

Objectives

- Understand the correspondence between a truth table and a Karnaugh map
- Understand how to fill out a Karnaugh map for a given expression
- Understand how to group items in a Karnaugh map
- Interpret the groupings in a Karnaugh map
- Simplify Boolean expressions with two, three or four variables using a Karnaugh map

Karnaugh maps

- We have seen how to complete truth tables for logic gates and logic circuits
- How do you simplify a complex Boolean expression?
- Karnaugh maps are similar to truth tables, and provide an alternative, often easier, method of simplifying expressions



Truth tables and Karnaugh maps

Compare the truth table with the Karnaugh map:

Α	В	Р
0	0	а
0	1	b
1	0	С
1	1	d

AB	0	1
0	а	b
1	С	d



Truth tables and Karnaugh maps

Fill in the Karnaugh map for the truth table below:

Α	В	Р
0	0	0
0	1	1
1	0	1
1	1	0

AB	0	1
0		
1		



Truth tables and Karnaugh maps

Fill in the Karnaugh map for the truth table below:

Α	В	Р
0	0	0
0	1	1
1	0	1
1	1	0

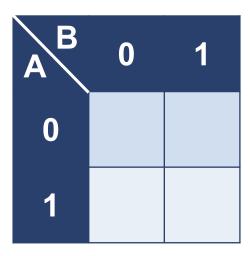
AB	0	1
0	0	1
1	1	0

- The top right square represents A = 0, B = 1
 - What logic gate does this represent?



• Fill in the Karnaugh map for the expression:

$$A \lor (A \land B)$$

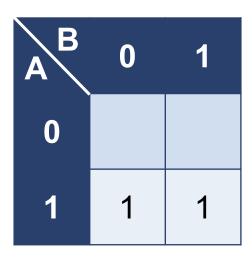


Step 1: Divide the expression into the two parts on each side of the OR symbol (V)

The whole expression is true if A is true OR if (A AND B) is true

Fill in the Karnaugh map for the expression

$$A \lor (A \land B)$$

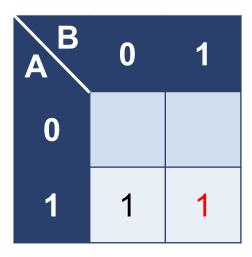


Step 2: Write a 1 in each square for which A is true



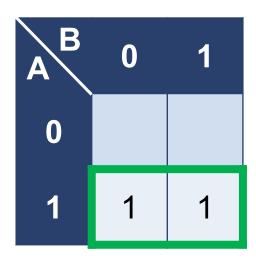
Fill in the Karnaugh map for the expression

$$A \lor (A \land B)$$



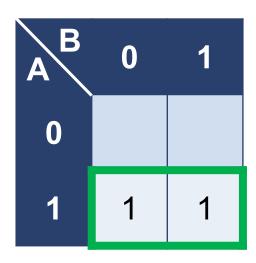
Step 3: Write a 1 in each square for which A ∧ B is true (The square, shown in red here, is already filled in, so nothing to do here)

Fill in the Karnaugh map for the expression
 A ∨ (A ∧ B)



Step 4: Draw a box around the group of two adjacent 1s What expression is represented by the group?

Fill in the Karnaugh map for the expression
 A ∨ (A ∧ B)



- The group outlined in green represents A
- This is one of the Absorption rules
- (A ∧ B) has been absorbed into the A group

A 3-variable Karnaugh map

 With three variables, we combine two of them across the columns

ABO	C 00	01	11	10
0				
1				

- The terms 00, 01, 11, 10 are arranged so that in each subsequent term *only one digit changes*
 - Note carefully that they are NOT in numerical sequence



Example

Use a Karnaugh map to represent (A ∧ ¬ B) ∨ C

A BO	00	01	11	10
0				
1	1	1		

Step 1: Put 1 in all squares in which (A ∧ ¬ B) is true



Example

Use a Karnaugh map to represent (A ∧ ¬ B) ∨ C

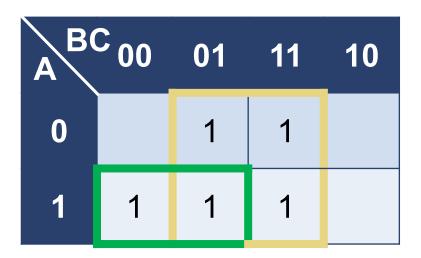
ABO	00	01	11	10
0		1	1	
1	1	1	1	

Step 2: Put 1 in all squares in which C is true



A 3-variable Karnaugh map

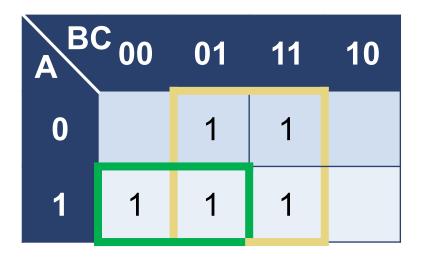
The map represents (A ∧ ¬ B) ∨ C



- We now group the items in groups of 1, 2, 4 or 8 (The groups can be overlapping)
- Make each group as large as possible
- Each 1 must be included in at least one group

A 3-variable Karnaugh map

The map represents (A ∧ ¬ B) ∨ C



- The green group represents A ∧ ¬ B
- The yellow group represents C
- NOTE: Never make groups of 3, 5, 6 or 7!
 Only groups of 1, 2, 4 or 8 are allowed



Exercise

• Fill in 1s to represent:

ABO	C 00	01	11	10
0				
1				



• Fill in 1s to represent:

ABO	C 00	01	11	10
0			1	
1				



• Fill in 1s to represent:

A BO	² 00	01	11	10
0			1	
1		1		



• Fill in 1s to represent:

ABO	C 00	01	11	10
0			1	
1		1		1



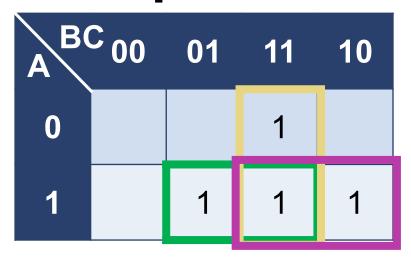
• Fill in 1s to represent:

$$(\neg A \land B \land C) \lor (A \land \neg B \land C) \lor (A \land B \land \neg C) \lor (A \land B \land C)$$

ABO	C 00	01	11	10
0			1	
1		1	1	1

- Now group them in overlapping pairs
- Remember to make each group as large as possible
- How many groups do you end up with?





- The green group represents A ∧ C
- The purple group represents A ∧ B
- The yellow group represents B ∧ C
- We have simplified the expression from:

(¬A
$$\wedge$$
 B \wedge C) \vee (A \wedge ¬B \wedge C) \vee (A \wedge B \wedge ¬C) \vee (A \wedge B \wedge C)

to: $(A \land C) \lor (B \land C) \lor (A \land B)$



Worksheet 3

Now try the questions in Task 1



A 4-variable Karnaugh map

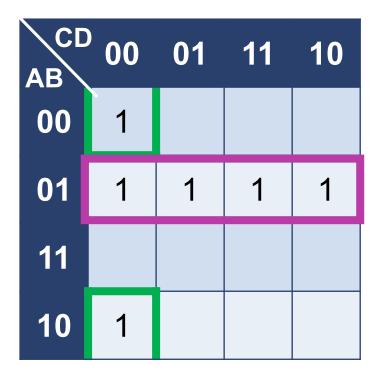
What does this map represent?

CD AB	00	01	11	10
00	1			
01	1	1	1	1
11				
10	1			



A 4-variable Karnaugh map

What does this map represent?

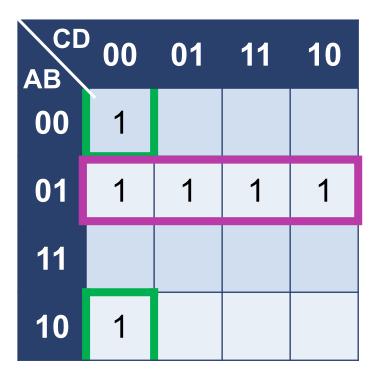


Notice that the green group "wraps" but is still a single group



A 4-variable Karnaugh map

This represents: (¬A ∧ B) V (¬B ∧ ¬C ∧ ¬D)



Notice that the green group "wraps" but is still a single group



Worksheet 3

Now try the questions in Task 2



Plenary

- Karnaugh maps provide a method of simplifying expressions
- This is often an easier method than using Boolean algebra (and more fun...)
- You need to know both methods



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