

DERIVATION

Consider 1 student in category i

Define a r.v

$$X_{ij} = \begin{cases} 1, & \text{if student } j \text{ attends lecture } i \\ 0, & \text{otherwise} \end{cases}$$

Since students attend independently,

$$P(X_{ij} = 1) = p_i \quad (1)$$

This is a Bernoulli r.v with parameter p_i ,

$$E[X_{ij}] = 1 \times p_i + 0 \times (1 - p_i) = p_i$$

The total number of students who attend is

$$O_i = \sum_{j=1}^{N_i} X_{ij}$$

Linearity of Expectation gives

$$E[O_i] = E\left[\sum_{j=1}^{N_i} X_{ij}\right] = \sum_{j=1}^{N_i} E[X_{ij}]$$

Substitute (1)

$$E[O_i] = \sum_{j=1}^{N_i} p_i = N_i p_i$$