Master 1 Informatique

Contrôle Continu

21/10/2019

Documents Non Autorisés

Total Number of Points: 20

Exercise I

(1) 1. Give the definition of $\mathcal{O}(2^n)$.

(1) 2. Give the definition of a polynomial-time functional reduction

(1) 3. Give the definition of a path in a directed graph.

(1) 4. Give the definition of a decision problem.

(1) 5. Give the definition of $\mathsf{NTIME}(f(n))$.

(1) 6. Given $\omega(\varphi_1)$ and $\omega(\varphi_2)$, what are the values of $\omega(\varphi_1 \vee \varphi_2)$ and $\omega(\varphi_1 \Leftrightarrow \varphi_2)$?

(1) 7. Give the definition of a deterministic Turing machine.

(1) 8. Give the definition of NP-completeness.

Exercise II

Multiple Choice Questions. For the following affirmations, there is **exactly one good answer**. You don't need to justify. For each question, a good answer is worth 0.5, no answer is worth 0, and a bad answer is worth −0.5.

(½) 1. $4n^3 + 3n^2 + 5n + 7 \in \mathcal{O}(n!)$.
☐ True ☐ False

(½) 2. For $f_1(n) = 2n^4 + 8 \times n^3 + 3 \times n^2 + 6 \times 2^3$, what is the smallest $\mathcal{O}$-class it belongs?
☐ $\mathcal{O}(n^5)$ ☐ $\mathcal{O}(n^4)$ ☐ $\mathcal{O}(2^n)$

(½) 3. For $f_2(n) = \log_3(n) + 4 \times n^4 + 7 \times n \log_2(n)$, what is the smallest $\mathcal{O}$-class it belongs?
☐ $\mathcal{O}(\ln(n))$ ☐ $\mathcal{O}(n^4)$ ☐ $\mathcal{O}(n \log(n))$

4. Let G be the following graph:

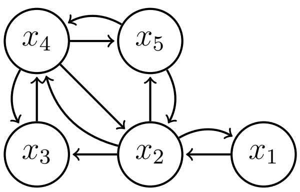

Figure 1: The directed graph G

(½) (a) What is number of nodes in the largest clique?
☐ 2 ☐ 3 ☐ 4

(½) (b) What is the length of the shortest path from x_1 to x_5?
☐ 2 ☐ 3 ☐ 4

(½) (c) What is the length of the longest path (without repetition) from x_1 to x_5?
☐ 2 ☐ 4 ☐ 6

(½) (d) To how many cycles does x_2 belong?
☐ 3 ☐ 4 ☐ 5 ☐ 6

(½) 5. The formula $\varphi_1 = (\neg p \wedge r \wedge \neg r) \vee (c \wedge b \wedge \neg a)$ is
☐ a CNF ☐ a DNF ☐ both ☐ neither of them

(½) 6. The formula $\varphi_2 = (a \vee c) \wedge \neg(\neg c \wedge b)$ is
☐ a CNF ☐ a DNF ☐ both ☐ neither of them

7. For each of these problems, determine which type of problem it is.

(½) (a) Given a directed graph $G = \langle N, E \rangle$, two nodes $n_1, n_2 \in N$, what is the largest clique in G that contains n_1 and n_2?
☐ function ☐ enumeration ☐ optimization ☐ decision

(½) (b) Given a list of integers L and an integer k, is k the greatest integer in L?
☐ function ☐ enumeration ☐ optimization ☐ decision

Exercise III

(1) 1. Prove with the formal definition that $f(n) = 4 \times n^2 + 2 \times n + 8 \in \mathcal{O}(n^2)$.

Exercise IV

For each formula φ_i and interpretation ω_i below, is the ω_i a model of φ_i? Justify.

(½) 1. $\varphi_1 = (x \vee \neg y \vee z) \wedge (x \vee \neg p)$ and $\omega_1 = \{x,p\}$

(½) 2. $\varphi_2 = (\neg x \vee \neg y) \wedge (\neg t \vee \neg z)$ and $\omega_2 = \{x,z\}$

(½) 3. $\varphi_3 = (\neg p \wedge \neg q \wedge r) \vee (a \wedge b \wedge \neg c)$ and $\omega_3 = \{a,b,c\}$

(½) 4. $\varphi_4 = (\neg a \vee b) \wedge (c \vee \neg b) \wedge (a \vee \neg c)$ and $\omega_4 = \{b,c\}$

(½) 5. $\varphi_5 = (a \wedge b) \vee (\neg c \wedge b)$ and $\omega_5 = \{a,b\}$

Exercise V

We suppose that the Turing machine starts on the first square of the input word (there are no blank symbols before it). There are (infinitely) many blank symbols after the input word.

(2) 1. Define a Turing Machine $\mathcal{M}_{sorted}$ which reads a sequence of numbers and decides if the sequence is sorted in increasing order. We consider only sequences of numbers in $\{1,2,3,4,5\}$. Example:

- $\mathcal{M}_{sorted}$ accepts the inputs 1234455, 345, and 122244;
- $\mathcal{M}_{sorted}$ rejects the inputs 1234452, 4142511, and 55544432111.

(1) 2. Give the sequence of transitions of the machine $\mathcal{M}_{sorted}$ on the input words 1352 and 112445.