

hw2_filter

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1 Part I Convolution

In the first part, you are required to implement a function that performs a 2D convolution on an image.

```
In [0]: import numpy as np
        from PIL import Image
        from matplotlib import pyplot as plt
        import math
```

Please load example1.jpg.

```
In [0]: # if you are using Google Colab, please use the below codes to load image
        from google.colab import files
        from io import BytesIO
        uploaded = files.upload()
        img = Image.open(BytesIO(uploaded['example1.jpg']))
```

```
In [0]: # if you are using local jupyter notebook, please use the below codes to load image
        img = Image.open('example1.jpg')
```

```
In [0]: # Show the image
        h,w,_ = np.shape(img)
        print('height:',h, ' width: ',w)
        plt.figure()
        plt.imshow(img)
        plt.show()
```

```
In [0]: # now convert the RGB image into the gray image for further process
        gray_image = np.array(img.convert('L'))
        plt.figure()
        plt.imshow(gray_image, cmap='gray')
        plt.show()
```

1.1 Question 1

Now you should implement your 2d convolution function.

The output image should have the same shape as the input.

For border strategy, you will assume that value of the pixels falling outside the input image is 0.

```
In [0]: def convolution_2d(image, filter):
        '''
        Write your own code here
        '''
        return output
```

Now we will check the result after a gaussian filter. If you write the 2d convolution correctly, you will find the image become vague

```
In [0]: # Gaussian filter
def get_gaussian(filter_size, sigma):
    # return a gaussian filter with a size of filter_size and parameter sigma
    dim_n = len(filter_size)
    gaussian_weight = np.zeros(shape=filter_size)
    if dim_n == 2:
        dim_x, dim_y = filter_size
        center_x = dim_x/2
        center_y = dim_y/2
        for id_x in range(dim_x):
            for id_y in range(dim_y):
                weight = (id_x-center_x) ** 2 + (id_y-center_y)**2
                gaussian_weight[id_x, id_y] = math.exp(-weight/(2*sigma**2))/(math.sqrt(2*pi))
    return gaussian_weight

gaussian_filter = get_gaussian([5, 5], 4)
output_gaussian = convolution_2d(gray_image, gaussian_filter)
plt.figure()
plt.imshow(output_gaussian, cmap='gray')
plt.show()
```

Compare your result with the output from the convolution function provided by scipy. You will get full credit if your output shape is right and the mse error is smaller than 1e-5

```
In [0]: from scipy import signal

        # check the shape of input and output
        shape_check = np.shape(output_gaussian) == np.shape(gray_image)
        if shape_check:
            print('The shape of the convolution output is the same as input')
        else:
            print('The shape of the convolution output and input do not match, please check your code')

        # check the mse error
        output_standard = signal.convolve2d(gray_image, gaussian_filter, mode='same', boundary='fill')
        mse = np.mean(((output_standard - output_gaussian)/255)**2)
        print('The mse error is:', mse)
```

2 Bonus

Write a function that can determine whether a square filter provided as input is separable. If the filter is separable, the function returns the two 1D vectors of the decomposed 2D filter, like [vector_h, vector_v]. If not, the function returns None

Hint: You can use `numpy.linalg.matrix_rank()` to determine the rank of a matrix.

2.1 Question2: Separate a filter

```
In [0]: # the filter is a 2D square matrix
        # if the filter is separable, it will return two 1d vectors in a form [vector_h, vector_v]
        # if not, it will return None
        def separate_filter(filter):
            '''
                Write your own code here
            '''
```

Now we will check the your code with a random image and a random filter. You will receive full credit if your mse error is less than $1e-5$.

```
In [0]: import time

        # generate random filter
        v = np.random.randint(1, 10, size=[5, 1])
        h = np.random.randint(1, 10, size=[1, 5])
        random_filter = h*v

        # generate a random image
        random_image = np.random.randint(0, 255, size=[1000, 1000])

        # original_convolution
        start_time = time.time()
        output_2d = convolution_2d(np.array(random_image), filter)
        end_time = time.time()
        time_1 = end_time - start_time
        print('execution time for 2D convolution is: %.3f s'%(time_1))

        # convolution with seperable filters
        separate_result = separate_filter(filter)
        if not separate_result:
            print('The filter is not separable')
        else:
            start_time = time.time()
            filter_h, filter_v = separate_result
            output_1d = convolution_2d(np.array(random_image), filter_h)
            output_1d = convolution_2d(np.array(output_1d), filter_v)
            end_time = time.time()
```

```
time_2 = end_time - start_time
print('execution time for two 1D convolution is: %.3f s'%(time_2))

# calculate the mse loss beblow, if your loss is less than 1e-3, meaning that the two
err = np.mean(np.power(output_2d - output_1d, 2))
print('The mse loss for the results is: %.6f' % err)
```

2.2 Question3: Which method runs faster? Give a brief explanation about your result.

You may write your answer here.