Data Assimilation Studies

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Studies about DAs

Lorenz-96 (Lorenz et al., 1998)

$$dX_j / dt = \underbrace{\left(X_{j+1} - X_{j-2}\right) X_{j-1}}_{\text{Advection term}} \underbrace{- X_j}_{\text{Dissipation term}} + \underbrace{F}_{\text{Forcing term}}$$

For j=1,…,J, $X_j=X_{j+J}$

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1. EKF: $0.01 \le \delta \le 0.30$ $\quad P^f \leftarrow P^f (1+\delta)^2$

2. 3DVAR: $0.20 \le b \le 1.00$ $\quad B_{ij} = \begin{cases} b & when(i=j) \\ 0 & when(i \ne j) \end{cases}$

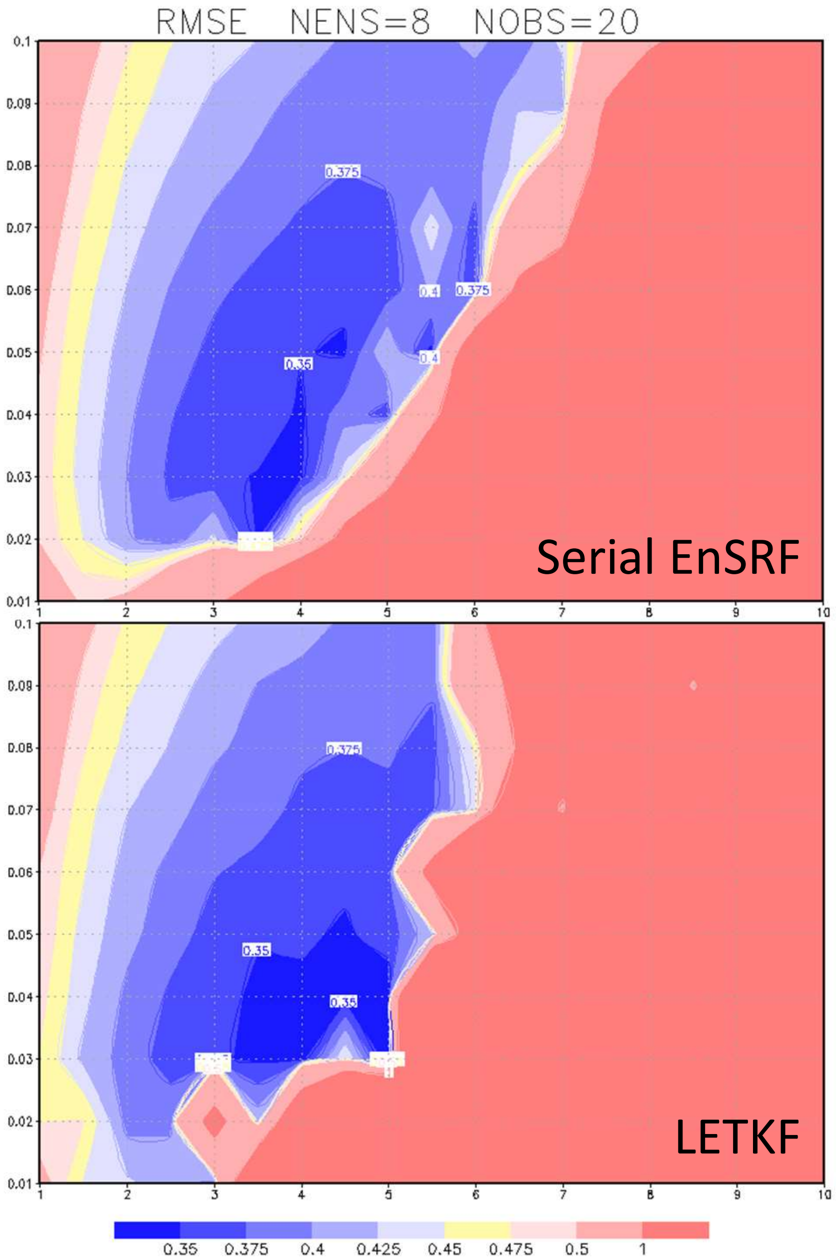
3. EnSRF & LETKF:

$0.01 \le \delta \le 0.10$ $\quad E^f \leftarrow E^f (1+\delta)$

$1.0 \le \sigma \le 10.0$ $\quad r = \dfrac{d}{\sqrt{10/3\sigma}}$

5. PF: $0.2 \le \gamma \le 0.5$ $\quad x_t^{a(l)} \leftarrow x_t^{a(l)} + \gamma \cdot N(0,1)$

EnSRF vs LETKF (ave. 10 simulations)



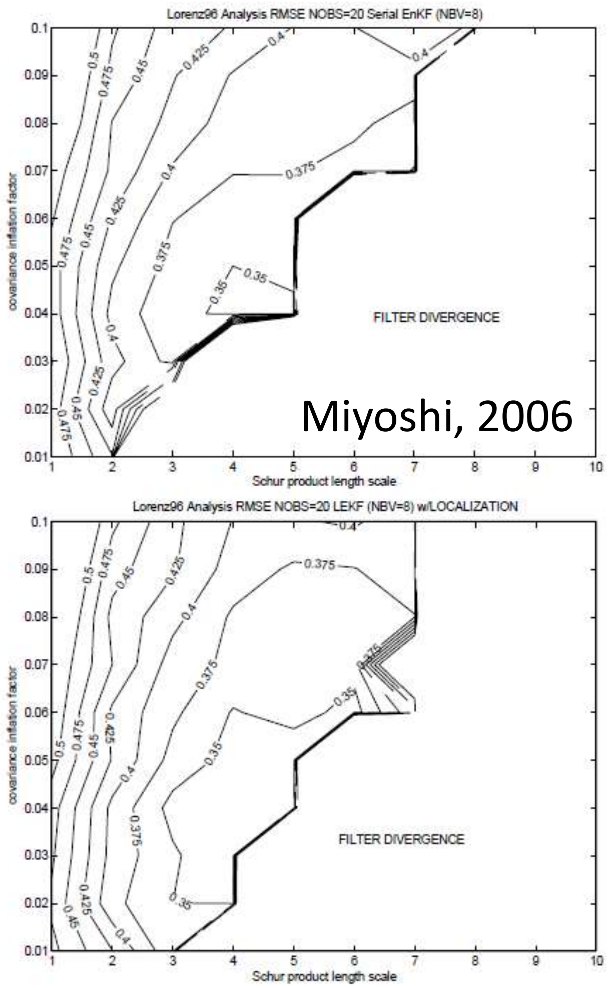
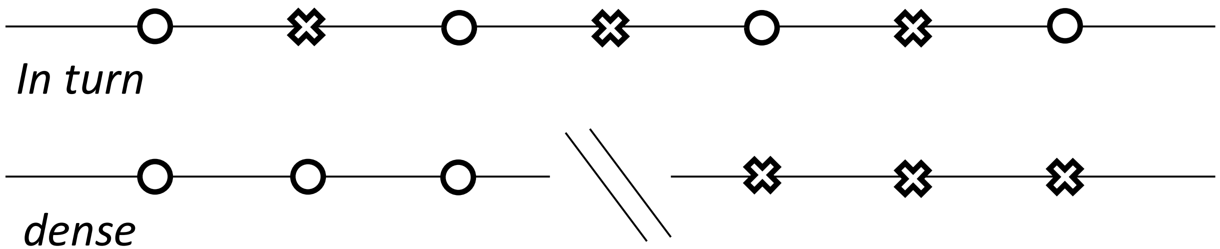


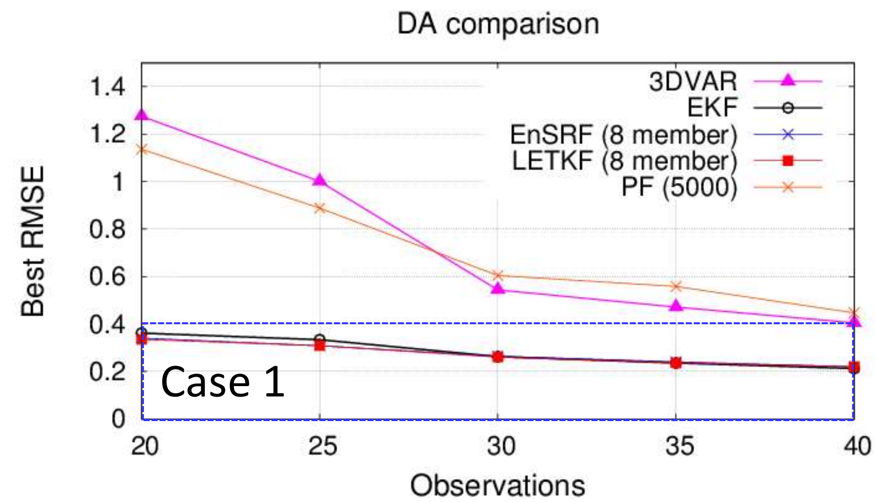
図 6.5.1　40 格子点のうち 20 点を観測し、アンサンブルメンバー数を 8 としたときの Lorenz モデルでの EnKF の解析 RMSE。縦軸は共分散膨張パラメータ、横軸は局所化パラメータを示す。上が Serial EnSRF（最小値は 0.34）、下が LEKF（最小値は 0.33）を表す。"FILTER DIVERGENCE" と書かれている領域は、誤差が発散しフィルタがうまく働かない領域を表す。

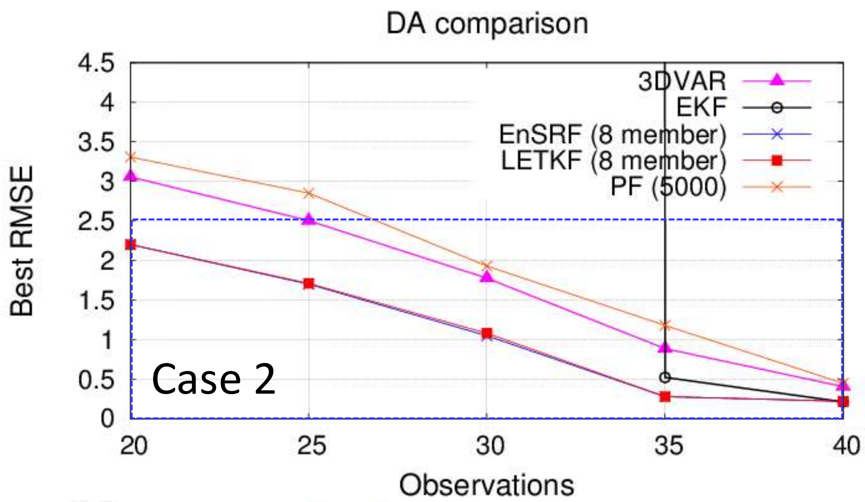
DA comparisons (without 4DVAR)

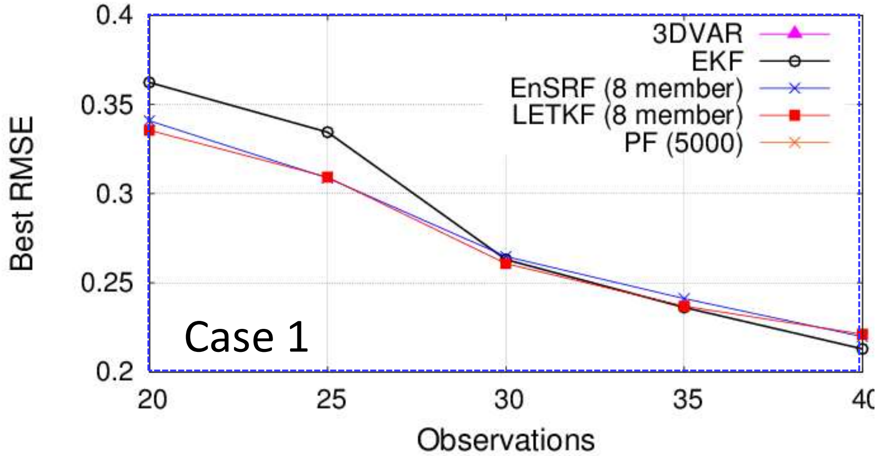
Case 1: Num. Obs. = X

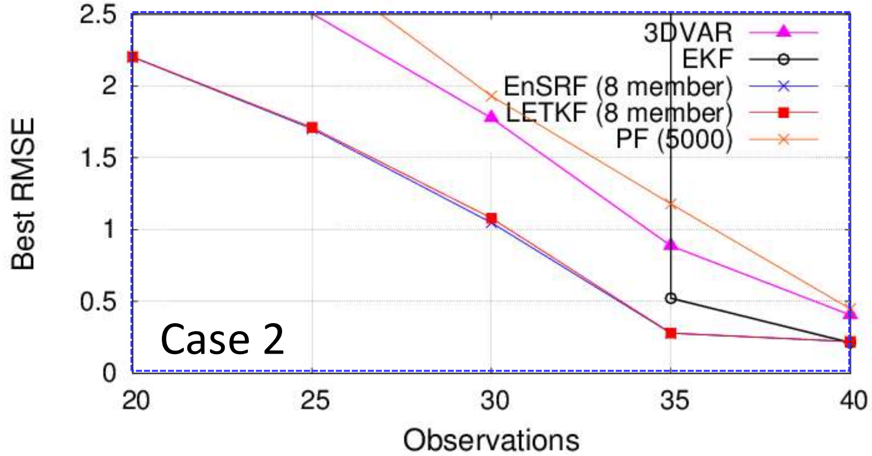
Case 2: Num. Obs. = X









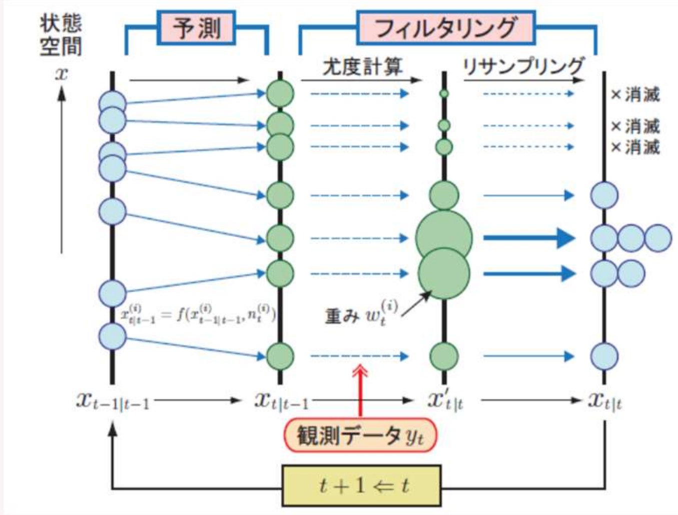


PF

(1) $$x_t^{f(l)} = M x_{t-1}^{a(l)}$$

(2) $$p\left(x_t \mid y_{1:t}\right) \approx \sum_{l=1}^{N} w_t^{(l)} \delta\left(x_t - x_{t|t-1}^{(l)}\right)$$

$$w_t^{(l)} = R\left(y_t \mid x_{t|t-1}^{(l)}\right) / \sum_{l} R\left(y_t \mid x_{t|t-1}^{(l)}\right)$$



Dont resampling (Tachikawa *et al.*, 2011)

$$R\left(y_t \mid x_{t|t-1}^{(l)}\right) \propto \exp\left[-\frac{1}{2}\left(Hx_t^{f(l)} - y_t\right)^T R^{-1}\left(Hx_t^{f(l)} - y_t\right)\right]$$

Observation covariance is approximated as Gaussian Function.

(3) $$x^a = \sum_{l=1}^{N} w_t^{(l)} x_{t|t-1}^{(l)}$$

(4) Resampling and perturbation $$x_t^{a(l)} \leftarrow x_t^{a(l)} + \gamma \cdot N(0,1)$$

（Dont method: very simple resampling）

error variance or likelihood estimation

KF, EnKF

Minimizing error variance estimation

$$P^a = \left\langle \delta x^a \cdot \delta x^{aT} \right\rangle$$

$$\frac{\partial}{\partial K}\left(trace(P^a)\right) = 0$$

$$\Rightarrow \quad K_t = P_t^f H^T \left[H P_t^f H^T + R \right]^{-1}$$

$$\underline{P_t^f = M P_{t-1}^a M^T}$$

P^f is time dependent.

3DVAR, PF

Maximizing likelihood estimation

$$p(x \mid y) = \frac{p(y \mid x)\, p(x)}{p(y)}$$

Bayes' theory

$$p(x) \propto \exp\left[-\frac{1}{2}\left(x - x^f\right)^T \underline{B^{-1}} \left(x - x^f\right) \right]$$

3DVAR

$$p(x_t \mid y_{1:t}) \approx \frac{1}{N} \sum_{l=1}^{N} \underline{\delta\left(x_t - x_{t|t-1}^{(l)}\right)}$$

PF

p(x) is time independent.

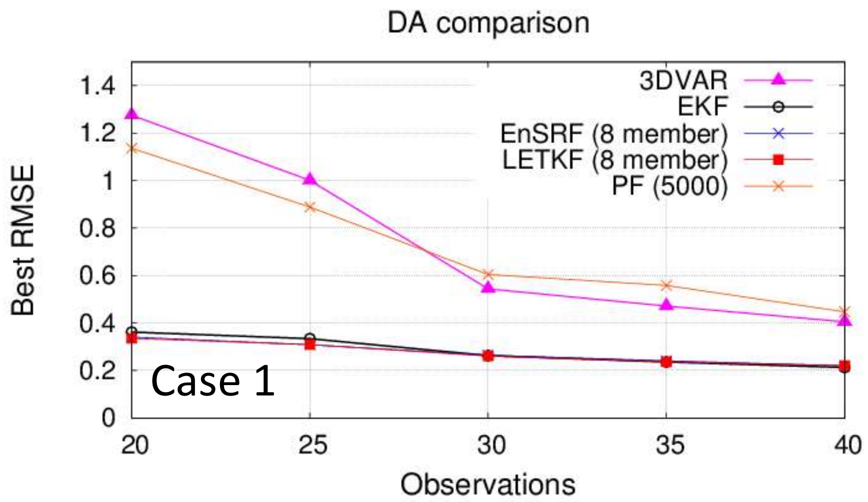
error variance or likelihood estimation

KF, EnKF

Minimizing error variance estimation

$$\underline{P_t^f = M P_{t-1}^a M^T}$$

<u>*P^f* is time dependent.</u>



3DVAR, PF

Maximizing likelihood estimation

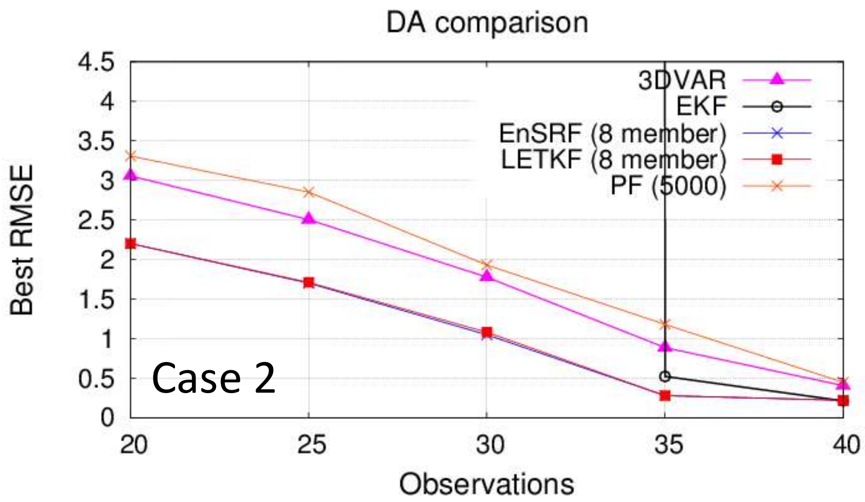
$$p(x) \propto \exp\left[-\frac{1}{2}\left(x - x^f\right)^T \underline{B^{-1}}\left(x - x^f\right)\right]$$

3DVAR

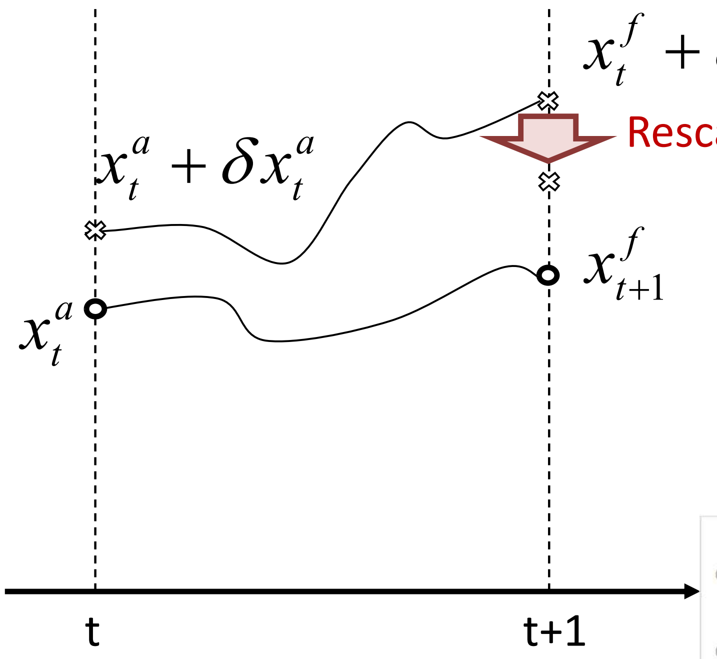
$$p\left(x_t \mid y_{1:t}\right) \approx \frac{1}{N}\sum_{l=1}^{N} \underline{\delta\left(x_t - x_{t|t-1}^{(l)}\right)}$$

PF

<u>*p(x)* is time independent.</u>



Breeding vectors with EKF cycle

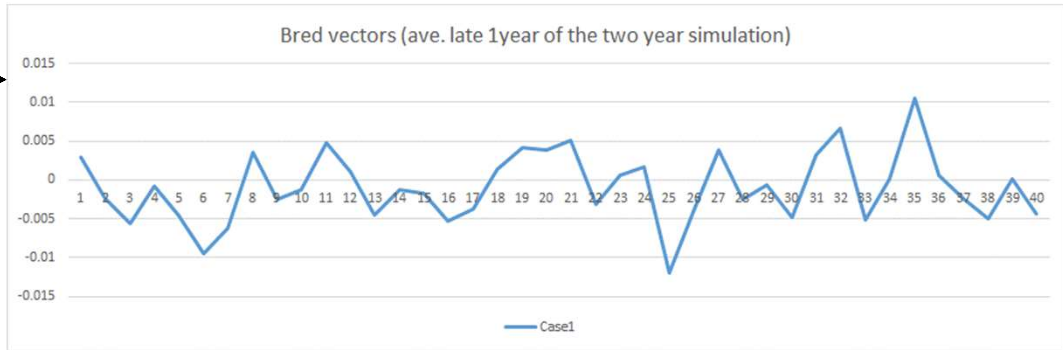


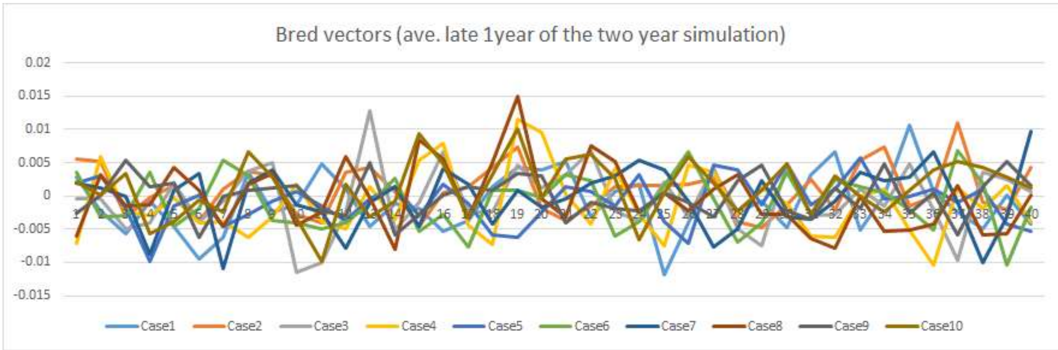
$$\delta x^a = \alpha \cdot \delta x^f$$

$$\alpha = \alpha_{init} / \left| \delta x^f \right|$$

For each grids, rescaling is performed when

$$\left| \delta x^f \right| > \alpha_{init}$$





The chaotic systems do not have same stable bred vectors.
(It's depends on the simulations.)

However, the chaotic system seems to have a stable bred vector.

Ongoing work

- Development of 4DVAR
  - On the way to coding the quasi Newton's method

$$J = \frac{1}{2}\left(x_0^a - x_0^f\right)^T B^{-1}\left(x_0^a - x_0^f\right) + \frac{1}{2}\left[H\begin{pmatrix} x_0^a \\ \vdots \\ x_N^a \end{pmatrix} - y^o\right]^T R^{-1}\left[H\begin{pmatrix} x_0^a \\ \vdots \\ x_N^a \end{pmatrix} - y^o\right]$$

$$\Rightarrow \quad g = \underline{B^{-1}}\left(x_0^a - x_0^f\right) + \sum_{t=0}^{T} \underline{M_0^T \cdots M_{t-1}^T} R_t^{-1}\left[H_t\left(M^t(x_0^a)\right) - y_t^o\right]$$

Are these B and joint models are time independent or not?

(Of course, it is better to change depends on time.)