Homework 1 of Computational Mathematics

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Problem 1. Use the intermediate value theorem and Rolle's theorem to show the graph of $f(x) = x^3 + 2x + k$ crosses the x-axis exactly once, regardless of the value of the constant k.

Solution.

Problem 2. Find $\max_{a \le x \le b} |f(x)|$ for the following functions and intervals.

a. $f(x) = \dfrac{2 - e^x + 2x}{3}$, $[0, 1]$

b. $f(x) = \dfrac{4x - 3}{x^2 - 2x}$, $[0, 5, 1]$

c. $f(x) = 2x\cos(2x) - (x - 2)^2$, $[2, 4]$

d. $f(x) = 1 + e^{-\cos(x-1)}$, $[1, 2]$

Solution.

Problem 3. Find the second Taylor polynomial $P_2(x)$ for the function $f(x) = e^x \cos(x)$ about $x_0 = 0$.

a. Use $P_2(0.5)$ to approximate $f(0.5)$. Find an upper bound for error $|f(0.5) - P_2(0.5)|$ using the error formula, and compare it to the actual error.

b. Find a bound for the error $|f(0.5) - P_2(0.5)|$ in using $P_2(x)$ to approximate $f(x)$ on the interval $[0, 1]$.

c. Approximate $\displaystyle\int_0^1 f(x)\,\mathrm{d}x$ using $\displaystyle\int_0^1 P_2(x)\,\mathrm{d}x$.

d. Find an upper bound for the error in (c) using $\displaystyle\int_0^1 |P_2(x)|\,\mathrm{d}x$.

Solution.

Problem 4. Let $f(x) = \dfrac{1}{1 - x}$ and $x_0 = 0$. Find the n-th Taylor polynomial $P_n(x)$ for $f(x)$ about x_0. Find a value of n necessary for $P_n(x)$ to approximate $f(x)$ to within 10^{-6} on $[0, 0.5]$.

Solution.

Problem 5. Find the largest interval in which p^* must lie to approximate p with relative error at most 10^{-4} for each value of p.

a. π

b. e

c. $\sqrt{2}$

d. $\sqrt[3]{7}$

Solution.

Problem 6. Let

$$f(x) = \frac{e^x - e^{-x}}{x}.$$

a. Find $\lim_{x \to 0} \frac{e^x - e^{-x}}{x}$.

b. Use three-digit rounding arithmatic to evaluate $f(x)$.

c. Replace each exponential function with its Maclaurin polynomial, and repeat part (b).

d. The actual value is $f(0.1) = 2.003335000$. Find the relative error for the values obtained in part (b) and (c).

Solution.