Quantum Computing and the CHSH Inequality: Circuit Simulation and Graph Analysis

RUTGERS–NEW BRUNSWICK
Douglass Residential College

Eileen Rashduni, Professor Emina Soljanin

Electrical and Computer Engineering, Rutgers University, Piscataway, New Jersey 08854

Abstract

The CHSH Inequality tests the limits of local realism versus quantum mechanics. We create and simulate quantum circuits for the CHSH game and analyze findings to observe and understand quantum phenomena and their impact on classical bounds. Violations of the classical bound reveals the influence of entanglement in quantum correlations.

Introduction

In Classical Computing, information is stored as bits, or what one could represent as 1's and 0's. When we migrate to Quantum Computing, we see the change of these being represented as qubits, which can be represented as 1's, 0's or any proportion of 0 and 1, which exists in a superposition state. This is the key component towards why Quantum Algorithms can process information faster than different Classical Systems.

The CHSH Game is a well-known problem in quantum information theory, designed to illustrate the power of quantum entanglement. In Quantum Variation, two players, Alice and Bob, receive binary inputs x and y, respectively, and produce outputs a and b. They win if:

$$x \cdot y = a \oplus b,$$

This means that:

For $x = y = 1$, they win $if\ a \neq b$.
For other input combinations, they win if $a = b$.

Classical Strategy: With a classical strategy where Alice and Bob independently choose their outputs a and b (e.g., both always output 0), the best they can achieve is a winning probability of 75%. This is optimal among all deterministic strategies.

Quantum Strategy: Using shared entanglement, Alice and Bob can achieve a higher winning probability. They share an entangled state of $|\phi AB\rangle$ and use specific measurement bases depending on their inputs:
Alice uses the computational basis for $x = 0$ and the Hadamard basis for $x = 1$.
Bob uses the computational basis rotated by $\frac{\pi}{8}$ for $y = 0$ and rotated by $-\frac{\pi}{8}$ for $y = 1$.

Measurement Probabilities:

When $x = y = 1$, the angle between Alice's and Bob's measurement bases is $\frac{3\pi}{8}$.
For other input combinations, the angle is $\frac{\pi}{8}$

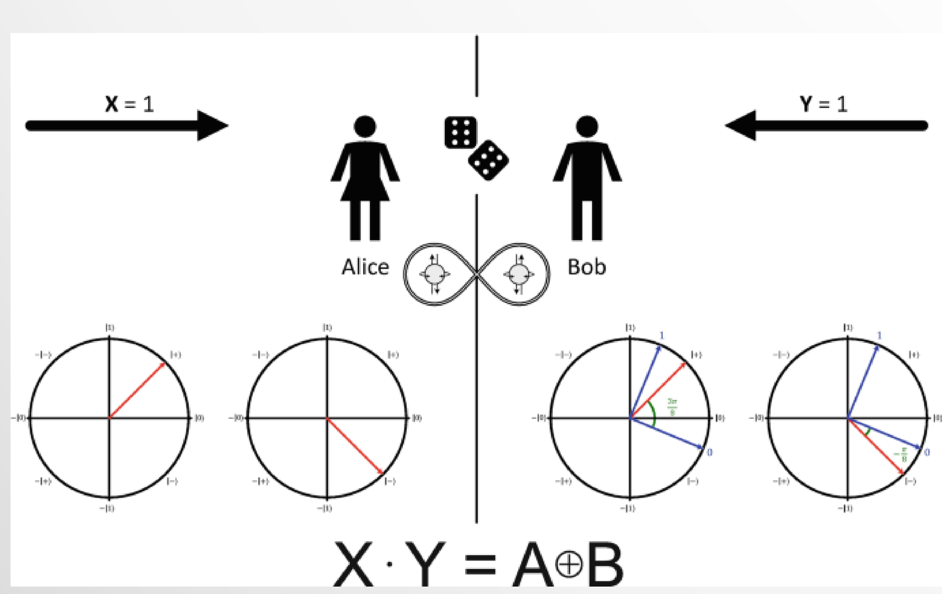
Probability of Matching Outputs: $P(a = b) = \cos^2\theta$, where θ is the angle between the measurement bases.

For $x = y = 1$, the winning probability is $\sin^2\frac{3\pi}{8}$
For other cases, it is $\cos^2\frac{\pi}{8}$

Overall Winning Probability:

$$Pwin = \frac{3}{4}\cdot\cos^2\frac{\pi}{8} + \frac{1}{4}\cdot\sin^2\frac{3\pi}{8}$$

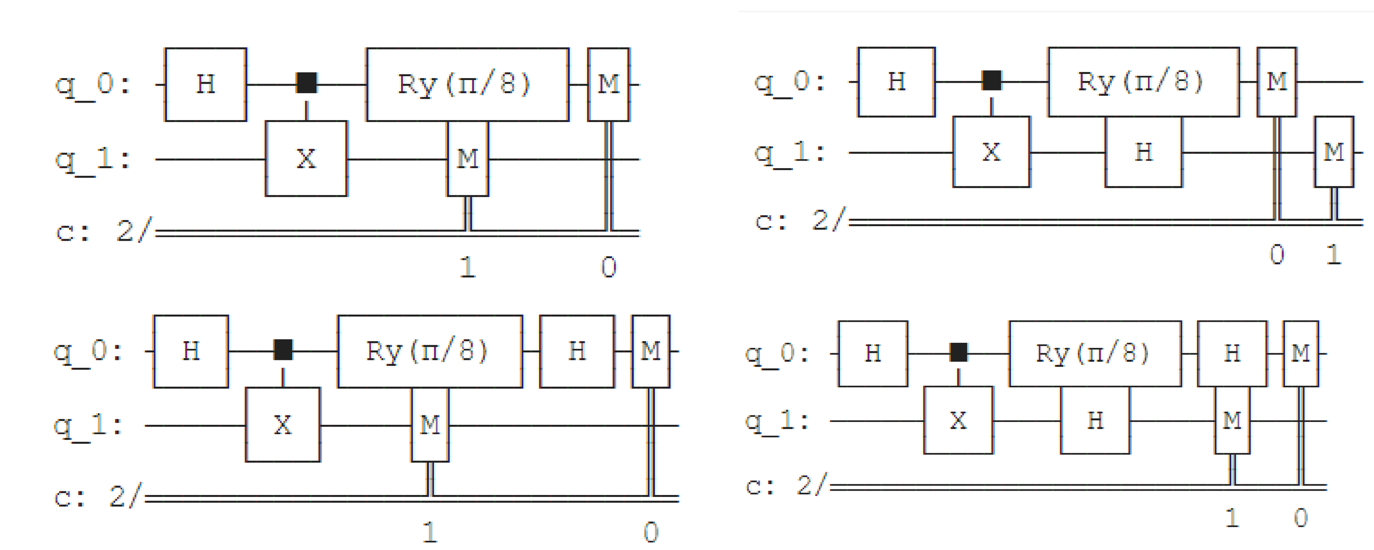
Evaluating this, we get approximately 0.853



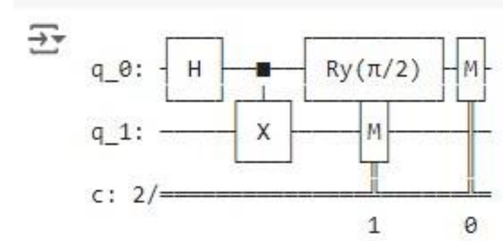
Methods

Using Qiskit to Provide Circuit Outcomes

Creating circuits with respect to angle values:
The function generates a list of quantum circuits for testing the CHSH inequality. Each circuit is configured for different combinations of measurement bases and rotation angles. The circuits prepare entangled qubits using Hadamard and CNOT gates, rotate qubits around the y-axis, and apply additional Hadamard gates based on the measurement basis before measuring both qubits.




```
[38] theta = np.pi / 2
     chsh_circuit_pi_2 = create_chsh_circuit(theta)
     print(chsh_circuit_pi_2)
```



Influence of Angles on Quantum CHSH

Inequality Formulation Classically:
For any local hidden variable model, the CHSH inequality is expressed as:

$$S = E(A1, B1) + E(A1, B2) + E(A2, B1) - E(A2, B2),$$

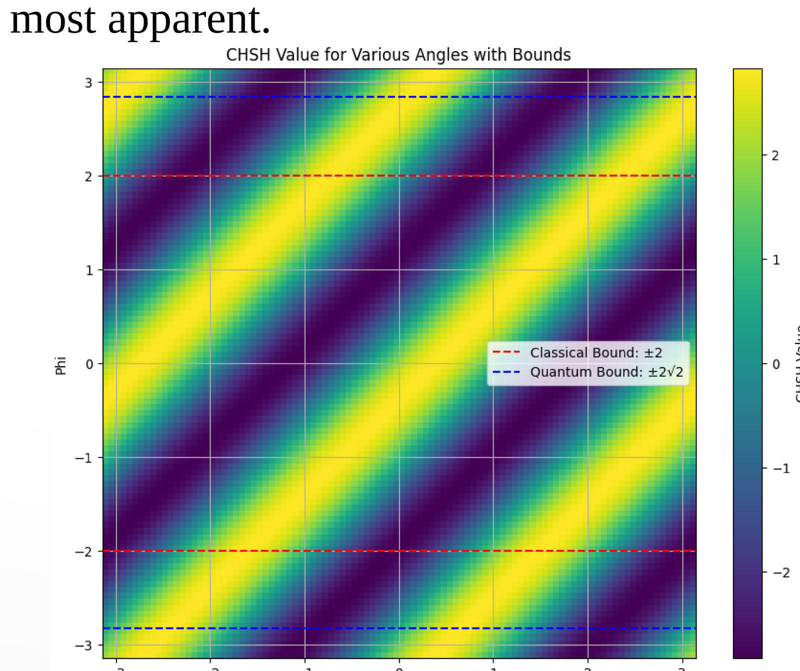
where S must satisfy:

$$|S| \leq 2$$

$E(A1, B1)$ represents the expected value of $(-1)^{a\oplus b}$whose probabilities are determined by measurements A and B.

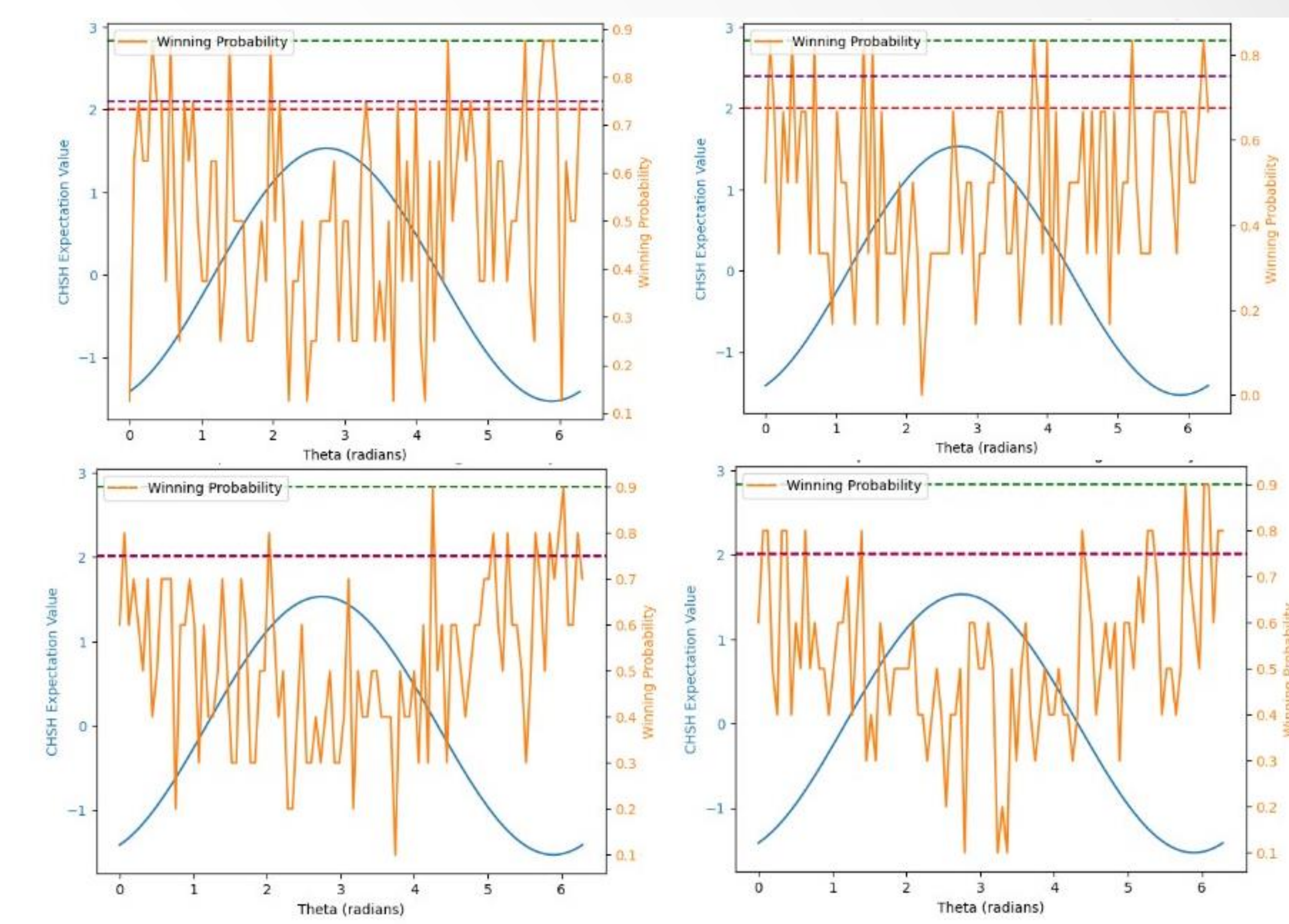
In Quantum, the bound can reach $2\sqrt{2}$ instead of 2. A bound of $2\sqrt{2}$ highlights the non-local nature of quantum mechanics, arising from the quantum correlations between entangled particles, which cannot be explained by local hidden variables.

Understanding the Diagonal Pattern:
The diagonal pattern observed in a CHSH heatmap is a result of the trigonometric relationships between the measurement angles θ and ϕ. The CHSH value is influenced by these angles through correlation functions that depend on phase differences such as θ − ϕ. This dependence generates diagonal patterns in the heatmap, which visually represent how quantum mechanical interference leads to correlations that can surpass classical limits. Consequently, these diagonal bands indicate significant angles where quantum effects are particularly strong, highlighting the regions where quantum correlations become most apparent.



Results

CHSH inequality Violation & Winning Probability Graph outcomes



Implications of Winning Probability Touching the Green Line:

- **Violation of Classical Bound:** If the winning probability reaches or exceeds the quantum bound (represented by the green dashed line), it indicates that the CHSH value is achieving the maximum quantum mechanical prediction, which violates the classical bound of 2 (red dashed line). When the winning probability exceeds the red dashed line, it reveals that the observed correlations cannot be accounted for by classical theories adhering to local realism. This breach highlights the non-classical, entangled nature of the quantum states under examination.
- **Optimal Settings for Quantum Violation:** Touching the quantum bound suggests that the angles used in the measurements are optimized for demonstrating quantum mechanical effects. In practice, this would mean that the chosen angles lead to the maximal violation of the CHSH inequality allowed by quantum mechanics.
- **Verification of Quantum Predictions**: If the winning probability aligns with the quantum bound, it verifies that the experimental setup is effectively capturing quantum mechanical correlations. This is a key result in experiments designed to test the validity of quantum mechanics versus classical theories of local realism.

Future Directions

In the future, I hope to expand these simulations to higher-dimensional quantum systems or more complex entangled states to explore if similar violations of classical bounds occur. Additionally, investigating the impact of noise and imperfections on the CHSH inequality in practical quantum systems could provide insights into the robustness of quantum correlations under realistic conditions.

References

- Soljanin, E. (2020). Quantum information processing: An essential primer. IEEE.
- Hooybergths, J. (2022). The CHSH Game. In: Introducing Microsoft Quantum Computing for Developers. Apress, Berkeley, CA.

Acknowledgments

I would like to express my sincere gratitude to Professor Emina Soljanin and PhD student Michael Schleppy for their invaluable guidance throughout this research project. Their assistance has been instrumental in navigating this complex topic. Thanks to their dedication, I have developed valuable research skills and look forward to building upon this project in the future.