Data Assimilation Studies

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4DVAR

4DVAR

Original formulation

$$J\left(\mathbf{x}_0\right)=\frac{1}{2}\left(\mathbf{x}_0-\mathbf{x}_0^b\right)^T\mathbf{B}^{-1}\left(\mathbf{x}_0-\mathbf{x}_0^b\right)+\sum_t\frac{1}{2}\left[\mathbf{H}\left(\underline{M(\mathbf{x}_0)}\right)-\mathbf{y}_t^o\right]^T\mathbf{R}^{-1}\left[\mathbf{H}\left(\underline{M(\mathbf{x}_0)}\right)-\mathbf{y}_t^o\right]$$

$$\nabla J\left(\mathbf{x}_0\right)=\mathbf{B}^{-1}\left(\mathbf{x}_0-\mathbf{x}_0^b\right)+\sum_{t=1}^{T}\underline{\mathbf{M}_0^T\cdots\mathbf{M}_{t-1}^T}\mathbf{H}^T\mathbf{R}_t^{-1}\left[\mathbf{H}\left(\underline{M(\mathbf{x}_0)}\right)-\mathbf{y}_t^o\right]$$

Nonlinear model

Tangent liner model

Increment formulation

$$\delta\mathbf{x}=\mathbf{x}_0-\mathbf{x}_0^b$$

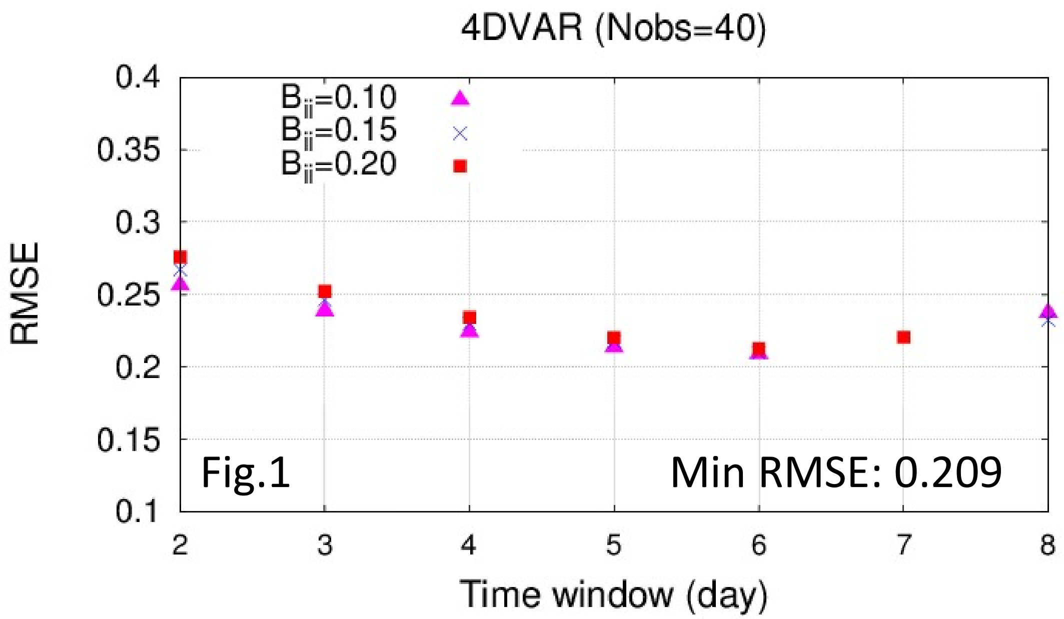
$$\mathbf{H}\left(M(\mathbf{x}_0)\right)-\mathbf{y}_t^o\approx\mathbf{H}\mathbf{M}_{t-1}\cdots\mathbf{M}_0\delta\mathbf{x}-\mathbf{d}$$

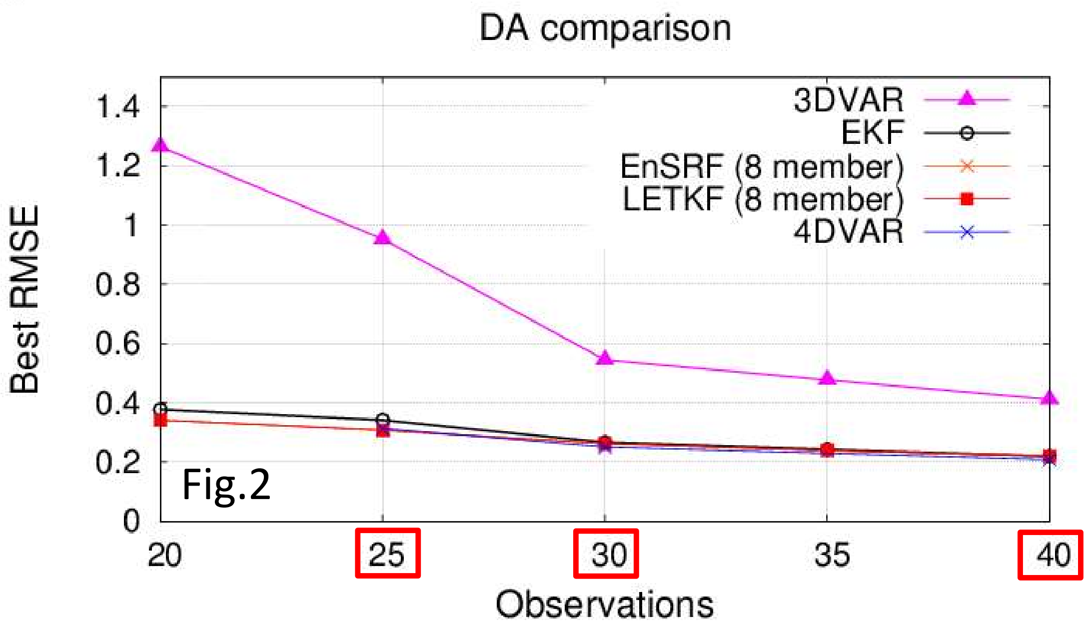
$$\mathbf{d}=\mathbf{y}_t^o-\mathbf{H}\left(M(\mathbf{x}_0^b)\right)$$

$$J\left(\delta\mathbf{x}\right)=\frac{1}{2}\left(\delta\mathbf{x}\right)^T\mathbf{B}^{-1}\left(\delta\mathbf{x}\right)+\sum_t\frac{1}{2}\left[\mathbf{H}\underline{\mathbf{M}_{t-1}\cdots\mathbf{M}_0\delta\mathbf{x}}-\mathbf{d}\right]^T\mathbf{R}^{-1}\left[\mathbf{H}\underline{\mathbf{M}_{t-1}\cdots\mathbf{M}_0\delta\mathbf{x}}-\mathbf{d}\right]$$

$$\nabla J\left(\delta\mathbf{x}\right)=\mathbf{B}^{-1}\left(\delta\mathbf{x}\right)+\sum_{t=1}^{T}\underline{\mathbf{M}_0^T\cdots\mathbf{M}_{t-1}^T}\mathbf{H}^T\mathbf{R}_t^{-1}\left[\mathbf{H}\underline{\mathbf{M}_{t-1}\cdots\mathbf{M}_0\delta\mathbf{x}}-\mathbf{d}\right]$$

comparison


4DVAR (Nobs=40)
B_{ii}=0.10
B_{ii}=0.15
B_{ii}=0.20
RMSE
Time window (day)
Fig.1
Min RMSE: 0.209


DA comparison
3DVAR
EKF
EnSRF (8 member)
LETKF (8 member)
4DVAR
Best RMSE
Observations
Fig.2