Calculus A II One-to-One Tutoring

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April 21, 2024

Definition 3.1 (Limit).

The limit

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

if for all $\varepsilon > 0$ (the greek letter ε stands for "error"), there exists a $\delta > 0$ (the greek letter δ stands for "difference") such that $\sqrt{(x-x_0)^2+(y-y_0)^2} < \delta$ implies $|f(x,y) - f(x_0,y_0)| < \varepsilon$.

Remark 3.2.

If there exist two path $C_1 : y = f_1(x)$ and $C_2 : y = f_2(x)$ such that

$$\lim_{\substack{(x,y)\to(x_0,y_0)\\(x,y)\in C_1}} f(x,y) \neq \lim_{\substack{(x,y)\to(x_0,y_0)\\(x,y)\in C_2}} f(x,y),$$

then the limit $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y)$ does not exist.

Example 3.3.

Determine the existance of $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ for each of the following functions:

1. $f(x,y) = \dfrac{x^2-y^2}{x^2+y^2}$;
2. $f(x,y) = \dfrac{xy}{x^2+y^2}$;
3. $f(x,y) = \dfrac{x^2y}{x^2+y^2}$.

Definition 3.4 (Continuity).

We say a function f is continuous at (x_0, y_0) if

$$\lim_{(x,y)\to(x_0,y_0)} f(x, y) = f(x_0, y_0).$$

We say a function is continuous if it is continuous at every point in its domain.