Calculus A II One-to-One Tutoring

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Question 2.1 (Some Basic Derivatives).

Find the derivative with respect to x for the following functions:

- 1. $y = x^n$;
- 3. $y = a^x$;
- 5. $y = \log_a x$;
- 7. $y = \cos x$; and

- 2. $y = e^x$;
- 4. $y = \ln x$;
- 6. $y = \sin x$;
- 8. $y = \tan x$;

Question 2.2 (Utilizing the Chain Rule).

Find the derivative of the following functions:

1.
$$y = \sqrt[3]{e^x + 1}$$
;

$$2. \ y = e^{\tan \theta};$$

$$3. \ y = \sin\left(\frac{e^x}{1 + e^x}\right);$$

4.
$$y = t \sin(\pi t)$$
;

5.
$$y = \sin(\ln x)$$
;

6.
$$y = \ln\left(\frac{x^a}{b^x}\right);$$

7. $y = \frac{1}{\ln x};$

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8.
$$y = \ln \left((\sin x)^2 \right)$$
; and
9. $y = \frac{\ln x}{1 + \ln x}$.

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 $\bf Question~2.3~(Applying~the~Chain~Rule).$

Use the fact that $|x| = \sqrt{x^2}$ to find $\frac{\mathrm{d}}{\mathrm{d}x} (|x|)$.

 ${\bf Question~2.4}$ (Applying the Chain Rule).

Find the derivative of $y = x^x$. (Hint: take log to both sides.)

Question 2.5 (Comprehensive Applications).

Find the derivative of the following functions:

$$1. \ y = \sqrt[4]{x\sqrt[3]{x\sqrt{x}}};$$

2.
$$y = (x-3)\sqrt{x^2+2x+3}$$
;

3.
$$y = x^{(\ln x)^{111}};$$

$$4. \ y = \cos(\sin 3x);$$

5.
$$y = e^t(1 + te^t);$$

$$6. \ y = x^3 e^x;$$

7.
$$y = \frac{x}{e^x}$$
; and

7.
$$y = \frac{x}{e^x}$$
; and
8. $y = \frac{e^x}{1 - e^x}$.

Example 2.6.

Compute $\frac{\mathrm{d}}{\mathrm{d}x} (\sin(\cos 5x))$.

Example 2.7.

Compute $\frac{\mathrm{d}}{\mathrm{d}x} \left(\sin \left(3\cos x \right) \right)$.

Example 2.8.

Compute $\frac{\mathrm{d}}{\mathrm{d}x} \left(\sin \left(\cos bx \right) \right)$ for $b \in \mathbb{R}^+$.

Example 2.9.

Compute $\frac{\mathrm{d}}{\mathrm{d}x} \left(\sin \left(a \cos x \right) \right)$ for $a \in \mathbb{R}^+$.

Example 2.10.

Compute $\frac{\mathrm{d}}{\mathrm{d}x} \left(\sin \left(a \cos bx \right) \right)$ for $a, b \in \mathbb{R}^+$.

Example 2.11.

Compute $\frac{\partial w}{\partial x}$ for $w = \sin(y \cos x)$.

Example 2.12.

Compute $\frac{\partial w}{\partial x}$ for $w = z \sin(\cos x)$.

Example 2.13.

Compute $\frac{\partial w}{\partial x}$ for $w = z \sin(y \cos x)$.

Example 2.14.

Find the first partial derivatives of the following functions:

- 1. $w = x^4 + 5xy^3$;
- 2. $w = x^2y 3y^4$;
- $3. \ w = x^3 \sin y;$
- 4. $w = e^{xt}$;
- 5. $w = \ln(x + t^2);$
- $6. \ w = \frac{e^x}{u + v^2};$
- 7. $w = x^y$; and
- 8. $w = \ln(x + 2y + 3z)$.