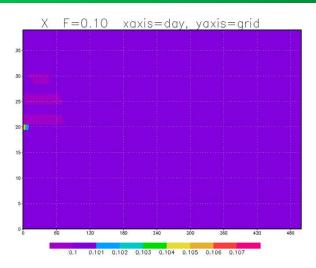
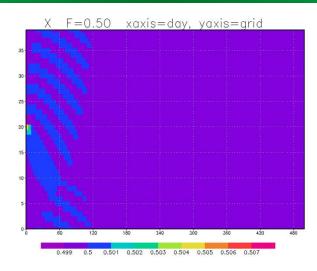
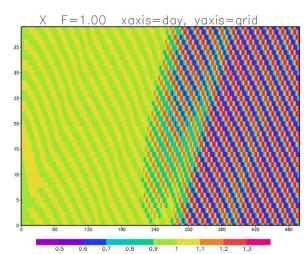
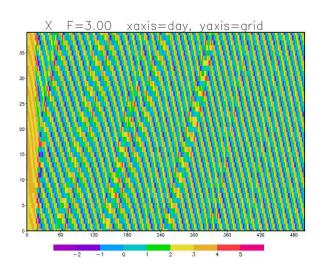


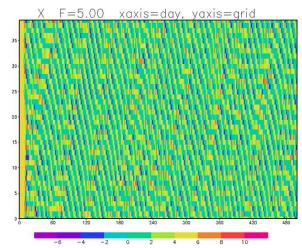
## Stable or chaotic?

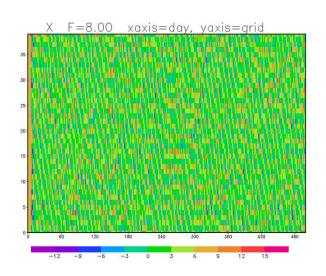




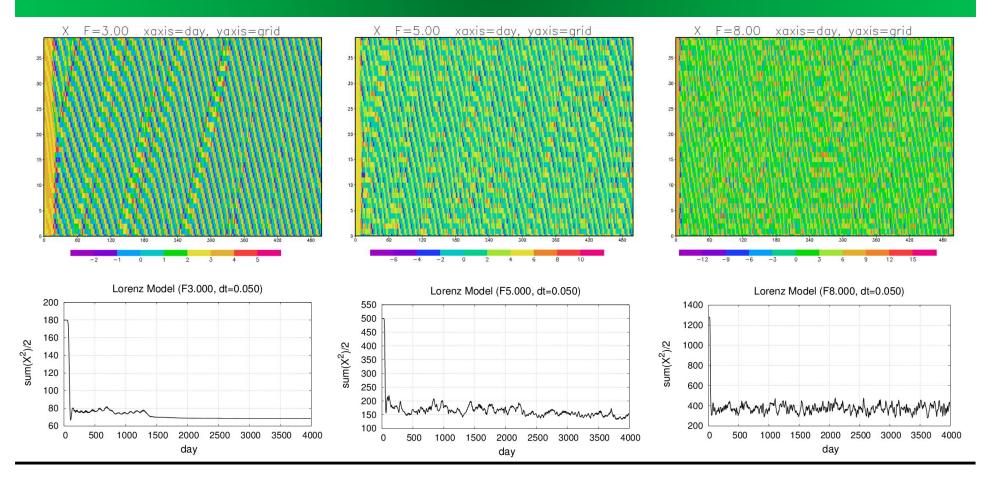








### Stable or chaotic?



$$dX_{j} / dt = (X_{j+1} - X_{j-2})X_{j-1} - X_{j} + F \quad \text{ For j=1,...,J, } X_{j} = X_{j+1} - X_{j} + K_{j} = X_{j+1} - X_{j} + X_{j} = X_{j+1} - X_{j} + X_{j+1} - X_{j} = X_{j+1} - X_{j} + X_{j+1} - X_{j} = X_{j+1} - X_{j} + X_{j+1} - X_{j+1} -$$

Advection term

Dissipation term

Forcing term



Advection term conserve the total energy defined as  $\sum (X_j^2)/2$ 

### **EnKF**

(1) 
$$X_t^{f(l)} = Mx_{t-1}^{a(l)}$$

(2) 
$$P_t^{f(l)} = E_t^f \left( E_t^f \right)^T = M' E_{i-1}^a \left( M' E_{i-1}^a \right)^T$$

$$M' E_{t-1}^a \approx \frac{1}{\sqrt{m-1}} \left[ M x_{t-1}^{f(1)} - x_{t-1}^{-f}, \dots, M x_{t-1}^{f(m)} - x_{t-1}^{-f} \right]$$

(3) 
$$K_t^f = E_t^f (HE_t^f)^T \Big[ HE_t^f (HE_t^f)^T + R \Big]^{-1}$$

$$K_t^f = E_t^f \Big[ I + (HE_t^f)^T R^{-1} HE_t^f \Big]^{-1} (HE_t^f)^T R^{-1}$$

(4) 
$$\bar{x}_{t}^{a} = \bar{x}_{t}^{f} + K_{t}^{f} \left( y - H \bar{x}_{t}^{f} \right)$$

- (5) Update Pa (Ea)
- (6) Update ensemble members

$$E_t^a \approx \frac{1}{\sqrt{m-1}} \left[ x_t^{a(1)} - x_t^{a(1)}, \dots, x_t^{a(m)} - x_t^{a(m)} \right]$$

#### Localization

$$r = \frac{d}{\sqrt{10/3}\sigma}$$

Schur product length scale

$$K \leftarrow S(r) \circ K$$

Fifth-order piecewise rational function (The Gaussian-like weighting function)

Serial EnSRF has been applied to the Lorenz-96 model.

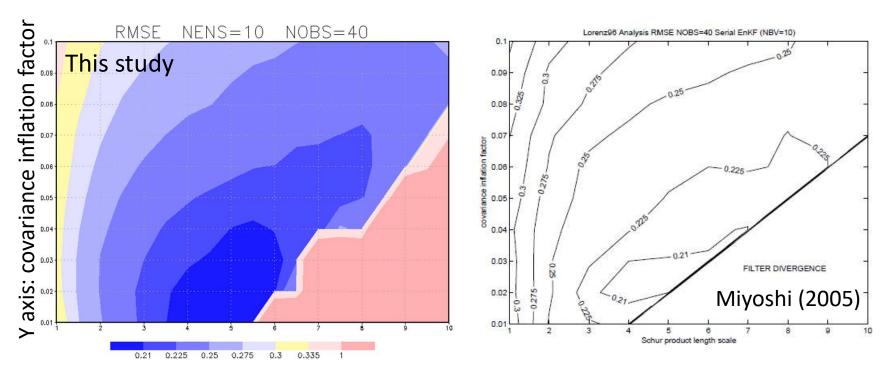
# Multiplicative covariance inflation

$$P^f \leftarrow P^f (1 + \delta)$$

$$P^f = E^f (E^f)^{\mathrm{T}}$$

$$E^f \leftarrow E^f \sqrt{1+\delta}$$

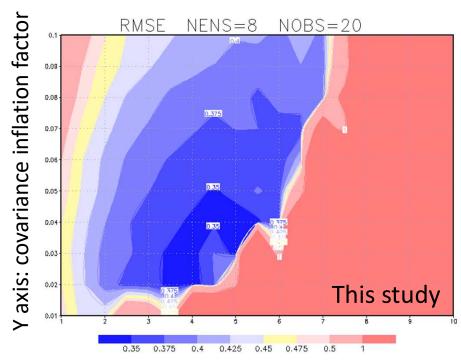
Only once for the first observation of the serial filtering loops.



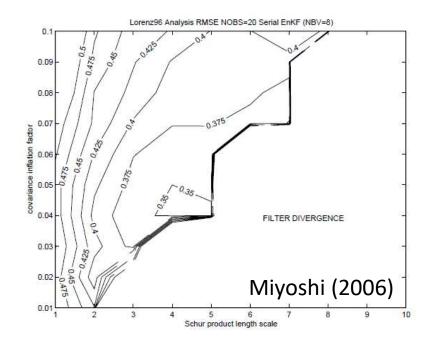
X axis: Schur product length scale

Figure 2.3: Parameter dependence of the analysis RMSE of serial EnSRF with 10 ensemble members on the Lorenz-96 model when the number of observations is 40, cf. Fig.3(b) of Whitaker and Hamill (2002). The horizontal and vertical axes show the localization length scale  $\sigma$  and the covariance inflation parameter  $\delta$ , respectively. The minimum error 0.20 is observed when  $\sigma=5$  and  $\delta=0.02$ . "FILTER DIVERGENCE" denotes the region with RMSE of more than 1.0.

※ Note: Single simulation result



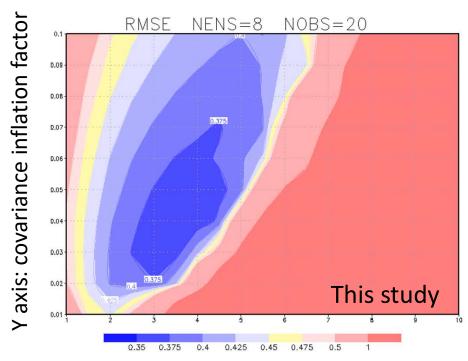
X axis: Schur product length scale



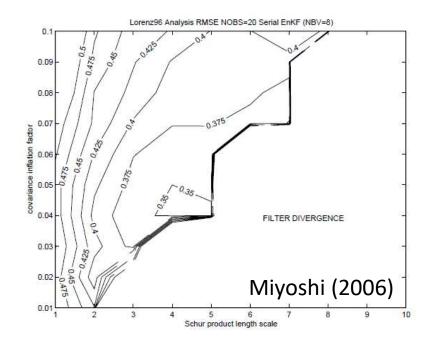
The differences of the figures may derive from following reasons.

- 1. The number of simulations
- 2. Intervals of X and Y axis.

※ Note: Single simulation result



X axis: Schur product length scale



The differences of the figures may derive from following reasons.

- 1. The number of simulations
- 2. Intervals of X and Y axis.

X Note: ave. 50 simulations

# Next step

- Remaining homework
  - To re-run KF with breeding method
  - To investigate dependency of filtering methods on observation density with KF, 3DVAR, and EnKFs.

- Code development
  - LEKF, LETKF

