



Functional Geometry

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Abstract. An algebra of pictures is described that is sufficiently powerful to denote the structure of a well-known Escher woodcut, Square Limit. A decomposition of the picture that is reasonably faithful to Escher's original design is given. This illustrates how a suitably chosen algebraic specification can be both a clear description and a practical implementation method. It also allows us to address some of the criteria that make a good algebraic description.

Keywords: Functional programming, graphics, geometry, algebraic style, architecture, specification.

A **picture** is an example
of a **complex object** that
can be described in terms
of its **parts**.

Let us define a picture as a **function** which takes three arguments, each being two-space **vectors** and returns **a set of graphical objects** to be rendered on the output device.

```
type Box = { a : Vector  
             b : Vector  
             c : Vector }
```

```
type Picture = Box -> Rendering
```