

Calculus A II One-to-One Tutoring

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Question 2.1 (Some Basic Derivatives).

Find the derivative with respect to x for the following functions:

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|----------------|------------------|---------------------|-----------------------|
| 1. $y = x^n$; | 3. $y = a^x$; | 5. $y = \log_a x$; | 7. $y = \cos x$; and |
| 2. $y = e^x$; | 4. $y = \ln x$; | 6. $y = \sin x$; | 8. $y = \tan x$; |

Question 2.2 (Utilizing the Chain Rule).

Find the derivative of the following functions:

1. $y = \sqrt[3]{e^x + 1}$;

2. $y = e^{\tan \theta}$;

3. $y = \sin \left(\frac{e^x}{1 + e^x} \right)$;

4. $y = t \sin(\pi t)$;

5. $y = \sin(\ln x)$;

6. $y = \ln \left(\frac{x^a}{b^x} \right)$;

7. $y = \frac{1}{\ln x}$;

8. $y = \ln \left((\sin x)^2 \right)$; and

9. $y = \frac{\ln x}{1 + \ln x}$.

Question 2.3 (Applying the Chain Rule).

Use the fact that $|x| = \sqrt{x^2}$ to find $\frac{d}{dx}(|x|)$.

Question 2.4 (Applying the Chain Rule).

Find the derivative of $y = x^x$. (Hint: take log to both sides.)

Question 2.5 (Comprehensive Applications).

Find the derivative of the following functions:

1. $y = \sqrt[4]{x^3 \sqrt{x \sqrt{x}}}$;

2. $y = (x - 3)\sqrt{x^2 + 2x + 3}$;

3. $y = x^{(\ln x)^{111}}$;

4. $y = \cos(\sin 3x)$;

5. $y = e^t(1 + te^t)$;

6. $y = x^3 e^x$;

7. $y = \frac{x}{e^x}$; and

8. $y = \frac{e^x}{1 - e^x}$.

Example 2.6.

Compute $\frac{d}{dx} (\sin (\cos 5x))$.

Example 2.7.

Compute $\frac{d}{dx} (\sin (3 \cos x))$.

Example 2.8.

Compute $\frac{d}{dx} (\sin (\cos bx))$ for $b \in \mathbb{R}^+$.

Example 2.9.

Compute $\frac{d}{dx} (\sin (a \cos x))$ for $a \in \mathbb{R}^+$.

Example 2.10.

Compute $\frac{d}{dx} (\sin (a \cos bx))$ for $a, b \in \mathbb{R}^+$.

Example 2.11.

Compute $\frac{\partial w}{\partial x}$ for $w = \sin (y \cos x)$.

Example 2.12.

Compute $\frac{\partial w}{\partial x}$ for $w = z \sin (\cos x)$.

Example 2.13.

Compute $\frac{\partial w}{\partial x}$ for $w = z \sin (y \cos x)$.

Example 2.14.

Find the first derivatives of the following functions:

1. $w = x^4 + 5xy^3;$

2. $w = x^2y - 3y^4;$

3. $w = x^3 \sin y;$

4. $w = e^{xt};$

5. $w = \ln(x + t^2);$

6. $w = \frac{e^x}{u + v^2};$

7. $w = x^y;$ and

8. $w = \ln(x + 2y + 3z).$