EECE 5550 HW2.

Eileen Wang

Problem 1

(a) $p_w = \Phi(0,0)\, p_r$

$$d\Phi(0,0) = \Phi(0,0) = \begin{pmatrix} R(0) & 0 \\ 0 & 1 \end{pmatrix} = I$$

(b) $t = (\dot{x}, \dot{y})$

$$\begin{pmatrix} \dot{t} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} (\dot\varphi_r + \dot\varphi_l)\frac{r}{2} \\ \frac{r}{w}(\dot\varphi_r - \dot\varphi_l) \end{pmatrix}$$

$$\dot{x}(\varphi_l, \varphi_r) = d\Phi(0,0) \begin{pmatrix} \dot{t} \\ \dot{\theta} \end{pmatrix} = I \cdot \begin{pmatrix} \frac{r}{2}(\dot\varphi_r + \dot\varphi_l) \\ \frac{r}{w}(\dot\varphi_r - \dot\varphi_l) \end{pmatrix} = \begin{pmatrix} \frac{r}{2}(\dot\varphi_r + \dot\varphi_l) \\ \frac{r}{w}(\dot\varphi_r - \dot\varphi_l) \end{pmatrix}$$

(c) $\frac{X}{|X|} = I. \quad X = I \cdot |X|$

$T_{RF} = \Phi(t, \theta)$

$T_{FW} = |X|$

$T_{RW} = T_{RF} \cdot T_{FW} = \Phi(t,\theta)\,|X|$

$$V(X, \varphi_l, \varphi_r) = T_{RW} \begin{pmatrix} \frac{r}{2}(\dot\varphi_r + \dot\varphi_l) \\ \frac{r}{w}(\dot\varphi_r - \dot\varphi_l) \end{pmatrix} = |X| \begin{pmatrix} R(\theta) & t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{r}{2}(\dot\varphi_r + \dot\varphi_l) \\ \frac{r}{w}(\dot\varphi_r - \dot\varphi_l) \end{pmatrix}$$

(d) $V_{(\dot\varphi_l, \dot\varphi_r)}(X) \triangleq V(X, \varphi_l, \varphi_r)$

$$= \begin{pmatrix} R(0) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{r}{2}(\dot\varphi_r + \dot\varphi_l) \\ \frac{r}{w}(\dot\varphi_r - \dot\varphi_l) \end{pmatrix} |X| = \begin{pmatrix} \frac{r}{2}(\dot\varphi_r + \dot\varphi_l) \\ \frac{r}{w}(\dot\varphi_r - \dot\varphi_l) \end{pmatrix} |X|.$$

With selected wheel speed, $\dot\varphi$ is fixed. This is a constant vector field.

(e) $\Upsilon(t) \triangleq (X(t), y(t), \theta(t)) \qquad X_0 = (x_0, y_0, \theta_0)$

$\dot\theta = \frac{r}{w}(\dot\varphi_r - \dot\varphi_l)$

$\theta(t) = \frac{r}{w}(\dot\varphi_r - \dot\varphi_l) \cdot t + \theta_0$

$\dot{x} = \frac{r}{2}(\dot\varphi_r + \dot\varphi_l)\cos\theta \qquad \dot{y} = \frac{r}{2}(\dot\varphi_r + \dot\varphi_l)\sin\theta.$

$$\Delta x = \int_0^t \frac{r}{2}(\dot\varphi_r + \dot\varphi_l)\cos\theta(t)\, dt = \frac{w(\dot\varphi_r + \dot\varphi_l)}{2(\dot\varphi_r - \dot\varphi_l)}(\sin\theta(t) - \sin\theta_0)$$

$x(t) = X_0 + \Delta X$

$$\Delta y = \int_0^t \frac{r}{2}(\dot\varphi_r + \dot\varphi_l)\sin\theta(t)\, dt = \frac{w(\dot\varphi_r + \dot\varphi_l)}{2(\dot\varphi_r - \dot\varphi_l)}(\cos\theta_0 - \cos\theta(t))$$

$y(t) = y_0 + \Delta y$

$$\Upsilon(t) = \begin{pmatrix} \frac{w(\dot\varphi_r + \dot\varphi_l)}{2(\dot\varphi_r - \dot\varphi_l)}(\sin\theta(t) - \sin\theta_0) + x_0 \\ \frac{w(\dot\varphi_r + \dot\varphi_l)}{2(\dot\varphi_r - \dot\varphi_l)}(\cos\theta_0 - \cos\theta(t)) + y_0 \\ \frac{r}{w}(\dot\varphi_r - \dot\varphi_l) + \theta_0 \end{pmatrix}$$

(f) $\Delta\theta = \theta(T) - \theta_0$

$\frac{r}{w}(\dot\varphi_r - \dot\varphi_l) = \frac{\Delta\theta}{T} \qquad (1)$

$\Delta X = X(T) - X_0$

$\frac{w(\dot\varphi_r + \dot\varphi_l)}{2(\dot\varphi_r - \dot\varphi_l)}(\sin\theta(T) - \sin\theta_0) = X(T) - X_0 \qquad (2)$

use (1) and (2) to calculate $\dot\varphi_r$ and $\dot\varphi_l$

2. (a) $\frac{r}{2}(\dot\varphi_r + \dot\varphi_l) \le 0.22$ m/s.

$\frac{0.033}{2}(\dot\varphi_r + \dot\varphi_l) \le 0.22.$

$\dot\varphi_r + \dot\varphi_l \le 13.33.$

$\dot\varphi_{max} \le 13.33$ m/s.

(b) $R = 1.5$ m. $V = 0.2$ m/s.

$\dot{x}_R = 0.2$ m/s.

$\dot\theta_R = V/R = 0.2/1.5 \approx 0.13$ rad/s.