



$$\begin{aligned} \|\vec{PH}\| &= h \\ \frac{h}{2} &= \sin\left(\frac{\pi-\theta}{2}\right)R \\ \sin\left(\frac{\pi-\theta}{2}\right) &= \frac{h}{2R} \\ \frac{\pi-\theta}{2} &= \text{Asin}\left(\frac{h}{2R}\right) \\ \theta &= -2\text{Asin}\left(\frac{h}{2R}\right) + \pi \\ &= -2\text{Asin}\left(\frac{h}{2R}\right) + 180 \end{aligned}$$

$$\begin{aligned} \gamma &= \pi - \left(\frac{\pi-\theta}{2}\right) + \varphi = \varphi + \frac{\pi}{2} + \frac{\theta}{2} \\ \varphi &= \gamma - \frac{\pi}{2} - \frac{\theta}{2} \end{aligned}$$

pour coordonnée polaires

$P(x,y)$ $x = h \cdot \cos(\gamma)$ $y = h \sin(\gamma)$

$$x = 2 \sin\left(\frac{\pi-\theta}{2}\right)R \cos\left(\varphi + \frac{\pi}{2} + \frac{\theta}{2}\right) = 2 \sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)R \sin\left(\varphi + \frac{\theta}{2}\right) = -2R \cos\left(-\frac{\theta}{2}\right) \sin\left(\varphi + \frac{\theta}{2}\right)$$

$$y = 2 \sin\left(\frac{\pi-\theta}{2}\right)R \sin\left(\varphi + \frac{\pi}{2} + \frac{\theta}{2}\right) = -2R \cos\left(-\frac{\theta}{2}\right) \cos\left(\varphi + \frac{\theta}{2}\right)$$

x and y knowing θ and φ

we look for $h(x,y)$ and $\gamma(x,y)$

$$h = \sqrt{x^2 + y^2} \rightarrow \theta = -2\text{Asin}\left(\frac{\sqrt{x^2 + y^2}}{2R}\right) \text{ good!}$$

$$\gamma = \text{Atan}\left(\frac{y}{x}\right) \rightarrow \varphi = \text{Atan}\left(\frac{y}{x}\right) - \frac{\pi}{2} + \frac{\theta}{2} \leftarrow \text{not so good but}$$

$$\varphi = \text{Atan}\left(\frac{y}{x}\right) - \frac{\pi}{2} - \text{Asin}\left(\frac{\sqrt{x^2 + y^2}}{2R}\right) \text{ good!}$$

we've got $\varphi(x,y)$ and $\theta(x,y)$

⚠ special points

if $\left|\frac{\sqrt{x^2 + y^2}}{2R}\right| > 1$, then Asin won't work! $\sqrt{x^2 + y^2}$ will always be positive

So $\frac{\sqrt{x^2 + y^2}}{2R} > 1$ means that $\|\vec{HP}\| > 2R \rightarrow$ we will implement that in the code

if $x=0$, $\text{Atan}\left(\frac{y}{x}\right)$ won't work. but we know that if $x=0$ then θ must be 180 or π .

what about φ ? it can be anything it wants but we don't want it to hit the floor:

our space

Asin returns

if $\frac{y}{x} < 0 \rightarrow$ return $\text{Asin} + \pi$

ok

not ok