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A journey to another planet

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Contents

0.1 Acknowledgements

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0.2 Introduction

I have been given a random seed, 23078, this seed represents my solar system, which consists of 1 star and 9 planets. I have decided to travel to the neighbouring planet, which I called G23 in this project. To travel to this planet we need an engine capable of reaching my planet's escape velocity, but as well as making maneuvers during the trip. With several simple boxes with holes in them, the engine is capable of doing these tasks.

In order to plan my satellite's trajectory, I need to know the motion of my solar system. Using Newton's second law and his law of gravity, I can predict the motion of the planet's. I now have an engine and an understading of how my solar system behaves, I can then give a little extra boost at a certain time, which will be enough to take off from my homeplanet and fly to G23.

When the satellite is in deep space, it is crucial to know what the position and velocity of the satellite is, to fix this issue I have implemented an orientation software. As time passes the satellite approaches G23, thereafter making an injection burn to enter G23's orbit. The satellite is now in a position where it can take orbital pictures and videos, however I am not yet finished, I want to explore the planet. With some simple physics and statistics, I can then find out what the atmosphere consists of, because as the satellite approaches G23, it gets warmer due to the drag forces in the atmosphere; I need to carefully direct it towards G23's surface so it does not crash or burned.

As the satellite approaches a safe height, so it can detatch its probe, the probe will approach G23's surface, and at some altitude it will approach terminal velocity (stops accelerating), and then deploy its parachute. As the lander have landed on the surface it will take pictures and send it back to its homeplanet.

Method

2.0.1 Engine

A simple model:

In the introduction I briefly explained how to build a rocket engine, in this section I will explain it in more detail. As I mentioned, the engine consists of several cubes with length d.

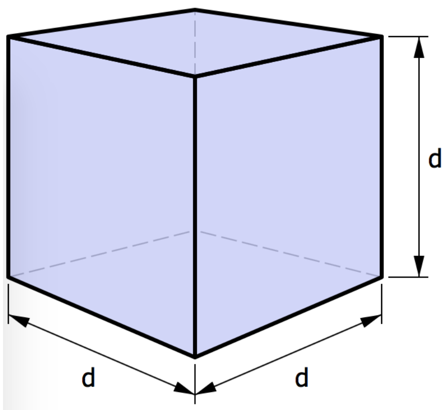


Figure 2.1: A example on how the cube will look like

Inside this cube there will be 100 000 random positioned hydrogen molecules, with Gaussian distributed velocity (I will explain this in more detail in the next section). In order to accelerate the rocket, I need to a implement a hole so the molecules can escape (Change in momentum will accelerate the rocket) the cube. This hole is located at the bottom of the cube, ($z = 0$), in the interval

$$\frac{d}{4} \leq x, y \leq \frac{3d}{4} \tag{2.1}$$

When a hydrogen molecule have a z component less or equal to 0,($z \leq 0$), it will escape through this hole. I will then replace this hydrogen molecule on the top side of the box with a random x, y position. I have implemented a sensor which counts amount of particle escaped, escaped momentum, pressure inside the box.

Motion of the Molecules:

As I mention in the previous section the hydrogen molecules are uniformly random positioned in the interval $x, y, z \in [0, L]$ with a gaussian distributed velocity. The motion of the molecules is described with Maxwell-Boltzmann distribution (skriv referanse her, Part1A)

$$P(\mathbf{v}) = (\frac{m_{H_2}}{2\pi k_b T})^{\frac{3}{2}} e^{-\frac{m_{H_2} v^2}{2k_b T}} = (\frac{1}{2\pi\sigma^2})^{\frac{3}{2}} e^{-(\frac{v_x^2+v_y^2+v_z^2}{2\sigma^2})} \tag{2.2}$$

due to large amount of particles, constant and high temperature, the most probable speed is
expressed as

$$v = \sqrt{\frac{k_b T}{m_{H_2}}} \tag{2.3}$$

I can then also use expression (2.3) to define it as the standard deviation, *sigma*, in expression (2.2). The speed of the molecules will now be in the range $[0, \sqrt{\frac{k_b T}{m_{H_2}}}]$.

Pressure and Momentum inside the cube:

First of all to simplify some problem for example change in momentum I will assume there is no interaction/collisions between the molecules, only elastic collision [1] between the walls. To a have fully functioning engine the pressure inside have to be stable. To find the pressure, I will look at the change in momentum when the molecules are colliding with the wall. The change is then (referanse fra part1A notater)

$$p_{\text{wall}} = \sum_{j=1}^{K} 2m_{\text{H}_2} v_{\text{x,j}} = 2\Delta p_x \tag{2.4}$$

Since I know the change in momentum, I can find the pressure exerted on the wall. The pressure is propotional to the force exerted on the wall in a time interval Δt. The pressure is then mathematically described as

$$P = \frac{2\Delta p_x}{A\Delta t} \tag{2.5}$$

From the previous section I defined the temperature to T (constant), and the highly probable speed is defined to be $\frac{k_b T}{m_{H_2}}$, the change in momentum on the wall is approximated to be constant therefore the pressure will also be constant.

[1] An elastic collision is a direct collision between two object where the total momentum and kinetic energy is consered.

Totalfroce and thrust from the engine:

To find the totalforce from the engine, I need to know how many cubes I need in order to accelerate to the Δv I want. Newton's second law states that

$$\sum_j F_j = ma \tag{2.6}$$

In this situation there is only one force, F_{engine}, which describes the totalforce from all the cubes. To simplify the problem, I assume that the satellite have reached the ΔV, and all the fuel is used up.

$$F_{\text{cube}} n_{\text{cube}} = m_r \frac{\Delta V}{\Delta t_{\text{launch}}} \tag{2.7}$$

solving with respect to n_{cube}, I will get

$$n_{\text{cube}} = \frac{m_r \Delta V}{F_{\text{cube}} \Delta t_{\text{launch}}} \tag{2.8}$$

Δt_{launch} describes how long it take to burn up all the fuel. The totalforce is therefore force per cube multiplied with number of cubes. To find out how effective a rocket engine is, I have to look at the thrust velocity. Thrust velocity is how much mass is expunged out of the engine per time. To find thrust, I can look at the force from one cube, which I can express it as

$$F_{cube} = \frac{p_e}{\Delta \tau} n_e \tag{2.9}$$

where p_e is the momentum escaped from the engine defined as $2m_H\mu$, where μ is the thrust velocity (I have multiplied with 2 because it is hydrogen molecules and not atoms), and n_e is the number of molecules escaped from the cube per $\Delta\tau = 10^{-9}$. Expresssion (2.9) can then be expressed as

$$F_{cube} = \frac{2m_H\mu}{\Delta \tau} n_e \tag{2.10}$$

solving for μ

$$\mu = \frac{F_{cube} \Delta \tau}{2m_H n_e} \tag{2.11}$$

Expression (2.11) defines how effective the rocket engine is.

2.0.2 Fuel Amount

To make the project more realistic the rocket engine needs fuel to function. I have found two ways to calculate the amount of fuel I need throughout the entire trip.

Method 1: Analytical Solution : Newton's Second Law states that force is proportional to acceleration (multiplied with an constant mass, m). Consider a situation where the mass changes as time passes in the z direction (upwards)

$$\sum_j F_j^{ext} = (\Delta m_f + m_r)a \tag{2.12}$$

where Δm_f describes the changing mass, which in this case is the amount of fuel used, and m_r describes the satellite's mass. Acceleration is the time derivative of the velocity, $a = \frac{dv}{dt}$, expression (2.12) can be expressed in terms of deltas

$$\sum_j F_j^{ext} = \frac{(\Delta m_f + m_r)\Delta v}{\Delta t} = \frac{\Delta p}{\Delta t} \tag{2.13}$$

where Δp is the change in momentum. The interesting part is how momentum changes with time. To make certain steps easier, I've assumed that I am starting at a time t with fuel Δm, and that I've reached burnout at time $t + \Delta t$. The momentum at time t is then $p(t) = \Delta m \mu_e + m_r v$
(μ_e is the velocity at which the particle is expunged out of the engine, this is a constant that also defines how efficient the engine is). At time $\Delta t + t$ the momentum has changed to
$p(\Delta + t) = m_r(v + \Delta v)$, the total change Δp is then

$$\Delta p = p(\Delta t + t) - p(t) = m\Delta v - \Delta m \mu_e \tag{2.14}$$

By inserting expression (2.14) in to (2.13) and taking the limit when $\Delta t \to 0$, expression (2.13) can then mathematically be described as

$$\sum_j F_j^{ext} + \mu_e \frac{dm_f}{dt} = m_r \frac{dv}{dt} \tag{2.15}$$

This equation is called Tsiolkovsky's rocket equation. I now have an equation with a time dependent mass. I can assume that the net external force $\sum_j F_j^{ext} = 0$ for simplicity's sake I will not include gravity (I will discuss this statement in depth in the discussion part). Expression (2.15) is now a separable differential equation, of the form

$$\mu_e dm = m dv \tag{2.16}$$

As I derived previously, μ_e is a constant which can be defined as

$$\mu_e = \frac{F\Delta\tau}{2n_e m_H} \tag{2.17}$$

where F is the force per box, $n_{escaped}$ is the number of hydrogen molecules that have escaped per box, and m_H is the mass of a single hydrogen atom (that's why I multiplied it by 2), and $\Delta\tau = 1ns$ Going back to expression (2.16), and integrating both sides with integration limits, it follows that:

$$\int_{m_r+m_r}^{m_r} \frac{dm}{m} = \int_0^{\Delta V} \frac{dv}{\mu_e} \tag{2.18}$$

$$\ln \frac{m_r}{m_r + m_f} = \frac{\Delta V}{\mu_e} \tag{2.19}$$

Solving with respect to m_f, the equation for the amount of fuel I need throughout the trip is given by

$$m_f = m_r(e^{-\frac{\Delta V}{\mu_e}} - 1) \tag{2.20}$$

Method 2: Numerical Solution: In order to do this numerically I will solve a differential equation of the form

$$\frac{d^2r}{dt^2} = \frac{F}{m_r + m_f} \tag{2.21}$$

where I guess how much fuel (m_f) I need, and I defined $F = \frac{\Delta p_e}{\Delta t_{period}} n_{box}$ in the part where I talked about the engine. I will use the numerical Euler-Cromer method

$$\begin{aligned} a_i(r,t) \\ v_{n+1} &= v_n + a_i\Delta t \\ x_{n+1} &= x_n + v_{n+1}\Delta t \\ \Delta t &= \frac{t}{N} \end{aligned}$$

where t is the launch period, and N reperesents the number of iterations. For every timestep I will subtract a mass of magnitude $m_{escaped} = \frac{m_{H_2} n_{escaped} n_{boxes}}{\Delta t_{period}}$, where I still use $\Delta t_{period} = 1ns$. I have also implemented an if test which breaks the for loop if the rocket has reached the Δv I want or used up all of its fuel. The algorithm will then look like this

Algorithm 1 Fuel finder

for i in xrange(N-1): **do**

$\quad a = \frac{F}{m_f + m_r}$

$\quad v_{i+1} = v_i + a_i \Delta t$

$\quad x_{i+1} = x_i + v_{i+1} \Delta t$

$\quad m_f - = m_{\text{escaped}}$

end for

2.0.3 Motion of the Solar System:

To make things more realistic, it is crucial to know the motion of the planet you are traveling to, but also the rest of the solar system. The reason being cost; it is not cheap to send a satellite out to another planet, but as well because of avoiding gravity from certain planets or using gravity assist to accelerate and guide the satellite to the destination. As I mentioned in the introduction, to predict the motion of the solar system I am going to use Newton's Second Law, his Law of Gravity, and numerically solve the following ordinary differential equation for an N-body system

$$m_p \frac{d^2\mathbf{r}}{dt^2} = -\frac{G m_p m_s}{r^2} \frac{\mathbf{r}}{r} \tag{2.22}$$

where $\frac{\mathbf{r}}{r}$ is a two dimensional unit vector in the xy plane, and $\mathbf{r}$ is the position vector relative to the star defined as $\mathbf{r}_{rel} = \mathbf{r}_{planet} - \mathbf{r}_{star}$. I will then assume that since the star is so massive compared to the planets, the star would not move. So I set $\mathbf{r_{star}} = \mathbf{0}$. The numerical method I used was the Euler-Cromer method, but I could use aslo Leap-Frog (I will discuss in the discussion part why I did not use it). The algorithm for solving this problem

Algorithm 2 Solving N-body system

for for i in xrange(N-1) **do**:

$\quad$ **for** for j in xrange(Number of planets) **do**:

$\quad\quad \mathbf{r^{rel}_{i+1,j}} = \mathbf{r}^{planet}_{i,j} - \mathbf{0}$

$\quad\quad a_{i,j} = (\frac{M_{star} G}{(r^{rel}_{i,j})^3})\mathbf{r}^{rel}$

$\quad\quad v_{i+1,j} = v_{i,j} + a_{i,j} \Delta t.$

$\quad\quad x_{i+1,j} = x_{i,j} + v_{i+1,j} \Delta t$

$\quad$ **end for**

end for

But in reality, so will the center of mass of the star slighty move in realtion to the planets. This can be shown mathematically and by observation.

2.0.4 Motion of the Star:

Observation

From observation one can show that the light intensity slightly vary, this is due to slight movements. If we look at the spectrum of the star, one will notice a slight periodic shift. One can find this shift by using the doppler effect

$$\frac{v_{rel}}{c} = \frac{\Delta\lambda}{\lambda_0} \tag{2.23}$$

In most cases these shift are caused by exoplanets orbiting around the star, causing the star move slight around its center of mass.

Theoretical:

I define the center of mass of the star to be $\mathbf{R}_{star}$, and the velocity is, $\mathbf{V}_{star} = \frac{d\mathbf{R}_{star}}{dt}$. I will now assume that the total momentum and energy of the solar system is conserved

$$\sum_{k=1}^{N} \mathbf{P}_j = \mathbf{0} \tag{2.24}$$

$\mathbf{P}_j$ is momentum. Expression (2.24) can be expanded to

$$M_{star}\mathbf{V}_{star} = \sum_{k=1}^{N-1} \mathbf{P}_j \tag{2.25}$$

The left hand side is the momentum of the star, and on the right hand side is the momentum of all the planets orbiting around the star. Dividing by the mass of the star, I will get the following expression

$$\mathbf{V}_{star} = \frac{\sum_{k=1}^{N-1} \mathbf{P}_j}{M_{star}} \tag{2.26}$$

Expression (2.26) shows that; planets orbiting around a star will cause the star to move slight around its center of mass.

2.0.5 Planet G23:

Surface Temperature of G23

I have now the knownledge of my solar systems motion, but I need further information about the planets. The most important one is the surface temperature. This temperature reveals if the planet is habitable planet or not. Looking at the energy recevied from the star

$$Fd\hat{A} = \frac{dE}{dAdt}d\tilde{A} = \frac{R_*^2\sigma T_*^4}{a^2}d\tilde{A} \tag{2.27}$$

The differential $d\tilde{A}$ is the area that absorbs the energy for the star. Assuming that the planet is black body, the planet will then follow Stefan-Boltzmann law

$$F = \sigma T^4 \tag{2.28}$$

From Stefan-Boltzmann law I can then find the energy sent out by the planet. The energy recevied from the star has to equal the energy the planet are sending out

$$E_{\text{inn}} = E_{\text{out}} \tag{2.29}$$

expressing equation (2.29) as

$$F_{\text{out}}dA_{\text{planet}} = \frac{R_*^2\sigma T_*^4}{a^2}d\tilde{A} \tag{2.30}$$

integrating with respect to area, I will get

$$4\pi r_{\text{planet}}^2 \sigma T_{\text{planet}}^2 = \pi r_{\text{planet}}^2 \frac{R_*^2\sigma T_*^4}{a^2} \tag{2.31}$$

The surface temperature of an arbitrary planet is (remember that r_{planet} is the radius of the planet)

$$T_{\text{Planet}} = T_* \sqrt{\frac{R_*}{2a}} \tag{2.32}$$

This explain the surface Temperature of a planet with no green house effect. I assume that habitable planet have a sufrace temperature in the interval $[260K, 390K]$. Calculating the surface temperature for G23, I will get that the temperature is $T_{\text{G23}} = 273,38K$ which means it is habitable. Expression (2.27) and (2.28) is aquired from [2]

Planning the journey to G23:

Now it is the time to launch the satellite in to deep space toward G23. In order to get to G23 with the smallest fuel consumption as possible, I will do a transfer orbit around my star and to G23. The reason is the gravity assist from the star, this effect will accelerate the satellite, and give the right path to G23 with minimal correction boost/burn. I am going to use Hohmann transfer[3]. When the satellite takes a Hohmann transfer orbit, it goes from a lower orbit to a higher circular orbit. This is a elliptic orbit which touches the initial orbit at perihelion and at aphelion (the final destination). In this proccess

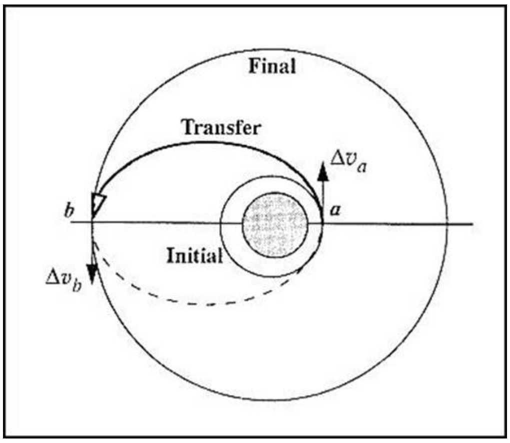


Figure 2.2: This figure illustrates Hohmann transfer orbit

the total kinetic energy and angular momentum is fully conserved.
The Δv required to get to G23's orbit is mathematically described as

$$\Delta V = \sqrt{\frac{m_{star}G}{r_1}}(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1) \tag{2.33}$$

to make the transfer orbit most efficient, I have to launch the satellite at a certain angle and at certain time when my homeplanet and G23 is aligned. To find this angle I will use the expression

$$\alpha = \pi[1 - \frac{1}{2\sqrt{2}}\sqrt{(\frac{r1}{r2} + 1)^3}] \tag{2.34}$$

and to find the time when this event happens I have made a algorithm which calculates it

The time it will take to reach G23 is calculated from the expression

$$t_H = \pi\sqrt{\frac{(r_1 + r_2)^3}{8GM_{star}}} \tag{2.35}$$

Solarpanels:

When the satellite is in deep space, and not using fuel to function, and to function it has to rely on a energy source. To solve this problem I can use energy from the light sent

[2] AST1100 lecture notes Part1D, author Proffesor Frode Hansen

[3] https://en.wikipedia.org/wiki/Hohmann_transfer_orbit
and
https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-07-dynamics-fall-2009/lecture-notes/MIT16_07F09_Lec17.pdf

Algorithm 3 Time align finder algorithm

$\alpha = \pi[1 - \frac{1}{2\sqrt{2}}\sqrt{(\frac{r1}{r2}+1)^3}]$
$c = 0$
$\phi = 0$
while $c < N$: **do**

  $\phi = \arccos\left(\frac{\mathbf{r_{home}} \cdot \mathbf{r_{G23}}}{|\mathbf{r_{home}}||\mathbf{r_{G23}}|}\right)|$
  $t = times[c]$
  $c+ = 1$
  if $\phi \leq \alpha$: **then**
    break
  end if
end while
return ϕ, t

out by star (or other stars), which means the satellite need solar panels, these panels have a efficieny of 12%, and a power of $P = 40W$. The question is how big these solar panels have to be. The area of the solar panels can by found from flux recieved from my star

$$F = \frac{dE}{dAdt} \tag{2.36}$$

Multiplying with the area of the solar panel, the power can be expressed as

$$FA_{panels} = \frac{dE}{dAdt}A \rightarrow P = \frac{L}{4\pi d^2}A_{panels} \tag{2.37}$$

where L is the luminosity is expressed $4\pi R_*^2 \sigma T_*^4$. The solar panel only have a efficiency of 12%, the power is then $\frac{P}{0.12}$. Solving with respect to A

$$A_{panel} = \frac{d^2 P}{0.12\sigma T_*^4 R^2} \tag{2.38}$$

The area of the solar panels needed are expressed with expression (2.38)

2.0.6 Orientation, Position and Velocity:

While the satellite is in deep space, it's very important to know its orientation, position and the velocity. To find this out I have one method to each of these issues.

Orientation:

To find the orientation of my satellite, I can compare existing picture of the sky or my path with pictures taken from the satellite. To do this I will do 360 projection of the sky and compare these pictures with the existing ones, and from that I can find my orientation angle. I will use the method stereographic projection. This method maps each position on the sphere, denoted by θ and ϕ, onto a point on the tangent plane of the surface about some point θ_0, ϕ_0, the opposite is also possible.

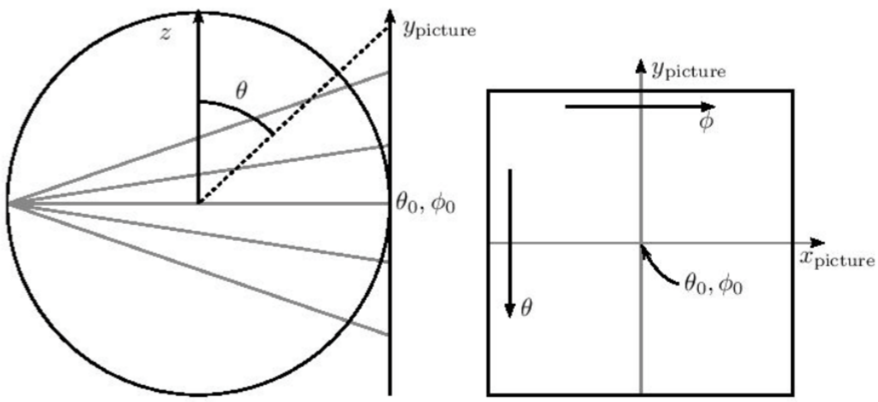


Figure 2.3: This picture illustrates how stereopgraphic works. It takes a spherical picture makes into a flat picture, $(\phi_0, \theta_0) \rightarrow (x_{\text{picture}}, y_{\text{picture}})$

I am going to define the points in the tagnet plane as x_{picture} and y_{picture}. Further I am going to use $(\phi - \phi_0)_{max} = \frac{\alpha_\phi}{2}$ and $(\theta - \theta_0)_{max} = \frac{\alpha_\theta}{2}$, these meassures the difference between the outer edge and the midtpoint in the picture measurred with the angles ϕ, θ. Using the equation

$$x_{max/min} = \pm \frac{2 \sin \frac{\alpha_\phi}{2}}{1 + \cos \frac{\alpha_\phi}{2}} \tag{2.39}$$

$$y_{max/min} = \pm \frac{2 \sin \frac{\alpha_\theta}{2}}{1 + \cos \frac{\alpha_\theta}{2}} \tag{2.40}$$

From these equations I can find the maximum and the minimum pixel placement in the picture at the midpoint and at the outer edge. Defining $x_{max/min}$ and $y_{max\,\mathrm{min}}$ as a interval $[x_{min}, x_{max}]$ and $[y_{min}, y_{max}]$, using values from these interval I can find the spherical positions θ and ϕ.Using the equations

$$\theta = \frac{\pi}{2} - \arcsin\left(\cos c \cos\theta_0 + \frac{y_{picture}\sin c \sin\theta_0}{\rho}\right) \tag{2.41}$$

$$\phi = \phi_0 + \arctan\left(\frac{x_{picture}\sin c}{\rho\sin\theta_0\cos c - y_{picture}\cos\theta_0\sin c}\right) \tag{2.42}$$

$$\rho = \sqrt{x_{picture}^2 + y_{picture}^2} \tag{2.43}$$

$$c = 2\arctan\frac{\rho}{2} \tag{2.44}$$

when the angles are found, I am sending these angles to a module which makes the angles into pixel with RGB colors. Putting the pixels together into a matrix, I can now compare these 360 pictures with the existing picture (In this case I have used a 640×480 matrix). To find the picture which is approximated to the existing one, I can use least squares method

$$Q = \sum_{j}^{360}(\text{Existing} - \text{Projection}[j])^2 \tag{2.45}$$

These equations and expression is aquired from [4].

Position finder:

To find the satellites position, I will implement a radar. This radar have the property to find the position x, y of the sallite by using three referance points in space, since I already know the position of the planets and the star at all time, I can use the planets and the star positions as referance points. I can then create three large circle around these points where the radius is the distance between the satellite and the referance point. This will then give me three circle equation which I need to solve

$$x^2 + y^2 = d_1^2 \tag{2.46}$$
$$(x - x_2)^2 + (y - y_2)^2 = d_2^2 \tag{2.47}$$
$$(x - x_3)^2 + (y - y_3)^2 = d_3^2 \tag{2.48}$$

[4] Part4 AST1100 Satellite project, author Proffesor Frode Hansen

The first equation represents the position of the star with a distance d_1 to the satellite. The other two equation reperesents an arbitrary planet with a distance d_2 and d_3 to the satellite. I am now interested to find x, y. multiplying out the parantheses in expression (2.47) and (2.48), so I get

$$x^2 + y^2 = d_1^2 \tag{2.49}$$
$$x^2 - 2xx_2 + x_2^2 + y^2 - 2y_2y + y^2 = d_2^2 \tag{2.50}$$
$$x^2 - 2xx_3 + x_3^2 + y^2 - 2y_3y + y^2 = d_3^2 \tag{2.51}$$

Inserting x^2 from expression (2.49) into expression (2.50) and (2.51), I will get

$$2x_2x + 2yy_2 = d_1^2 - d_2^2 + x_2^2 + y_2^2 \tag{2.52}$$
$$2x_3x + 2yy_3 = d_1^2 - d_3^2 + x_3^2 + y_3^2 \tag{2.53}$$

expression (2.52) and (2.53) and can be written as a matrix equation

$$\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{(d_1^2 - d_2^2 + x_2^2 + y_2^2)}{2} \\ \frac{(d_1^2 - d_3^2 + x_3^2 + y_3^2)}{2} \end{pmatrix} \tag{2.54}$$

solving equation (2.54), I will find the x, y position of the satellite.

velocity of the satellite:

To do maneuvers during the trip it is crucial to know the velocity of the satellite. One way to find the velocity is to look at the wavelength at a certain angle ϕ_1 and ϕ_2 from two distant stars. When light travels a long distance or if the satellite approaches a star, the light will be slightly shifted. This is the doppler effect.

$$\frac{v}{c} = \frac{\Delta\lambda}{H_\alpha} \tag{2.55}$$

where $H_\alpha = 656.3nm$ is the non shifted light sent out by the reference star, and $\Delta\lambda$ is the shift measured relative to my star. Using this I will find out the velocity relative to my star and not the satellite. Receving the same light beam, but meassuring another doppler shift, using this shift I can meassure the velocity of the reference star relative to my satellite. The velocity of my satellite is then expressed as

$$v_{satellite} = v_{\text{refstar}\to\text{star}} - v_{\text{refstar}\to\text{satellite}} \tag{2.56}$$

where

$$v_{\text{refstar}\to\text{star}} = \frac{\Delta\lambda}{H_\alpha}c \qquad v_{\text{refstar}\to\text{satellite}} = \frac{\Delta\lambda_{\text{satellite}}}{H_\alpha}c \tag{2.57}$$

The expression on the left expresses the velocity of the reference star1 relative to my star. The expression on the right expresses the exact same but relative to my satellite.

I will now use the same on reference star2. The velocity of my satellite is then given as; Described with Referance star 1

$$\mathbf{v}_1 = v_{satellite} = v_{\text{refstar1}\rightarrow\text{star}} - v_{\text{refstar1}\rightarrow\text{satellite}} \tag{2.58}$$

Described with Referance star 2

$$\mathbf{v}_2 = v_{satellite} = v_{\text{refstar2}\rightarrow\text{star}} - v_{\text{refstar2}\rightarrow\text{satellite}} \tag{2.59}$$

expression (2.58) and (2.59) is just a matrix equation, and it is given as

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \frac{1}{\sin(\phi_1 - \phi_2)} \begin{pmatrix} \sin\phi_2 & -\sin\phi_1 \\ -\cos\phi_2 & \cos\phi_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \tag{2.60}$$

solving this matrix equation, I will find the velocity of the satellite.

2.0.7 Orbit around G23:

Minimal Distance

When the satellite gets closer to G23 it have to a injection burn. This injection burn occours when the satellite is at its closest to the planet. I then need to know this distance. To find the distance I can look at the ratio between of the force excerted on th satellite from G23 and the planet. The force from the star

$$F_* = \frac{m_{sat} M_* G}{R^2} \tag{2.61}$$

where R is the distance between satellite and the star. The force from G23

$$F_{G23} = \frac{m_{sat} M_{G23}}{r^2} \tag{2.62}$$

finding the ratio, and defining the magnitude as k, I will get the expression

$$\frac{R_*^2 m_{G23}}{r^2 M_*} = k \tag{2.63}$$

Solving expression (2.63) with respect to the distance between G23 and the satellite, r, gives me

$$r = \sqrt{\frac{R_* m_{G23}}{M_* k}} \tag{2.64}$$

Using expression 2.64 I can find out the distance needed to do a injection burn.

Correction Boost/Burn:

In general to get close enough to the planet, one has to do an correction boost/burn. A correction boost is a boost/burn which changes the direction of the velocity vector with increasing or decreasing speed, it depens on the situation. I will now define the correction burn as a some scalar factor Q multiplied with the position vector.

$$\mathbf{v_{burn}} = -Q\frac{\mathbf{r}}{|\mathbf{r}|} \tag{2.65}$$

Since I know the position of G23 at all time, I will define the position vector to the satellite relative to G23 as

$$\mathbf{r_{rel}} = \mathbf{r_{sat}}(t) - \mathbf{r_{G23}}(t + \Delta t) \tag{2.66}$$

Expression (2.66) will then ensure that I boost/burn towards G23. If the scalar factor Q is negative, it will be a boost toward the planet, and positive it will be a burn.

Injection Boost:

After reaching the closest distance to G23, I can finally do a injection boost in to a stable circular orbit. To find the velocity needed in order to achieve a stable circular orbit, I have to look at Newton's second law expressed with sentripetal accelartion, $a = \frac{v_{so}^2}{|\mathbf{r}|}$

$$F = m(\frac{v_{SO}^2}{|\mathbf{r}|}) \tag{2.67}$$

At this distance the dominating force is from G23, I can therefore neglect neighbouring planet and the star. Using Newton's law of gravity, I can solve expression (2.67) with respect to v_{so}. I will then get the expression

$$v_{so} = \sqrt{\frac{m_{G23}G}{|\mathbf{r}_{rel}|}} \tag{2.68}$$

But even though I have the value of the velocity I need, I still need the change in velocity relative to G23 and v_{so} have to be a velocity vector and not a scalar. To make expression 2.68 in to a vector, I can place G23 in the center in a another coordinate system, and find a angle θ between the satellite and G23. The angle θ is only the coordinates of $\mathbf{r_{rel}}$ expressed in polar coordinates. To find v_{so}, I can express the position vector in polar coordinates

$$\mathbf{r_{rel}} = |\mathbf{r_{rel}}|(\cos\theta, \sin\theta) = |\mathbf{r_{rel}}|\hat{i}_\theta \tag{2.69}$$

Differentiating expression (2.69) at a time $t + \Delta t$, the velocity relative to G23 is given by

$$\frac{d}{dt}(|\mathbf{r}_{rel}|\hat{i_r}) = \frac{d}{dt}|\mathbf{r}_{sat} - \mathbf{r}_{g23}|\hat{i}_r + |\mathbf{r_{rel}}|\frac{d}{dt}\hat{i_r} \tag{2.70}$$

which gives me

$$\frac{d}{dt}(|\mathbf{r}_{rel}|\hat{i_r}) = \frac{d}{dt}|\mathbf{r}_{sat} - \mathbf{r}_{g23}|\hat{i}_r + |\mathbf{r_{rel}}|\hat{i}_\phi \tag{2.71}$$

here is $\hat{i}_r = (\cos\phi, \sin\phi)$ and $\hat{i}_\phi = (-\sin\phi, \cos\phi)$. Assuming the distance $|\mathbf{r_{rel}}|$ at a time $t + \Delta t$ is constant, the term $\frac{d}{dt}|\mathbf{r}_{sat} - \mathbf{r}_{g23}|\hat{i}_r = \mathbf{0}$. I have then the velocity at a time $t + \Delta t$

$$\frac{d}{dt}(|\mathbf{r}_{rel}|\hat{i_r}) = |\mathbf{r_{rel}}|\frac{d}{dt}\hat{i_r} \tag{2.72}$$

which is also the same as

$$\frac{d}{dt}(|\mathbf{r}_{rel}|\hat{i_r}) = -|\mathbf{r}_{rel}|\hat{i}_r\frac{d\phi}{dt} = |\omega\mathbf{r}_{rel}|\hat{i}_\phi \tag{2.73}$$

The right handside of expression (2.73) is just v_{so}. That means that

$$\frac{d}{dt}(|\mathbf{r}_{rel}|\hat{i_r}) = \sqrt{\frac{m_{G23}G}{|\mathbf{r}_{rel}|}}\hat{i}_\phi = \sqrt{\frac{m_{G23}G}{|\mathbf{r}_{rel}|}}(-\sin\phi, \cos\phi) \tag{2.74}$$

The change in velocity from time t to $t + \Delta t$ is expressed as

$$\Delta v = v_{so}\hat{i}_\phi - (\mathbf{v}_{sat} - \mathbf{v}_{planet}) = \mathbf{v}_{so} - (\mathbf{v}_{sat} - \mathbf{v}_{planet}) \tag{2.75}$$

Using expression (2.75) the satellite manages to get into a stable orbit around G23.

2.0.8 Analyzing the Atmosphere:

The satellite is now in orbit, Before the satellite go to a lower orbit and detatch its landerprobe, I have to know what type of gas molecules the atmosphere consist of, this is because of air resistance. To find out what kind of gas molecule the atmosphere consist of, I will look at the spectras. Assuming that I have a list of the noise, wavelength and its flux sent out of G23 atmosphere, I can create a model of the flux and compare it with the real values. From previous section I know that, when a moving molecule emits light, it will occour doppler shift will occour.

Assuming that the movement is caused by a temperature in the interval $T \in [150k, 450k]$ and I know that particle motion is Gaussian, I can find the standard diviation of the doppler shift from a arbitrary gas molecule

$$\sigma = \frac{2\lambda_{center}}{c}\sqrt{\frac{k_bT}{m_{gas}}} \tag{2.76}$$

The wavelengths are centered around the arbsorption lines λ_{center}. Trying different standard deviation, the model of the flux can be expressed as

$$F^{model}(\lambda_{center}, \lambda, \sigma, F_{min}, F_{max}) = F_{max} + (F_{min} - F_{max})e^{-\frac{(\lambda-\lambda_{center})^2}{2\sigma^2}} \tag{2.77}$$

Using the χ^2 method, I can determine which absorption lines are real or just noise. χ^2 method is expressed as

$$\chi^2 = \frac{\sigma_{\lambda_m in}^{lambda_e nd}(F^{real} - F^{model})^2}{\sum_n^K \sigma_n^2} \tag{2.78}$$

I will now define:

• I will set F_{min} in the interval $[0.7, 1]$
• If $F^{model} = F_{min} = 0.7$, I have then detected that $\lambda = \lambda_{center}$ which means its highly probable that I have detected a absoprtionline.
• If $F^{model} = F_{max} = 1$, I have then found that $\lambda \neq \lambda_{center}$, which mean the standard diviation is large, and it is highly probable that I have detected noise.

Expression (2.76), (2.77) and (2.78) is aquired from (skriv referansenummer her)

2.0.9 Modelling G23s Atmosphere:

I will now assume the atmosphere consist of two layers. The first layer is the adiabatic layer and the second layer is isothermal.

Adiabatic Layer:

First of all, a adiabatic process means that a system does not exchange heat nor energy with its surruonding. I assume a that G23 has a adiabatic atmosphere at the distance $R_{planet} \leq r \leq d$. R_p is at the surface of G23 and d is the distance from the surface to part of the atmosphere where the adiabatic layer is transitioning to the isothermal layer. The temperature is varying from $T_0 \leq T \leq T_d$.

To find a analytcal solution of the temperature, density and pressure of this layer, I have to solve the equation of hydrostatic equilibrium which is expressed as

$$\frac{dP}{dr} = -\rho(r)g(r) \tag{2.79}$$

P is pressure varying with the distance r, $\rho(r)$ is the density of the atmosphere and $g(r)$ is the gravitational acceleration (which also is varying with distance). I am also assuming the atmosphere follows the ideal gas law, which is mathematically expressed as

$$P = \frac{\rho k_b T}{\mu m_H} \tag{2.80}$$

k_b is boltzmans constant, m_H is the mass of a single hydrogen atom. Before I solve the ODE I have to express dP adiabaticly. From Thermodynamics a adiabatic process is expressed as $P^{1-\gamma}T^{\gamma} = Q$, where γ is the adiabatic index and Q is a constant. I will now differentiate $P^{1-\gamma}T^{\gamma} = Q$ implicitly, and it follows

$$\partial((P^{1-\gamma}T^{\gamma}) = 0 \rightarrow (1-\gamma)P^{-\gamma}T^{\gamma}dP + \gamma T^{\gamma-1}P^{1-\gamma}dT = 0 \tag{2.81}$$

Doing some algebra, expression (2.81) can be expressed as

$$\frac{dP}{P} = \frac{\gamma}{\gamma-1}\frac{dT}{T} \tag{2.82}$$

inserting expression (2.82) into the equation of hydrostatic equilibrium and redefining $\rho(r)$ using the ideal gas law, I will get following differential equation so solve;

$$dT = -\frac{\gamma-1}{\gamma}\frac{\mu m_H m_p G}{k_b(r+r_p)^2}dr \tag{2.83}$$

The solution of this equation after integrating on both sides, is given by

$$T = E + \frac{\gamma-1}{\gamma}\frac{\mu m_H m_p G}{k_b(r+r_p)} \tag{2.84}$$

where E is constant which can be found by solving for the conditions I talked about.

At the surface, $r = r_p$, the temperature is defined to be T_0, inserting these values into expression (2.84), the constant E can be defined as

$$E = T_0 - \frac{\gamma-1}{\gamma}\frac{\mu m_H m_p G}{k_b(r_p)} \tag{2.85}$$

expression (2.84) is then defined as

$$T(r) = T_0 + \frac{\gamma-1}{\gamma}\frac{\mu m_H m_p G}{k_b}(\frac{1}{r+r_p} - \frac{1}{r_p}) \tag{2.86}$$

The temperature for the atmosphere is then given by

$$T(r) = T_0 + \frac{\gamma-1}{\gamma}\frac{\mu m_H m_p G}{k_b}(\frac{1}{r+r_p} - \frac{1}{r_p}) \tag{2.87}$$

To find the height when transitioning layer between the adiabatic and isothermal layer, I say at a height d the temperature is T_0

$$\frac{T_0}{2} = T_0 + \frac{\gamma-1}{\gamma}\frac{\mu m_H m_p G}{k_b}(\frac{1}{r_p+d} - \frac{1}{r_p}) \tag{2.88}$$

$$-\frac{d}{r_p(d+r_p)} = -\frac{k_b T_0}{2\mu m_H m_p G}(\frac{\gamma}{\gamma-1}) \tag{2.89}$$

solving for d

$$d = \frac{dk_bT_0r_p}{2\mu m_H m_p G}(\frac{\gamma}{\gamma-1}) + \frac{k_bT_0r_p^2}{2\mu m_H m_p G}(\frac{\gamma}{\gamma-1}) \tag{2.90}$$

From now on I will define Ω as

$$\Omega = \frac{k_bT_0r_p}{2\mu m_H m_p G}(\frac{\gamma}{\gamma-1}) \tag{2.91}$$

And then I will get that the transitioning d is

$$d = \frac{\Omega r_p}{1-\Omega} \tag{2.92}$$

I have now a equation which describes the temperature at the height $r_p \leq r \leq d$, but I want to know what the density of the atmosphere is. Looking at the equation for an adiabatic process

$$P^{1-\gamma}T^{\gamma} = Z \tag{2.93}$$

where Z is an constant. Redefining expression (2.93)

$$\frac{P}{T} = ZT^{\frac{1}{\gamma-1}} \tag{2.94}$$

Now I will insert the ideal gas law and the temperature which describes the adiabatic layer into expression (2.94), and I will get

$$\rho(r)\frac{k_bT}{\mu m_H T} = Z[T_0 + \frac{\gamma-1}{\gamma}\frac{\mu m_H m_p G}{k_b}(\frac{1}{r_p+d} - \frac{1}{r_p})]^{\frac{1}{\gamma-1}} \tag{2.95}$$

solving for $\rho(r)$, I will get

$$\rho(r) = \frac{Z\mu m_h}{k_b}[T_0 + \frac{\gamma-1}{\gamma}\frac{\mu m_H m_p G}{k_b}(\frac{1}{r_p+d} - \frac{1}{r_p})]^{\frac{1}{\gamma-1}} \tag{2.96}$$

Here I have a unkown constant Z, to find this constant, I know at the surface of G23,$r = r_p$, the density of the atmosphere is ρ_0, using this information I will get

$$Z = \frac{\rho_0 k_b}{\mu m_h T_0^{\frac{1}{\gamma-1}}} \tag{2.97}$$

Inserting Z into expression (2.96), I will get

$$\rho(r) = \rho_0[1 + \frac{\gamma-1}{\gamma}\frac{\mu m_H m_p G}{k_b}(\frac{1}{r_p+r} - \frac{1}{r_p})]^{\frac{1}{\gamma-1}} \tag{2.98}$$

which describes the density of the atmosphere at the interval $r_p \leq r \leq d$.

Isothermal Layer

First of all, an isothermal process means that the temperature is the same throughout the system. To find the density for this layer, I have to solve the the equation for hydrostatic equilibrium and use Ideal gas law. The following steps would look like this

$$\frac{dP}{dr} = -\rho(r)g(r) \tag{2.99}$$

$$P = \frac{k_b T_0}{2\mu m_H}\rho(r) \tag{2.100}$$

Since the temperature is the same throughout this layer and when $r > d$, the temperature is $T_0/2$. Inserting expression (2.100) into (2.99), and making expression (2.99) into an seperable differential equation, I will get

$$\frac{d\rho}{\rho} = -\frac{2\mu m_H m_p G}{k_b T_0 (r + r_p)^2} dr \tag{2.101}$$

solving the differential equation, I will get

$$\ln\rho = C + \frac{2\mu m_H m_p G}{k_b T_0 (r + r_p)} \tag{2.102}$$

Solving expression (2.102) with respect to density, ρ, I will get

$$\rho(r) = \hat{C} e^{\frac{2\mu m_H m_p G}{k_b T_0 (r + r_p)}} \tag{2.103}$$

To find the unkown constant $\hat{C}$, I have to find the density at height d, this will also make the transitioning between the adiabatic and isothermal layer continous.

$$\rho(d) = \hat{C} e^{\frac{2\mu m_H m_p G}{k_b T_0 (d + r_p)}} \tag{2.104}$$

Using the density equation which describes the adiabatic layer and inserting the height d(transitioning height) in to the equation, I get that the density at height d is

$$\rho(d) = \rho_0 [1 + \frac{\gamma - 1}{\gamma} \frac{\mu m_H m_p G}{k_b} (\frac{1}{r_p + d} - \frac{1}{r_p})]^{\frac{1}{\gamma - 1}} \tag{2.105}$$

Inserting expression (2.105) into (2.104), I will get

$$\hat{C} = \rho(d) e^{-\frac{2\mu m_H m_p G}{k_b T_0 (d + r_p)}} \tag{2.106}$$

Inserting $\hat{C}$ in to expression (2.103), I will get that the density at the isothermal layer is expressed as

$$\rho(r) = \rho(d) e^{\frac{2\mu m_H m_p G}{k_b T_0} (\frac{1}{r + r_p} - \frac{1}{r_p})} \tag{2.107}$$

Defining the constants in the exponents as η, I will get that expression (2.107) can be expressed as

$$\rho(r) = \rho(d) e^{\eta(\frac{1}{r + r_p} - \frac{1}{r_p})} \tag{2.108}$$

This equation describes the density when $r > d$.

Summary

Density:

$$\rho(r) \begin{cases} \rho_0[1+\frac{\gamma-1}{\gamma}\frac{\mu m_H m_p G}{k_b}(\frac{1}{r_p+r}-\frac{1}{r_p})]^{\frac{1}{\gamma-1}} & ,0 \le r \le d \\ \rho(d)e^{\eta(\frac{1}{r+r_p}-\frac{1}{r_p+d})} & ,r > d \end{cases} \tag{2.109}$$

Temperature:

$$T(r) = \begin{cases} T_0 + \frac{\gamma-1}{\gamma}\frac{\mu m_H m_p G}{k_b}(\frac{1}{r+r_p}-\frac{1}{r_p}) & ,0 \le r \le d \\ \frac{T_0}{2} & ,r > d \end{cases} \tag{2.110}$$

where d is the transitioning altitude is given as

$$d = \frac{\Omega r_p}{1-\Omega} \approx 51.3km \qquad \Omega = \frac{k_b T_0 r_p}{2\mu m_H m_p G}(\frac{\gamma}{\gamma-1}) \tag{2.111}$$

2.0.10 Landing on G23:

General information needed in order to land

Landing on a planet is very difficult considering the dragforce from the atmosphere on the lander. If one does not carefully plan this part, the lander will burn. The dragforce is mathematically described as

$$F_d = \frac{1}{2}\rho(r)C_d A|\mathbf{v}_{sat}| \tag{2.112}$$

$\rho(r)$ is the density of the atmosphere, C_d is dimensionless friction constant which is set to be 1, A is the cross-sectional area of the satellite, and v is the satellites velocity. Looking at the expression it is crucial to find a altitude where $\rho \approx 0$, if not, the velocity will be large, when hitting the atmosphere which will incinerate the probe. Therefore I have to find this altitude when $\rho \approx 0$. Since the satellite is above the isothermal layer, I have to to use the equation

$$\rho(r) = \rho(d)e^{\eta(\frac{1}{r+r_p}-\frac{1}{r_p+d})} \tag{2.113}$$

Since equation (2.103) is not solveable for $\rho = 0$, I can define $\hat{\rho} = 10^{-12}$ at the altitude h, I will then get

$$\hat{\rho} = \rho(d))e^{\eta(\frac{1}{h+r_p}-\frac{1}{r_p+d})} \tag{2.114}$$

Since I am interested in the altitude h, I have to solve equation with respect to h. Taking the inverse logarithm on both sides, dividing by η, I will get

$$\ln(\frac{\hat{\rho}}{\rho_d})\frac{1}{\eta} = \frac{1}{h+r_p} - \frac{1}{r_p+d} \tag{2.115}$$

I will now for simplicity define Ψ as

$$\Psi = \frac{r_p}{r_p + d} + \ln\left(\frac{\hat{\rho}}{\rho_d}\right)\frac{1}{\eta} \tag{2.116}$$

Expression (2.115) can be expressed as

$$\frac{1}{h + r_p} = \Psi \tag{2.117}$$

solving for h, I will get

$$h = \frac{1 - \Psi r_p}{\Psi} \tag{2.118}$$

When calculating h the safe altitude is approximated to be $h \approx 545km$ above G23 surface. This means the satellite has to get an altitude of h, to do this I can do another hohmann transfer orbit. Using the expression from previous

$$\Delta V = \sqrt{\frac{m_p G}{r_1}}\left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right) \tag{2.119}$$

I will come closer to G23. The satellite is now in a low and stable circular orbit around G23, and is ready to detatch its landerprobe. Before detatching I have to find out what the terminal velocity is, because the area of the parachute depends on it (The air resistance is equal to gravity, $a = 0$)

$$F_G = F_d \rightarrow \frac{G m_{G23} m_{sat}}{r_{G23}^2} = \frac{1}{2} C_d \rho_0 A v_{sat}^2 \tag{2.120}$$

solving with respect to v_{sat} (Here is A the area before parachute is deployed, and r_{G23} is the radius of G23) I will get

$$v_{term} = \sqrt{\frac{2G m_{G23} m_{sat}}{r_{G23}^2 \rho_0 C_d A}} \tag{2.121}$$

I can now assume that I will deploy the parachute when the terminal velocity is $v = 3m/s$. Solving expression (2.121) with respect to A, I will get the expression

$$A = \frac{v_{term}^2 r_{G23}^2 \rho_0 C_d}{2G m_{g23} m_{sat}} \tag{2.122}$$

Important!

Before I detatch the landerprobe I have to include the atmospheres velocity in air resistance. Assuming that the atmosphere moves with G23's rotation, the speed of the atmosphere at some point (where the satellite is) is expressed as

$$v_{atmosphere} = \omega R = \frac{2\pi}{T_{G23}} R \tag{2.123}$$

where T_{G23} is the period of a complete circulation around its axis. R is the distance between the center of G23 and the satellite. Vectorizing expression (2.123), I can multiply with the unitvector $\hat{u}$, which can be found by taking the cross product between rotation axis and the satellites position

$$\hat{u} = \frac{\mathbf{k} \times \mathbf{r_{sat}}}{|\mathbf{k} \times \mathbf{r_{sat}}|} \tag{2.124}$$

this unit vector is tangential to the circular movement. I will then multiply expression (2.124) with expression (2.123), and I will get the expression

$$\mathbf{v}_{atmosphere} = \omega R \hat{u} \tag{2.125}$$

The satellites velocity relative to the atmosphere is expressed as

$$\mathbf{v_{rel}} = \mathbf{v}_{sat} - \mathbf{v}_{atmosphere} \tag{2.126}$$

Inserting this velocity in to the air resistance equation

$$\mathbf{F}_d = \frac{1}{2} C_d A \rho_0 |\mathbf{v}_{sat} - \mathbf{v}_{atmosphere}| \mathbf{v_{sat}} \tag{2.127}$$

To have a succesful landing, $\mathbf{F}_d$ must never exceed $250000N$, and it is recommended to stay below $25000N$ at all time.

Results and Discussion

In this section I will present my result and discuss if needed.

3.0.11 Engine and Fuel consumed:

The effectivity of the rocket engine:

In the section where I introduced the method I used , I derived a general form of thrust velocity

$$\mu_e = \frac{F_{cube}\Delta\tau}{2m_H n_e} = \frac{p_e}{2m_H n_e} \tag{3.128}$$

Looking at the engine status

```
###############################################
      Engine status (Numerical values)
-----------------------------------------------
The amount of particle escaped 63945
Amount of particles collided with one wall 254755
Momentum escaped 1.71348e-18 kgm/s
Kinetic energy per particle 2.07787e-19j
Total kinetic energy 2.07787e-14j
Pressure inside the engine is 13697.389375
momentum on the wall 2.74047e-17
total force 20299.9
###############################################
```

Figure 3.4: The engine status from the satellite at time 1ns

Using the escaped momentum at the time period $\Delta\tau$ gives me the thrust velocty

$$\mu = 8020ms^{-1} \tag{3.129}$$

this means at for every $1ns$ the mass is expunged out of the engine at the velocity $8020ms^{-1}$. Comparing the thrust velocity from my rocket with different vehicles with thrust technology (The table is given down below)

Specific impulse of various propulsion technologies

Engine	Effective exhaust velocity (m/s)	Specific impulse (s)	Exhaust specific energy (MJ/kg)
Turbofan jet engine (*actual* V is ~300 m/s)	29,000	3,000	~0.05
Solid rocket	2,500	250	3
Bipropellant liquid rocket	4,400	450	9.7
Ion thruster	29,000	3,000	430
VASIMR[12][13][14]	30,000–120,000	3,000–12,000	1,400
Dual-stage 4-grid electrostatic ion thruster[15]	210,000	21,400	22,500

Figure 3.5: A table which compares different vehicles with each other. These vehicles have jet technology, this data is taken from (skriv referanse her)

I can therefore conclude that my rocket engine is pretty effective in the simulation (since it has double thrust velocity than a Bipropellant liquid rocket). Of course this would look much different in reality, because I have not included gravity when I launch the rocket, and there is several other important factor to include, for example the shape of the nozzle and the chamber plays a huge role. I use pure liquid hydrogen, but in reality NASA use liquid hydrogen with liquid oxygen.

Fuel equation with no gravity:

When I derived the equation for the fuel needed for the whole trip I did not include Gravity. The reason is I have assumed that in the launch period, the velocity change is instantaneous, and assumed that the force from the engine is so dominant that gravity does not effect the system that much.

In reality this equation would be ok estimation on the ground level, but as it reaches higher altitude, g will decrease very fast

$$g = \frac{Gm_{planet}}{r^2} \tag{3.130}$$

and gravity (absolute value) will be very small compared to the engines force, the equation I derived will be more accurate at this stage.

3.0.12 The journey to G23:

Time and Angle:

Using the angle expression I wrote down in the section of "Planning the journey to G23"

$$\alpha = \pi[1 - \frac{1}{2\sqrt{2}}\sqrt{(\frac{r1}{r2} + 1)^3)}] \tag{3.131}$$

the angle is calculated to

$$\alpha = 45.8^\circ \tag{3.132}$$

but in my simulations I approximated it 46° so I get a little gravity assist from G23, so I dont have to do many correction burn. Launching at the angle 46°, the algorithm calculated the time to be $t = 5.9055 years$. This means I have to wait $5.9055 years$ in order to launch the satellite.

Orbit around G23

In this part I will discuss how I achieved a stable orbit around G23 in my simulation and in the module (the real launch). In my simulation I only need one boost in the begining when launching the rocket into deep space, and one injection burn when the satellite is at its closest to G23.

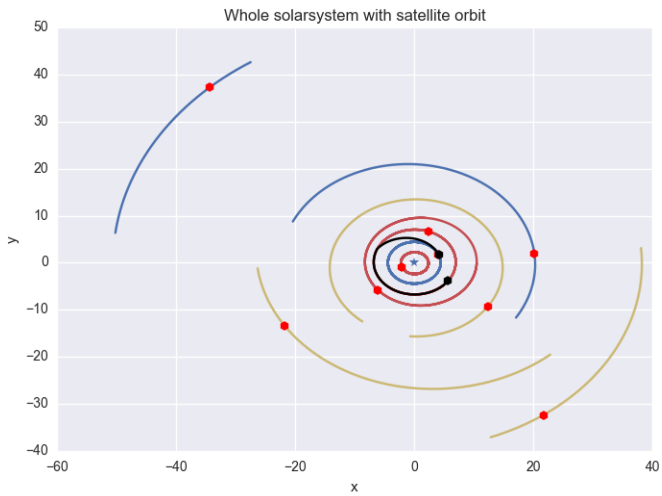


Figure 3.6: the black line shows the trajectory of the satellite, and the pentagon is the satellite.

but in the module I need two boost, one launch boost and one correction boost as the satellite approach the G23, and injenction burn when the satellite is at its closest to G23. It is cleary that my simulation have a small error in the trajectory. In the simulation I had to use 14.84% of the hohmann transfer boost, in order to get the right ellipse trajectory I wanted (reducing the hohmann boost only affect how elliptic the trajectory of your satellite is going to be, if $\Delta v_{hohmann} = 0$, the satellite wont fly towards the targetplanet), a small error can occour here.

I also used the nummerical integration Euler-Cromer in my simulation. This nummerical method is not as effective and accurate as the Leapfrog integration. If I had used Leapfrog integration it would be possible that my initial boost would be different, but not at least the number of boost. In both of the simulation the satellite manages to get into a stable circular orbit around G23 without falling out of the orbit.

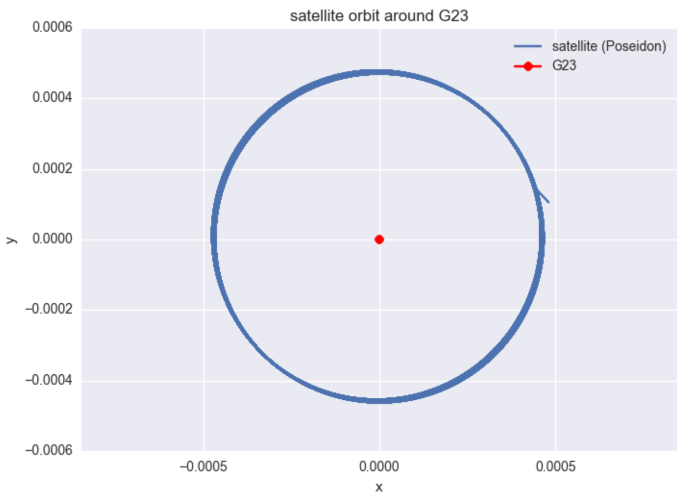


Figure 3.7: The satellites orbit around G23

Numerical or analytic fuel finder?:

In this section I will compare the Nummerical fuel finder method with analytical solution. Since I have the total boost (launch, correction burn, and injection burn), I can calculate the fuel I need throughout the journey. Using the analytical solution I will get

$$m_f = 12957kg \tag{3.133}$$

adding 1% to the fuel (in case of facing problem during the trip), I will get

$$m_f = 14252.7kg \tag{3.134}$$

Here I used $\Delta v = 20401 m/s$ and $\mu = 8020 m/s$. Using the nummerical method, I will now guess an amount of fuel. I will choose m_f as

$$m_f = 1300 kg \tag{3.135}$$

almost the same as the analytical. I will now start the engine and launch the rocket to see if it reaches the $\Delta v = 20401 m/s$. A graph is given down below which shows the fuel consumption and the velocity gain Analyzing the graph it looks that the satellite reaches

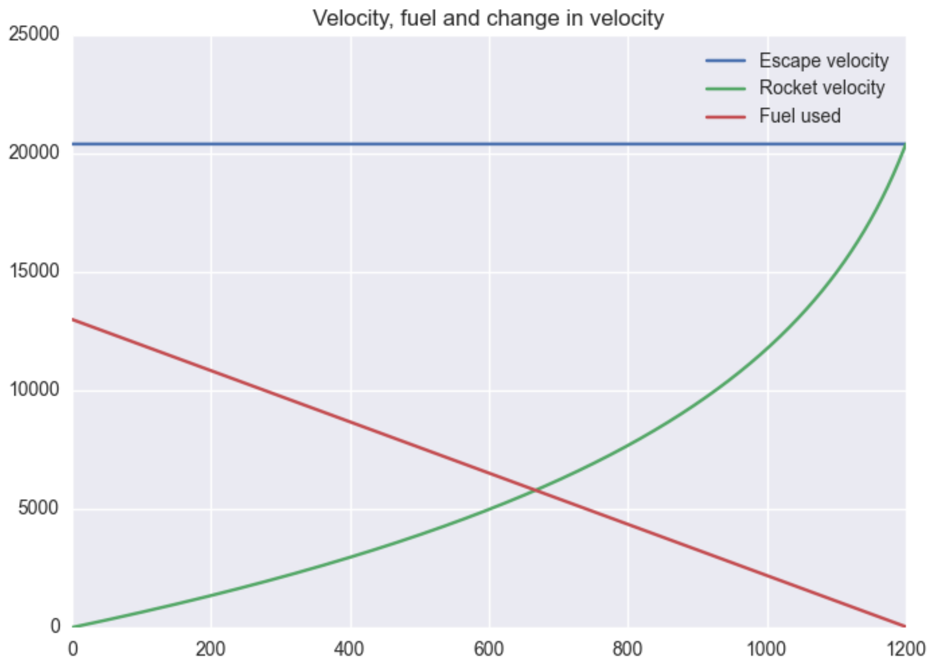


Figure 3.8: A graph showing the fuel consumption and the velocity gain

the $\Delta v = 20402 m/s$, which means that the nummerical method and the analytical solution agree with each other. I therefore conclude these two methods gives the same result. It can be easier to calculate with the analytical solution if the person know the thrust of its rocket, if not I would choose the nummerical method.

Comment: I calculated the ammount of fuel needed wrong the first time, the reason was due to some miscalculating, where I took the wrong value of the velocity. Since the analytical solution is a exponential, small changes results a huge change in the result. The fuel I need throughout this journey is calculated to $m_f = 14252.7 kg$.

3.0.13 Analyzing the atmosphere:

When I analyzed G23 atmosphere, I was looking for 6 different type of gas molecules, Oxygen, Carbon monooxide, Carbon dioxide, Water vapor, Nitrous oxide and methane. In the method section I defined that: If the model finds a absoprtionline $F^{model} = F_{min} \approx 0.7$ but also have small standard diviation, and when $F^{model} = F_{max} = 1$ I have detected noise with a large standard diviation.

Oxygen:

Oxygen have three absorption lines at $630nm, 690nm$ and $760nm$. The model I recieved for these absoprtionlines is shown down below Where the width black graph is the

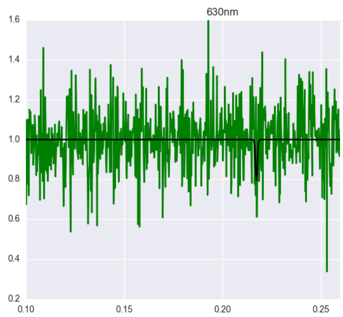


Figure 3.9: Absoptionline at 630nm

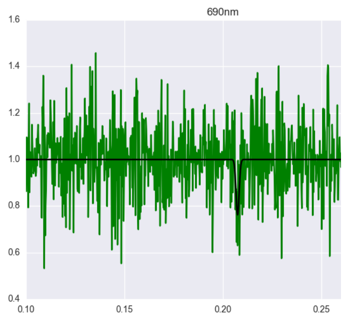


Figure 3.10: Absoptionline at 690nm

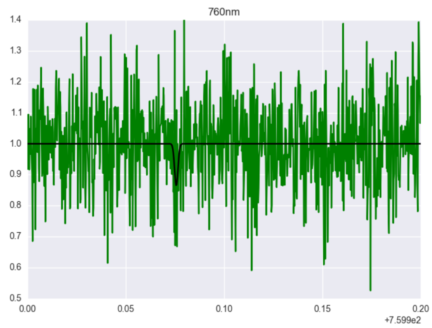


Figure 3.11: Absoptionline at 760nm

standard diviation. Looking at the graph, I see the spectra in the middle stands out. The reason is the width of the graph, but also $F \approx 0.7$ (the flux). Looking at the two other spectra, the diviation is also small, but flux is around $0.85 - 0.87$. This means that Oxygen is very highly probable to find in the atmosphere. Since the absorptionline at $690nm$ have a flux approximated to 0.7, I therefore conclude that Oxygen do exist in the atmosphere.

Methane:

Methane have two absorption lines at $1660nm$ and $2200nm$. The spectra is given down below

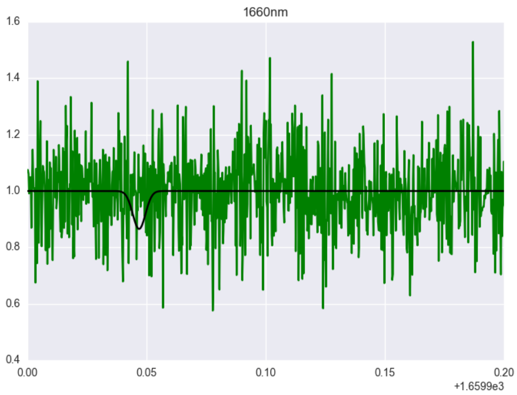


Figure 3.12: Absoptionline at 1660nm

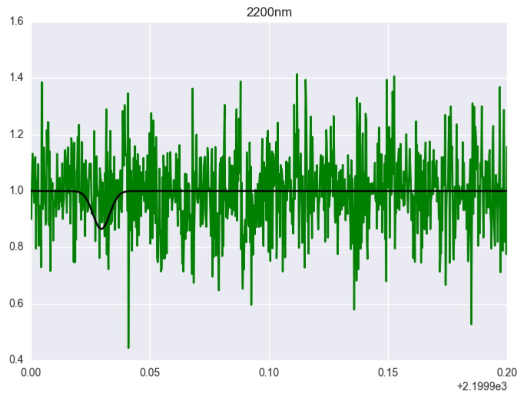
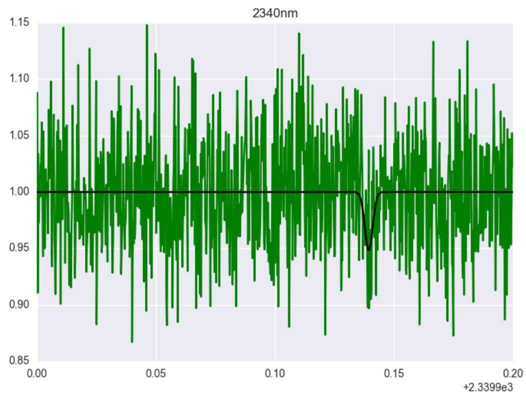


Figure 3.13: Absoptionline at 2200nm

Looking at the spectra, I notice that the standard diviation is quite large, and the measured is around 0.9, which indicates that I caught noise. I therefore conclude that methane do not exist in the atmosphere.

Carbon monooxide:

Carbon monooxide have absorptionline at the wavelength $2340nm$. The spectra is given down below



Looking at the spectra, I see that the stanard diviation is quite small, but F^{model} is around $0.93 - 0.96$ which makes me question that I have detected carbon monooxide. Since the $F^{model} \approx 1$, I will therefore conclude that carbon monooxide do not exist in the atmosphere.

Water vapor:

Water vapor have absorption lines at $720nm, 820nm$ and 940. when looking at the

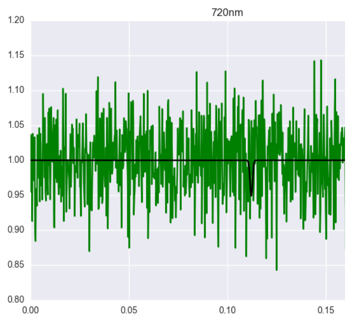


Figure 3.14: Absoptionline at 720nm

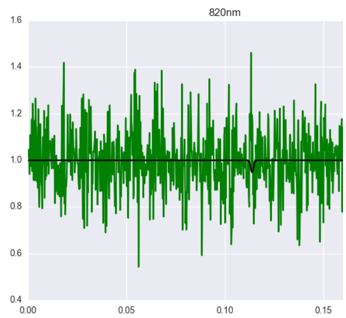


Figure 3.15: Absoptionline at 820nm

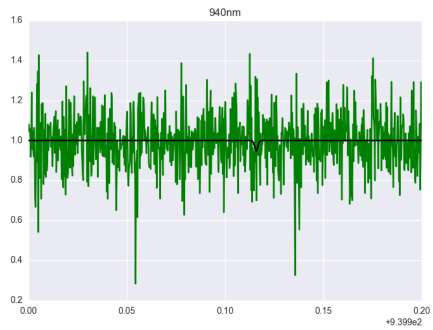
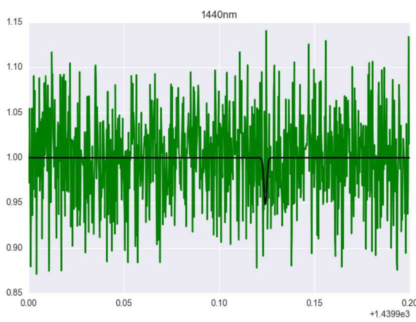


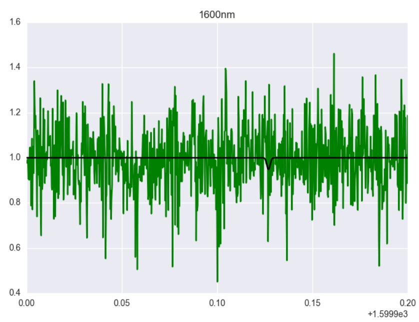
Figure 3.16: Absoptionline at 940nm

spectras it looks like that I have not detected water vapor in the atmosphere. Since all the spectra shows that $F^{model} \approx 1$, which means no absorption line.

Carbon dioxide:

Carbon dioxide have two absoprtionlines at $1440nm$ and $1600nm$.





Looking at the spectra I have not detected any Carbon dioxide in the atmosphere, even though the standard diviation on the spectra to the left is small, but $F^{model} \approx 0.95$ which indicates that I have not detected carbon monooxide.

Nitrous oxide:

Nitrous oxide have a absorption line at $2870nm$. Looking at the spectra it seems that

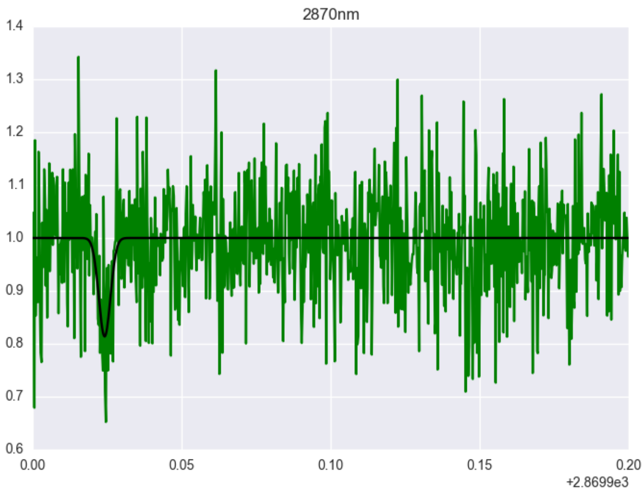


Figure 3.17: A diagram over flux seen and model at the wavelength 2870nm

I have detected Nitrous oxide in the atmosphere, since $F^{model} \approx$ and have quite small standard divation, which means this is not noise but a absorption line. I therefore conclude Nitrous oxide do exist in the atmosphere.

Conclusion of the data:

Looking through the spectras I recieved, I conclude that G23's atmosphere highly consist of oxygen and nitrous oxide. The mean molar mass of G23's atmosphere is

$$\mu_{mean} = 30.00245 \tag{3.136}$$

3.0.14 Modeling the atmosphere:

There are several methods to model the atmosphere. From previous sections I derived a analytical solution of G23 atmosphere, using the mean molar mass $\mu_{mean} = 30.00245$ the density of the atmosphere would like

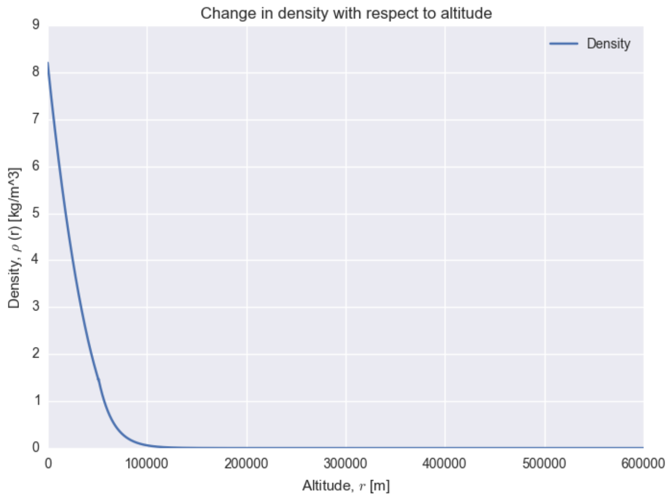


Figure 3.18: A graph showing how the atmosphere changes as the altitude changes

Looking at the graph it makes with the calculations done. At the height $51.3km$ it will go over to the isothermal part of the atmosphere. Since the isothermal is exponential function, when the altitude increases the atmospheric density decreases rapidly.

Conclusion

References