin\left(a\cos bx\right)\right)$ for $a, b \in \mathbb{R}^+$.

Example 2.11.

Compute $\dfrac{\partial w}{\partial x}$ for $w = \sin\left(y\cos x\right)$.

Example 2.12.

Compute $\dfrac{\partial w}{\partial x}$ for $w = z\sin\left(\cos x\right)$.

Example 2.13.

Compute $\dfrac{\partial w}{\partial x}$ for $w = z\sin\left(y\cos x\right)$.

Question 2.14.

Find the first partial derivatives of the following functions:

1. $w = x^4 + 5xy^3$;
2. $w = x^2y - 3y^4$;
3. $w = x^3 \sin y$;
4. $w = e^{xt}$;
5. $w = \ln(x + t^2)$;
6. $w = \dfrac{e^x}{u + v^2}$;
7. $w = x^y$; and
8. $w = \ln(x + 2y + 3z)$.