

Calculus A II One-to-One Tutoring

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Theorem 1.1 (Formulae about Derivatives).

The symbol x denotes a variable. The symbols a, r are constant and are in \mathbb{R}^+, \mathbb{R} , respectively.

1. $\frac{d}{dx}(x^r) = rx^{r-1}$;
2. $\frac{d}{dx}(a^x) = \ln a \cdot a^x$;
3. $\frac{d}{dx}(\ln x) = \frac{1}{x}$;
4. $\frac{d}{dx}(\sin x) = \cos x$; and
5. $\frac{d}{dx}(\cos x) = -\sin x$.

Theorem 1.2 (Operations on Derivatives).

Let $f(x)$ and $g(x)$ be differentiable. Then

1. $\frac{d}{dx}(f(x) + g(x)) = \frac{df}{dx}(x) + \frac{dg}{dx}(x)$;
2. $\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{df}{dx}(x)$;
3. $\frac{d}{dx}(f(x) \cdot g(x)) = \frac{df}{dx}(x) \cdot g(x) + f(x) \cdot \frac{dg}{dx}(x)$; and
4. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{\frac{df}{dx}(x) \cdot g(x) - f(x) \cdot \frac{dg}{dx}(x)}{(g(x))^2}$.

Example 1.3.

Find the derivative of x^4 with respect to x .

Example 1.4.

Find the derivative of t^{100} with respect to t .

Example 1.5.

Find the derivative of $\frac{1}{u}$ with respect to u .

Example 1.6.

Find the derivative of $x\sqrt{x}$ with respect to x .

Example 1.7.

Find the derivative of $x^4 - 6x^2 + 4$ with respect to x .

Example 1.8.

Find the derivative of $e^t + t^e$ with respect to t .

Example 1.9.

Find the derivative of x^{1000} with respect to x .

Example 1.10.

Evaluate $\lim_{x \rightarrow 1} \frac{x^{1000} - 1}{x - 1}$.

Example 1.11.

Find the derivative of $\frac{x^2 + x - 2}{x^3 + 6}$ with respect to x .

Example 1.12.

Find the derivative of $\frac{e^x}{1 - e^x}$ with respect to x .

Example 1.13.

Find the derivative of $x^3 e^x$ with respect to x .

Example 1.14.

Find the derivative of $\sin \theta + \cos \theta$ with respect to θ .

Example 1.15.

Find the derivative of $\sin \theta \cdot \cos \theta$ with respect to θ .

Example 1.16.

Find the derivative of $\frac{\sin x}{\cos x}$ with respect to x .

Example 1.17.

Find the derivative of $\frac{\cos \theta}{e^\theta}$ with respect to θ .

Question 1.18.

Find the derivative with respect to the corresponding variable for the following functions:

1. $f_1(x) = \pi^{25}$;

2. $f_2(x) = (4x^2 + 3)(2x + 5)$;

3. $f_3(x) = \frac{x}{e^x}$;

4. $f_4(u) = \frac{6u^4 - 5u}{u + 1}$;

5. $f_5(t) = \frac{t^4 - 5t^3 + \sqrt{t}}{t^2}$;

6. $f_6(\theta) = \tan(\theta)$;

7. $f_7(\theta) = \sec(\theta)$; and

8. $f_8(x) = e^x \cos x + \sin x$.

Theorem 1.19 (Chain Rule).

If g is differentiable at x and f is differentiable at $g(x)$, then $(f \circ g)(x)$ is differentiable at x and

$$\frac{d}{dx} ((f \circ g)(x)) = \frac{df}{dx}(g(x)) \cdot \frac{dg}{dx}(x).$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{u}{x}.$$

Example 1.20.

Find the derivative of $(x^3 - 1)^{100}$ with respect to x .

Example 1.21.

Find the derivative of $\left(\frac{t-1}{2t+1}\right)^9$ with respect to t .

Example 1.22.

Find the derivative of $\frac{1}{\sin u}$ with respect to u .

Example 1.23.

Find the derivative of $x^4 - 6x^2 + 4$, where $x = \sin v$, with respect to v .

Example 1.24.

Find the derivative of $(e^{\sin x})^{100}$ with respect to x .

Example 1.25.

Find the derivative of e^{e^x} with respect to x .

Example 1.26.

Find the derivative of $\sin(\cos x)$ with respect to x .

Example 1.27.

Find the derivative of $\sin\left(\frac{e^x}{1-e^x}\right)$ with respect to x .

Question 1.28.

Find the derivative with respect to the corresponding variable for the following functions:

1. $f_1(x) = (x^2 + 1)^{25}$;
2. $f_2(x) = \cos(x^2)$;
3. $f_3(x) = (\cos x)^2$;
4. $f_4(u) = e^{u^2 - u}$;
5. $f_5(t) = e^{at} \sin(bt)$;
6. $f_6(\theta) = \cos(\sin(3\theta))$;
7. $f_7(\theta) = \sqrt{\theta + \sqrt{\theta}}$;
8. $f_8(x) = 5^{\sqrt{x} + \sin(x)}$;
9. $f_9(x) = \ln(\sin x)$; and
10. $f_{10}(x) = e^{x^2 \ln x}$.

