Master 1 Informatique

Examen

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Documents Non Autorisés

Exercise I

(3) 1. For the following affirmations, answer by true or false. You don't need to justify. For each affirmation, a good answer is worth 0.5, no answer is worth 0, and a bad answer is worth -0.5.

(a) $n^3 + 2n^2 - 8n + 4 \in \mathcal{O}(2^n)$.

(b) Let $\mathcal{P}$ be the problem : "Given a directed graph $G = \langle N,E \rangle$, and two nodes $n_1, n_2 \in N$, is there a path from n_1 to n_2 ?". $\mathcal{P}$ is recognizable.

(c) We know that SAT $\in \Sigma_2^{\mathsf{P}}$.

(d) If a problem $\mathcal{P}$ is in PSPACE, then $\mathcal{P} \in \Sigma_3^{\mathsf{P}}$.

(e) If $\mathcal{P}$ is coNP-complete and $\mathcal{P} \leqslant_f^{\mathsf{P}} \mathcal{P}'$, then we are sure that $\mathcal{P}' \in \mathsf{coNP}$.

(f) If $\mathcal{P}$ is coNP-complete and $\mathcal{P} \leqslant_f^{\mathsf{P}} \mathcal{P}'$, then we are sure that $\mathcal{P}'$ is coNP-complete.

(1) 2. Give a problem which is Π_2^{P}-complete. If this is not a problem presented during the classes, you must prove that it is Π_2^{P}-complete.

(1) 3. Give the definition of Σ_k^{P}.

Exercise II

1. We have two algorithms A and B to solve a problem. They need respectively $t_A(n) = 48 \times n$ and $t_B(n) = 6 \times n \times \log_5(n)$ seconds to finish when n is the size of the problem.

(½) (a) Which algorithm is better (with respect to the $\mathcal{O}$ notation) ?

(1) (b) For which problem size does it become better ?

Exercise III

We suppose that the Turing machine starts on the first square of the input word (there are no blank symbols before it). There are (infinitely) many blank symbols after the input word. The whole definition of the Turing machine is expected (set of states, vocabulary, input vocabulary, blank symbol, initial state, final states, transition function).

(3) 1. Define a Turing Machine $\mathcal{M}_{even}$ which reads a sequence of numbers and replaces each number by 0 if the number is even and 1 if the number is odd. We consider an alphabet with only five symbols $\{1,2,3,4,5\}$.
Example : on the input 1234455, the execution of $\mathcal{M}_{even}$ gives on the tape 1010011.

Exercise IV

We call VALID the problem : "Given a propositional formula φ, is φ valid ?". φ is valid if every possible interpretation ω is a model of φ.

(½) 1. Reformulate VALID as a language.

(1) 2. A negative certificate for VALID is an interpretation ω such that $\omega(\varphi) = 0$. Write a polynomial algorithm for such a certificate.

(1) 3. From the previous question, for which complexity class C of the polynomial hierarchy can we say that $\mathsf{VALID} \in \mathsf{C}$? Why ?

(2) 4. Prove that $\mathsf{UNSAT} \leqslant_f^{\mathsf{P}} \mathsf{VALID}$.

(1) 5. From the previous questions, for which complexity class C of the polynomial hierarchy can we say that VALID is C-complete ? Why ?

Exercise V

We call DEDUCE the problem : "Given two propositional formulas φ, ψ, is $\varphi \vdash \psi$ true ?". $\varphi \vdash \psi$ means that ψ is a consequence of φ, or equivalently that ψ can be deduced from φ. Concretely, it means that all the models of φ are also models of ψ.

(5) 1. Prove that DEDUCE is coNP-complete.