$\boxed{1\text{-}15}$

Since $f(n) = n^2 + 3n^3$, we consider $h(n) = n^3$. By definition, we can find $c_1 = 3$, $c_2 = 4$, and $N = 1$ such that $c_1 \cdot h(n) \le f(n) \le c_2 \cdot f(n)$ holds. We vertify as follows:

$f(n) = n^2 + 3n^3 \le 4n^3 = O(n^3)$ if $n \geqslant N = 1$ and

$f(n) = n^2 + 3n^3 \geqslant 3n^3 = \Omega(n^3)$ if $n \geqslant N = 1$

That is, $f(n)$ is in both $O(n^3)$ and $\Omega(n^3)$

$\boxed{1\text{-}16}$

To show $T(n) = 6n^2 + 20n = O(n^3)$, by definition, we can find $c = 1$ and $N = 9$ such that $6n^2 + 20n < n^3$ if $n \geqslant N = 9$. Thus, $T(n) \in O(n^3)$

For proving $T(n) \notin \Omega(n^3)$, we need to show that for any given positive constant c, there exists an integer $N > 0$ shch that $T(n) < c \cdot g(n)$ when $n \geqslant N$.

We consider $T(n) = 6n^2 + 20n$ and $g(n) = n^3$. Then, for any given constant $c > 0$, we choose N

$$N = \begin{cases} \lceil \frac{9}{c} \rceil, & \text{if } 0 < c < 1 \\ 9, & \text{if } c \geqslant 1 \end{cases}$$

It's easy to vertify that $6n^2 + 20n < c \cdot n^3$ if $n \geqslant N$. Thus, $6n^2 + 20n \notin \Omega(n^3)$

$\boxed{1\text{-}18}$

To show $g(n) = 5n^5 + 4n^4 + 6n^3 + 2n^2 + n + 7 \in \Theta(n^5)$, by definition,

$c \cdot n^5 \le g(n) \le d \cdot n^5 \Rightarrow c = 5, d = 6$.

Then $6n^5 \geqslant 5n^5 + 4n^4 + 6n^3 + 2n^2 + n + 7 \Rightarrow n^5 \geqslant 4n^4 + 6n^3 + 2n^2 + n + 7 \Rightarrow N = 6$.

Thus, $5n^5 + 4n^4 + 6n^3 + 2n^2 + n + 7 \in \Theta(n^5)$

$\boxed{1\text{-}22}$ $\square(n^n)$ $\square(n \ln n)$

$n^n = n^n + \ln n > 2^{n!} > 10^n + n^{20} > 4^n > e^n > (\lg n)! > n! > n^{5/2} > 5^{\lg n} > 5n^2 + 7n > (\lg n)^2 > n \ln n$
$= \lg(n!) > 8n + 12 > \sqrt{n}$