To show that every even length palindrome is divisible by 11:

1. $Let\ x, y, c \in \mathbb{Z}^+$
   a. $x + y = k\ where\ k \in \mathbb{Z}^+$
   b. $x = ck_1\ and\ y = ck_2\ where\ k_1, k_2 \in \mathbb{Z}^+\ then$
   c. $ck_1 + ck_2 = c(k_1 + k_2) = k$
   d. $\frac{k}{c} = k_1 + k_2 = k_3$ where $k_3 \in \mathbb{Z}^+\ therefore$
   e. $\frac{k}{c} \in \mathbb{Z}^+$

2. $Let\ a \in \mathbb{Z}^+\ and\ aaa \ldots a\ be\ an\ even\ length\ integer\ of\ repeating\ digits.$
   a. $aaa \ldots a\ = 10^{n-1}a + 10^{n-2}a + \cdots + 10^0 a$
   b. $\quad = a(10^{n-1} + 10^{n-2} + \cdots + 10^0)$
   c. $\quad = a\left(10^{n-2}(10^1 + 1) + \cdots + (10^1 + 1)\right)$
   d. $\quad = 11(10^{n-2} + 10^{n-4} + 10^{n-6} + \cdots + 10^2)$
   e. $\quad = 11\left(10^2(k + 1)\right)$ where $k \in \mathbb{Z}^+$

   $If\ the\ integer\ of\ repeating\ digits\ has\ an\ odd\ length\ then\ it\ expands\ to$:
   $aaa \ldots a\ = a\left(10^{n-2}(10^1 + 1) + \cdots + 10^2 + (10 + 1)\right)$
   which is not divisible by 11.

3. *Let abc…dd…cba be a palindrome of integers, which itself is an integer.*
   a. *abc…dd…cba* = a(100…001) + b(100…010) + c(100…0100) + … + d(110…0)
   b. = a(99…90 + 11) + b(99…900 + 110) + c(99…9000 + 1100) + … + d(110…0)
   c. = a(99…9 × 10 + 11) + b(99…9 × 100 + 110) + c(99…9 ×1000 + 1100) + … + d(11 × 10…0)

By the second theorem, if the coefficients which are of repeating digits in these expansions are an even number of digits, then they are divisible by 11, *rendering every part of the sum also divisible by* 11. *By the first theorem, the sum of these parts would then also be divisible by* 11. *Therefore, only the even length palindromes are divisible by* 11. *For palindromes of even lengths, these repeating integers of 9 are always even in length. To show this consider an arbitrary even length palindrome, abccba. This yields*

a. *abccba* = a (100001) + b(10010) + c(1100)
b. = a(9999 × 10 + 11) + b(99 × 100 + 110) + c(11 × 100)

This intuitively makes sense since any even length number where the digits on both ends are removed, leaves an even number of digits. The last remaining value, c, is adjacent to c. Any even number palindrome has this feature since each half is the same as the other half in reverse order. Therefore, this middlemost part, c in our example, is some number 11×10^{k} *which is certainly divisible by* 11. *The remaining preceding pairs follow the form of:*

$10\ldots01 \times 10^{k}$

This concludes the proof.							*Chris Carter 2/17/2017*