Master 1 Informatique **Contrôle Continu** 6/11/2018

Documents Non Autorisés

Total Number of Points: 20

**Exercise I**

(1) 1. Give the definition of a recognizable problem.

(1) 2. Give the definition of $\mathcal{O}(n^3)$.

(1) 3. Give the definition of an enumeration problem.

(1) 4. Give the definition of a chain in a non-directed graph.

(1) 5. Given $\omega(\varphi_1)$ and $\omega(\varphi_2)$, what are the values of $\omega(\varphi_1 \wedge \varphi_2)$ and $\omega(\varphi_1 \Rightarrow \varphi_2)$?

(1) 6. Give the definition of a cube.

(1) 7. Give the definition of $\mathsf{DTIME}(f(n))$

**Exercise II**

Multiple Choice Questions. For the following affirmations, there is **exactly one good answer**. You don't need to justify. For each question, a good answer is worth 0.5, no answer is worth 0, and a bad answer is worth −0.5.

(½) 1. UNSAT is decidable.
☐ True ☐ False

(½) 2. $n^4 + 3n + 4 \in \mathcal{O}(3^n)$.
☐ True ☐ False

(½) 3. If a problem $\mathcal{P}$ can be decided by a deterministic Turing machine, then it can be recognized by a deterministic Turing machine.
☐ True ☐ False

(½) 4. For $f_1(n) = n^4 + 8 \times n^3 + 7 \times n^2 + 6$, what is the smallest $\mathcal{O}$-class it belongs?
☐ $\mathcal{O}(n^5)$ ☐ $\mathcal{O}(n^4)$ ☐ $\mathcal{O}(n^2)$

(½) 5. For $f_2(n) = \log_3(n) + 4 \times n^4 + 7 \times (\ln(n))^{300}$, what is the smallest $\mathcal{O}$-class it belongs?
☐ $\mathcal{O}(\ln(n))$ ☐ $\mathcal{O}(n^4)$ ☐ $\mathcal{O}(\log_3(n))$

6. Let $G$ be the following graph:

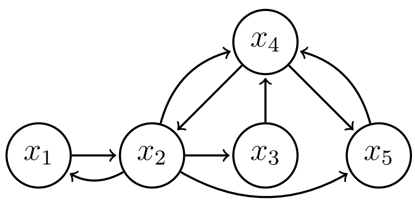


Figure 1: The directed graph $G$

(½) (a) What is number of nodes in the largest clique?
☐ 2 ☐ 3 ☐ 4

(½) (b) What is the length of the shortest path from $x_1$ to $x_5$?
☐ 2 ☐ 3 ☐ 4

(½) (c) What is the length of the longest path (without repetition) from $x_1$ to $x_5$?
☐ 2 ☐ 4 ☐ 6

(½) (d) To how many cycles does $x_2$ belong?
☐ 2 ☐ 3 ☐ 4 ☐ 5

(½) 7. The formula $\varphi_1 = \neg(\neg p \land r \land \neg r) \lor (c \land b \land \neg a)$ is
☐ a CNF ☐ a DNF ☐ both ☐ neither of them

(½) 8. The formula $\varphi_2 = (a \lor c) \land (\neg c \land b)$ is
☐ a CNF ☐ a DNF ☐ both ☐ neither of them

(½) 9. The formula $\varphi_3 = (x \lor \neg y \land z) \land (\neg x \lor p)$ is
☐ a CNF ☐ a DNF ☐ both ☐ neither of them

(½) 10. The formula $\varphi_4 = (\neg x \lor \neg y) \land (z \lor \neg x)$ is
☐ a CNF ☐ a DNF ☐ both ☐ neither of them

11. For each of these problems, determine which type of problem it is.

(½) (a) Given a directed graph $G = \langle N, E\rangle$ and two nodes $n_1, n_2 \in N$, is there a clique in $G$ that contain $n_1$ and $n_2$?
☐ function ☐ enumeration ☐ optimization ☐ decision

(½) (b) Given a list of integers $L$ and an integer $i$, give the $i^{th}$ element of $L$.
☐ function ☐ enumeration ☐ optimization ☐ decision

**Exercise III**

(1) 1. Prove with the formal definition that $f(n) = 3 \times n^2 + 6 \times n + 7 \in \mathcal{O}(n^2)$.

**Exercise IV**

For each of these pairs of formulas and interpretations, is the interpretation a model of the formula?

(½) 1. $\varphi_1 = (x \lor y \lor \neg z) \land (\neg x \lor p)$ and $\omega_1 = \{y,p\}$

(½) 2. $\varphi_2 = \neg(\neg x \lor y) \land (t \lor \neg z)$ and $\omega_2 = \{x,t\}$

(½) 3. $\varphi_3 = (\neg p \land q \land r) \lor (a \land \neg b \land c)$ and $\omega_3 = \{a,c\}$

(½) 4. $\varphi_4 = (\neg a \lor b) \land (c \lor b)$ and $\omega_4 = \{a,c\}$

(½) 5. $\varphi_5 = (\neg a \land b) \lor (c \land b)$ and $\omega_5 = \{a,b\}$

**Exercise V**

We suppose that the Turing machine starts on the first square of the input word (there are no blank symbols before it). There are (infinitely) many blank symbols after the input word.

(2) 1. Define a Turing Machine $\mathcal{M}_{pos}$ which reads a sequence of letters and replaces the value of each letter by its position in the alphabet. We consider an alphabet with only five letters $\{a,b,c,d,e\}$.
Example: on the input *adeabaceca*, the execution of $\mathcal{M}_{pos}$ gives on the tape 1451213531.