

OCR

A Level

Computer Science
H446 – Paper 1

3

Karnaugh maps

Unit 8

Boolean Algebra



PG ONLINE

Objectives

- Understand the correspondence between a truth table and a Karnaugh map
- Understand how to fill out a Karnaugh map for a given expression
- Understand how to group items in a Karnaugh map
- Interpret the groupings in a Karnaugh map
- Simplify Boolean expressions with two, three or four variables using a Karnaugh map

Karnaugh maps

- We have seen how to complete truth tables for logic gates and logic circuits
- How do you simplify a complex Boolean expression?
- Karnaugh maps are similar to truth tables, and provide an alternative, often easier, method of simplifying expressions

Truth tables and Karnaugh maps

- Compare the truth table with the Karnaugh map:

A	B	P
0	0	a
0	1	b
1	0	c
1	1	d

		B	
		0	1
A	0	a	b
	1	c	d

Truth tables and Karnaugh maps

- Fill in the Karnaugh map for the truth table below:

A	B	P
0	0	0
0	1	1
1	0	1
1	1	0

		B	
		0	1
A	0		
	1		

Truth tables and Karnaugh maps

- Fill in the Karnaugh map for the truth table below:

A	B	P
0	0	0
0	1	1
1	0	1
1	1	0

		B	
		0	1
A	0	0	1
	1	1	0

- The top right square represents $A = 0, B = 1$
 - What logic gate does this represent?

Simplifying an expression

- Fill in the Karnaugh map for the expression:

$$A \vee (A \wedge B)$$

A \ B	0	1
0		
1		

Step 1: Divide the expression into the two parts on each side of the OR symbol (\vee)

The whole expression is true if A is true OR if (A AND B) is true

Simplifying an expression

- Fill in the Karnaugh map for the expression

$$A \vee (A \wedge B)$$

A \ B	0	1
0		
1	1	1

Step 2: Write a 1 in each square for which A is true

Simplifying an expression

- Fill in the Karnaugh map for the expression

$$A \vee (A \wedge B)$$

		B	
		0	1
A	0		
	1	1	1

- Step 3: Write a 1 in each square for which $A \wedge B$ is true
(The square, shown in red here, is already filled in, so nothing to do here)

Simplifying an expression

- Fill in the Karnaugh map for the expression

$$A \vee (A \wedge B)$$

		B	
		0	1
A	0		
	1	1	1

- Step 4: Draw a box around the group of two adjacent 1s
- What expression is represented by the group?

Simplifying an expression

- Fill in the Karnaugh map for the expression

$$A \vee (A \wedge B)$$

		B	
		0	1
A	0		
	1	1	1

- The group outlined in green represents A
- This is one of the Absorption rules
- $(A \wedge B)$ has been absorbed into the A group

A 3-variable Karnaugh map

- With three variables, we combine two of them across the columns

A \ BC	BC			
	00	01	11	10
0				
1				

- The terms 00, 01, 11, 10 are arranged so that in each subsequent term *only one digit changes*
 - Note carefully that they are NOT in numerical sequence

Example

- Use a Karnaugh map to represent $(A \wedge \neg B) \vee C$

A \ BC	BC			
	00	01	11	10
0				
1	1	1		

Step 1: Put 1 in all squares in which $(A \wedge \neg B)$ is true

Example

- Use a Karnaugh map to represent $(A \wedge \neg B) \vee C$

A \ BC	BC			
	00	01	11	10
0		1	1	
1	1	1	1	

Step 2: Put 1 in all squares in which C is true

A 3-variable Karnaugh map

- The map represents $(A \wedge \neg B) \vee C$

A \ BC	BC			
	00	01	11	10
0		1	1	
1	1	1	1	

- We now group the items in groups of 1, 2, 4 or 8 (The groups can be overlapping)
- Make each group as large as possible
- Each 1 must be included in at least one group

A 3-variable Karnaugh map

- The map represents $(A \wedge \neg B) \vee C$

A \ BC	BC			
	00	01	11	10
0		1	1	
1	1	1	1	

- The green group represents $A \wedge \neg B$
- The yellow group represents C
- **NOTE: Never make groups of 3, 5, 6 or 7!**
Only groups of 1, 2, 4 or 8 are allowed

Exercise

- Fill in 1s to represent:

$$(\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C)$$

A \ BC	BC			
	00	01	11	10
0				
1				

Exercise – Step 1

- Fill in 1s to represent:

$$(\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C)$$

A \ BC	BC			
	00	01	11	10
0			1	
1				

Exercise – Step 2

- Fill in 1s to represent:

$$(\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C)$$

A \ BC	BC			
	00	01	11	10
0			1	
1		1		

Exercise – Step 3

- Fill in 1s to represent:

$$(\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C)$$

A \ BC	BC			
	00	01	11	10
0			1	
1		1		1

Exercise – Step 4

- Fill in 1s to represent:

$$(\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C)$$

A \ BC	BC			
	00	01	11	10
0			1	
1		1	1	1

- Now group them in overlapping pairs
- Remember to make each group as large as possible
- How many groups do you end up with?

Exercise – Step 5

A \ BC		BC			
		00	01	11	10
A	0			1	
	1		1	1	1

- The green group represents $A \wedge C$
- The purple group represents $A \wedge B$
- The yellow group represents $B \wedge C$
- We have simplified the expression from:

$$(\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C)$$

$$\text{to: } (A \wedge C) \vee (B \wedge C) \vee (A \wedge B)$$

Worksheet 3

- Now try the questions in **Task 1**



A 4-variable Karnaugh map

- What does this map represent?

AB \ CD	CD			
	00	01	11	10
00	1			
01	1	1	1	1
11				
10	1			

A 4-variable Karnaugh map

- What does this map represent?

CD \ AB		CD			
		00	01	11	10
AB	00	1			
	01	1	1	1	1
	11				
	10	1			

- Notice that the green group “wraps” but is still a single group

A 4-variable Karnaugh map

- This represents: $(\neg A \wedge B) \vee (\neg B \wedge \neg C \wedge \neg D)$

AB \ CD	CD			
	00	01	11	10
00	1			
01	1	1	1	1
11				
10	1			

- Notice that the green group “wraps” but is still a single group

Worksheet 3

- Now try the questions in **Task 2**



Plenary

- Karnaugh maps provide a method of simplifying expressions
- This is often an easier method than using Boolean algebra (and more fun...)
- You need to know both methods

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