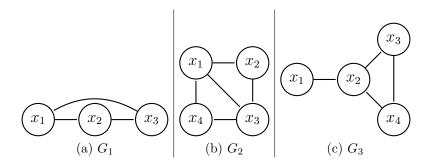
Exercises: Graphs and Logic

2022

I Graphs

Exercise I

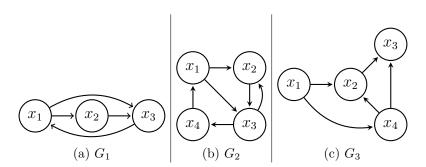
Let G_1, G_2, G_3 be the following graphs: For each of these graphs, answer the following questions:



- 1. (a) Give all the chains between x_1 and x_3 .
 - (b) Give all the cycles.
 - (c) Give all the cliques.
- 2. Give the subgraph induced by $\{x_1, x_2, x_3\}$.

Exercise II

Let G_1, G_2, G_3 be the following graphs: For each of these graphs, answer the following questions:



- 1. (a) Give all the paths from x_1 to x_3 .
 - (b) Give all the cycles.
 - (c) Give all the cliques.
- 2. Give the subgraph induced by $\{x_1, x_2, x_3\}$.

II Logic

Exercise III

Which of these formulas are well-formed? For those which are not well-formed, why?

1.
$$\varphi_1 = x$$

2.
$$\varphi_2 = x \wedge (y \vee)$$

3.
$$\varphi_3 = x \wedge (y \vee z)$$

$$4. \varphi_4 \Longrightarrow a$$

5.
$$\varphi_5 = (a \lor x) \land (c \lor \neg (d \land \neg p))$$

6.
$$\varphi_6 = (p \lor q) \Rightarrow (a \land \neg)$$

7.
$$\varphi_7 = x \vee (y \wedge) \vee z$$

8.
$$\varphi_8 = a \Rightarrow (\neg b \land \neg b \land \neg c)$$

9.
$$\varphi_9 = a \Leftrightarrow \neg a$$

10.
$$\varphi_{10} = x \vee (a \wedge (b \Leftrightarrow (c \Rightarrow (d \Rightarrow e))))$$

11.
$$\varphi_{11} = \neg(a \land (c \lor x))$$

12.
$$\varphi_{12} = \neg(a \Leftrightarrow (\land d))$$

Exercise IV

For each of these formulas, is it a CNF formula, a DNF formula, or neither of them?

1.
$$\varphi_1 = (x \vee \neg y \wedge z) \wedge (\neg x \vee p)$$

2.
$$\varphi_2 = \neg(x \vee \neg y) \wedge (t \vee \neg z)$$

3.
$$\varphi_3 = (p \land q \land \neg r) \lor (a \land \neg b \land \neg c)$$

4.
$$\varphi_4 = (\neg a \lor c) \land (a \lor b)$$

5.
$$\varphi_5 = (\neg a \lor b \lor c) \land (\neg b \lor a)$$

6.
$$\varphi_6 = (a \land \neg b) \lor (c \land \neg d)$$

7.
$$\varphi_7 = (x \wedge \neg x) \vee (\neg y \wedge y)$$

8.
$$\varphi_8 = (t \vee u \vee \neg v) \wedge (a \vee b) \wedge c$$

9.
$$\varphi_9 = a \wedge b \wedge c$$

10.
$$\varphi_{10} = a \lor b \lor c$$

11.
$$\varphi_{11} = \neg(a \wedge b \wedge c)$$

12.
$$\varphi_{12} = a \vee \neg c \vee (x \wedge y)$$

Exercise V

For each of these pairs of formulas and interpretations, is the interpretation a model of the formula?

1.
$$\varphi_1 = (x \vee \neg y \vee z) \wedge (\neg x \vee p)$$
 and $\omega_1 = \{x,y\}$

2.
$$\varphi_2 = \neg(x \vee \neg y) \wedge (t \vee \neg z)$$
 and $\omega_2 = \{x, t\}$

3.
$$\varphi_3 = (p \land q \land \neg r) \lor (a \land \neg b \land \neg c)$$
 and $\omega_3 = \{p,q,r\}$

- 4. $\varphi_4 = (\neg a \lor c) \land (a \lor b)$ and $\omega_4 = \{a,b\}$
- 5. $\varphi_5 = (\neg a \lor b \lor c) \land (\neg b \lor a) \text{ and } \omega_5 = \{a\}$
- 6. $\varphi_6 = (a \wedge \neg b) \vee (c \wedge \neg d)$ and $\omega_6 = \{a, d\}$
- 7. $\varphi_7 = (x \wedge \neg x) \vee (y \wedge \neg y)$ and $\omega_7 = \emptyset$
- 8. $\varphi_8 = (t \lor u \lor \neg v) \land (a \lor b) \land c \text{ and } \omega_8 = \{b,c\}$