Knowledge Representation and Reasoning

Computational Argumentation

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M1 Computer Science

Outline

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- **John**: "Moreover, the other restaurants in the streets are closed."

Formalizing Arguments

Formally, an argument is made of a support which allows to deduce a given claim

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Conflicts between Arguments

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- claim vs claim (rebuttal): "I'm hungry, let's go to this restaurant." vs "Its grades on Tripadvisor are bad, let's go somewhere else."
- claim vs support (undercut): "The Tripadvisor grades are old and there is a new chef, so it should be better now." vs "I've seen on Tripadvisor that the food is bad, let's go somewhere else."

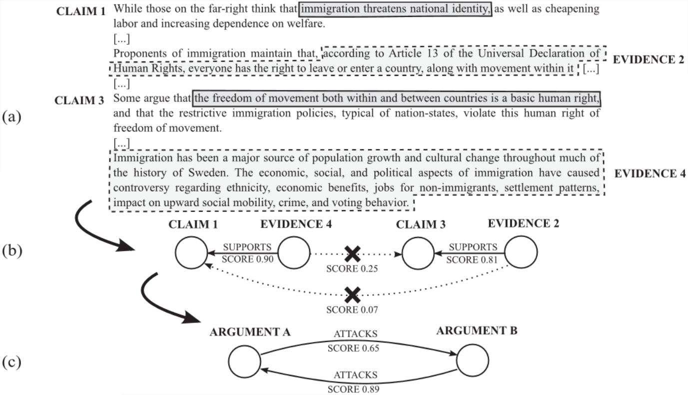
Different argumentation formalisms

Several **formalisms** capture the nature of **arguments** and **attacks**

- Deductive argumentation [Besnard and Hunter 2008]
- Rule-based argumentation
  [Kakas and Moraitis 2003, Toni 2014, Modgil and Prakken 2014]

Support and claims are represented as logical formulas or rules, then arguments and attacks are built

Argument mining



[Lippi and Torroni 2016]

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- A. Kakas, P. Moraitis, *Argumentation based decision making for autonomous agents*. AAMAS'03, pp 883-890, 2013.
- S. Modgil, H. Prakken, *The ASPIC+ framework for structured argumentation: a tutorial*. Argument & Computation 5(1), pp 31-62, 2014.
- M. Lippi and P. Torroni, *Argumentation mining: State of the art and emerging trends*. ACM Transactions on Internet Technology (TOIT), 2016.

Outline

Abstracting arguments and relationships

Dung's Argumentation Framework [Dung 1995]

Argumentation Framework (**AF** for short): $F = \langle A, R \rangle$ where

- A is a set of arguments
- $R \subseteq A \times A$ represents attacks between arguments

Abstracting arguments and relationships

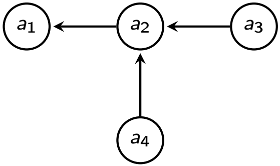
Dung's Argumentation Framework [Dung 1995]

Argumentation Framework (**AF** for short): $F = \langle A, R \rangle$ where

- A is a set of arguments
- $R \subseteq A \times A$ represents attacks between arguments

- Example:
  - a_1: (John) "I'm hungry, let's go to this restaurant."
  - a_2: (Yoko) "I've seen on Tripadvisor that the food is bad, let's go somewhere else."
  - a_3: (John) "The Tripadvisor grades are old and there is a new chef, so it should be better now."
  - a_4: (John) "Moreover, the other restaurants in the streets are closed."

$F = \langle A, R \rangle$ with
$A = \{a_1, a_2, a_3, a_4\}$,
$R = \{(a_2, a_1), (a_3, a_2), (a_4, a_2)\}$



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Given an argumentation framework $F = (A, R)$:

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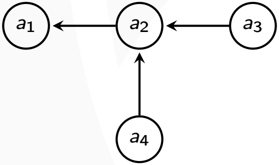

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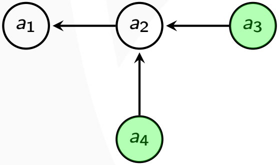

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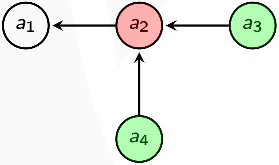

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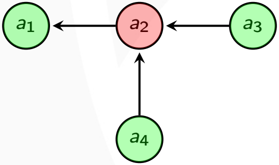

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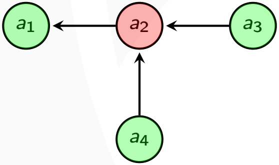

- We say that a_1 is defended by a_3 and a_4 against a_2

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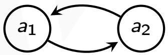
Cycles: Dilemmas

- a_1: "I like this restaurant, let's eat here." (John)
- a_2: "I don't like this restaurant, let's go somewhere else." (Yoko)



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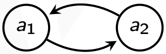
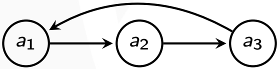


What can we accept?

Cycles: Paradoxes

- a_1: "John says that Paul is a liar."
- a_2: "Paul says that George is a liar."
- a_3: "George says that John is a liar."



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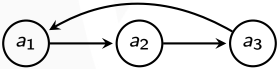


What can we accept?

Extension-based Semantics [Dung 1995]

- **Extension**: set of jointly acceptable arguments
  - "solution" of the debate
  - "point of view" about the situation
- A set of arguments has to satisfy some properties to be an extension: *e.g.* we don't accept arguments a_1 and a_2 together if there is an attack between them
- **Semantics**: Function σ which maps an AF $F = \langle A, R \rangle$ to a set of extensions $\sigma(F) \subseteq 2^A$

Dung's Semantics

Basic principles

A set $S \subseteq A$ is

- conflict-free (**cf**) w.r.t. F if $\nexists a_i, a_j \in S$ s.t. $(a_i, a_j) \in R$
- admissible (**ad**) w.r.t. F if S is cf and S defends each $a_i \in S$ (*i.e.* $\forall a_i \in S$, $\forall a_j$ s.t. $(a_j, a_i) \in R$, $\exists a_k \in S$ s.t. $(a_k, a_j) \in R$)

Classical semantics

A set $S \subseteq A$ is

- complete (**co**) w.r.t. F if S is ad and S contains all the arguments that it defends
- preferred (**pr**) w.r.t. F if S is a maximal co extension (w.r.t. $\subseteq$)
- stable (**st**) w.r.t. F if S is cf and S attacks every $a_j \in A \setminus S$
- grounded (**gr**) w.r.t. F if S is a minimal co extension (w.r.t. $\subseteq$)

Arguments' Acceptance

Skeptical Acceptance

Given $F = \langle A, R \rangle$ and σ, $a \in A$ is skeptically accepted by F w.r.t. σ iff $\forall S \in \sigma(F)$, $a \in S$

$skep_\sigma(F)$ is the set of skeptically accepted arguments

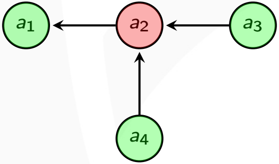
Credulous Acceptance

Given $F = \langle A, R \rangle$ and σ, $a \in A$ is credulously accepted by F w.r.t. σ iff $\exists S \in \sigma(F)$, s.t. $a \in S$

$cred_\sigma(F)$ is the set of credulously accepted arguments

Dung's Semantics

Example:



- $\{a_1, a_3, a_4\}$ is the unique complete (resp. preferred, stable, grounded) **extension**.
- This set is then also the set of **skeptically** and of **credulously** accepted arguments under these semantics.

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grounded	$\{\emptyset\}$	$\emptyset$	$\emptyset$

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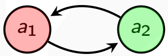

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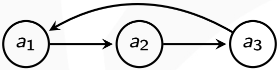

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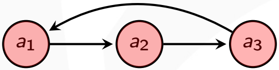

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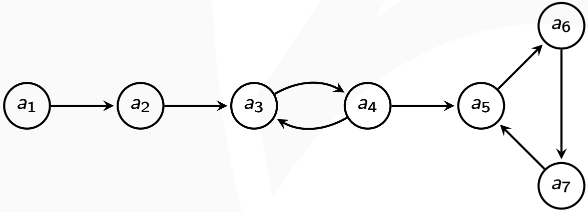
Exercise



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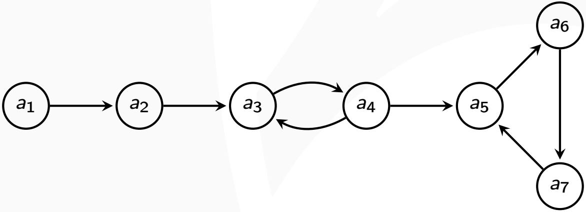
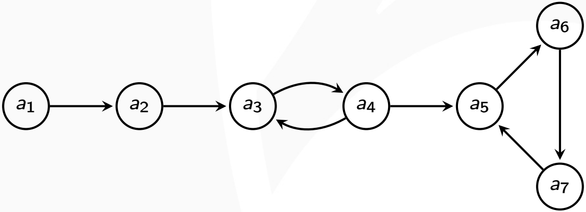


Questions:

- What are the **extensions** under the various semantics (grounded, stable, preferred, complete)?
- Under each semantics, which arguments are **credulously** accepted? Which ones are **skeptically** accepted?

Example: Semantics Comparison



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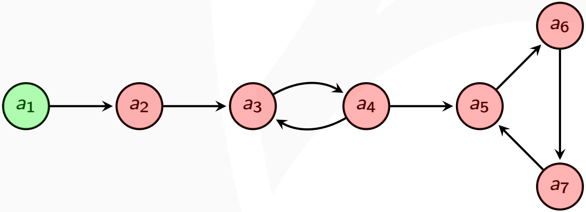

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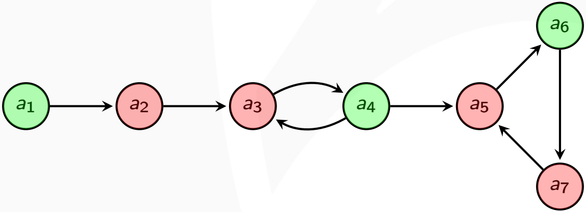

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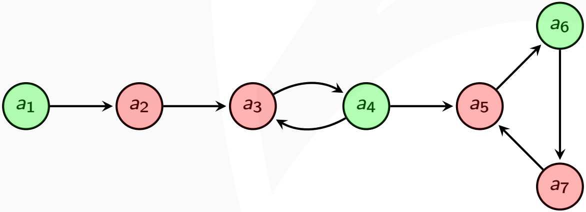

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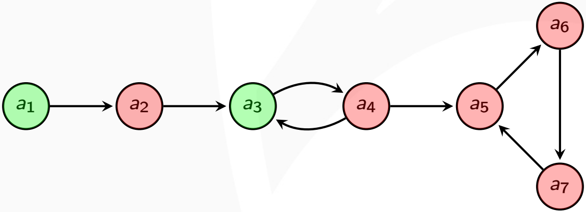

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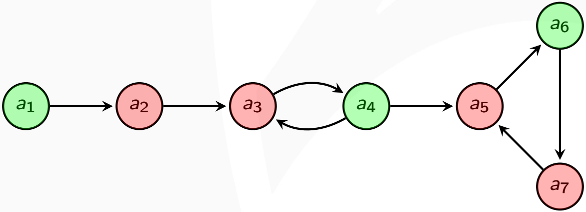

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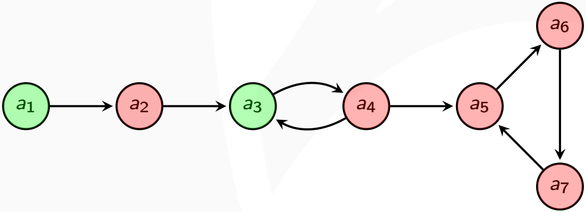

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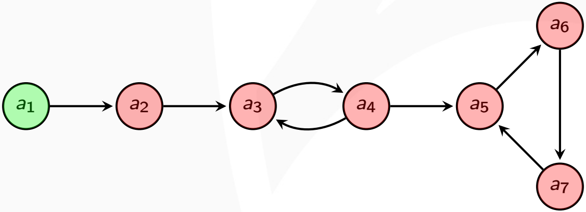

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Computational Complexity

- Reasoning with AFs is generally hard [Dvorak and Dunne 2018]

Problem	Grounded	Stable	Preferred	Complete
Exist	Trivial	NP-c	Trivial	Trivial
Exist^{NT}	L	NP-c	NP-c	NP-c
Verif	P-c	L	coNP-c	L
Cred	P-c	NP-c	NP-c	NP-c
Skep	P-c	coNP-c	Π_2^P-c	P-c

- Exist: For F and σ, is $\sigma(F) \neq \emptyset$?
- Exist^{NT}: For F and σ, is $\sigma(F) \neq \emptyset$ s.t. F has at least one non-empty extension?
- Verif: For F, S and σ, is $S \in \sigma(F)$?
- Cred: For F, a and σ, is there some $S \in \sigma(F)$ s.t. $a \in S$?
- Skep: For F, a and σ, is $a \in S$ for each $S \in \sigma(F)$?

A (very) naive approach

- Apply the definitions: compute all the **cf** sets, and then choose among them
  - Choose the ones that attack everything else: **st**
  - Choose the ones that defend themselves: **ad**
    - Choose the ones that contain everything they defend: **co**
    - Choose the $\subseteq$-maximal **ad** sets: **pr**
    - Choose the $\subseteq$-minimal **co** extension: **gr**

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    - Choose the ones that contain everything they defend: **co**
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    - Choose the $\subseteq$-minimal **co** extension: **gr**
- **Not efficient at all!**
  - The number of conflict-free sets can be exponential

Dung's Characteristic Function

Definition

The characteristic function of an AF $\langle A, R \rangle$ is $\mathcal{F} : 2^A \to 2^A$ defined, for $S \subseteq A$, by

$$\mathcal{F}(S) = \{a \in A \mid S \text{ defends } a\}$$

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We can use the characteristic function to determine the extensions for all the semantics based on admissibility

co-Extensions and Characteristic Function

Proposition

Let S be a conflict-free set. $S \in \text{co}(F)$ iff $S = \mathcal{F}(S)$

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- To compute a complete extension, consider a conflict-free set S, and apply iteratively the characteristic function until a fixed point is reached

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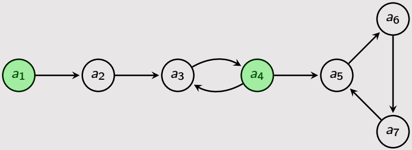

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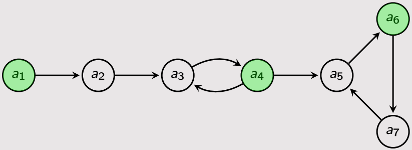

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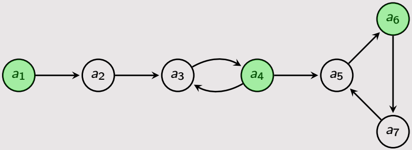

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pr-Extensions and Characteristic Function

- To compute a preferred extension with the characteristic function, first compute a complete extension S
- Then, define $A^S = \{a \in A \setminus S \mid S \cup \{a\} \in \mathbf{cf}(F)\}$
- If $A^S = \emptyset$, then S is a preferred extension of F
- Otherwise, choose some $a \in A^S$, and apply iteratively $\mathcal{F}$ from $S \cup \{a\}$ until a new complete extension S^2 is obtained
- Repeat the process: if $A^{S^2} = \emptyset$, then S^2 is a preferred extension, otherwise choose some $a \in A^{S^2}$, etc.

Computing **pr**-Extensions

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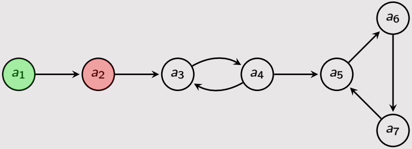

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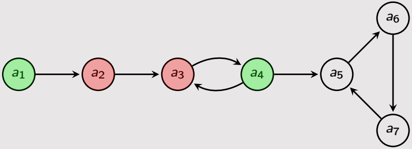

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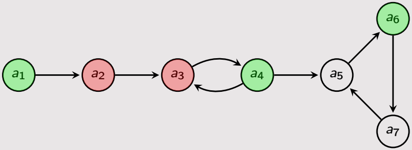

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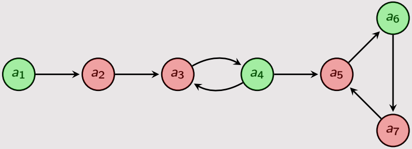

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- $A^{S^2} = \emptyset$: $S^2 \in \text{pr}(F)$

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The grounded extension of F is the fixed-point obtained when $\mathcal{F}$ is applied iteratively from $\emptyset$

In short: take the unattacked arguments, and then propagate

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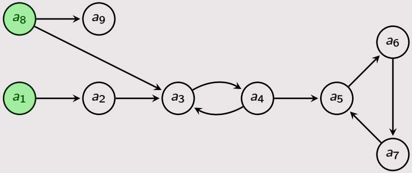

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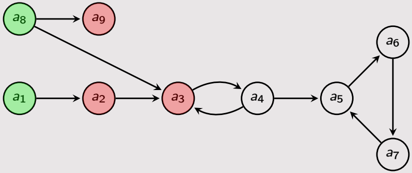

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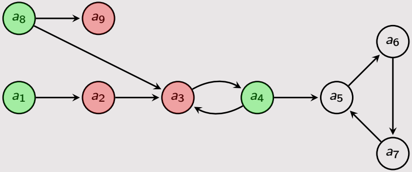

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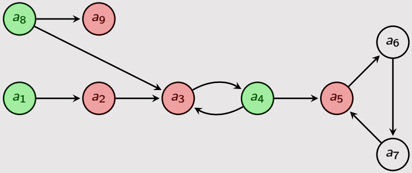

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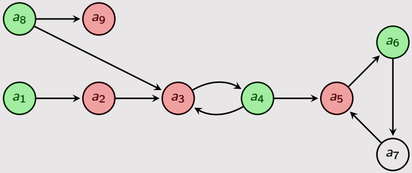

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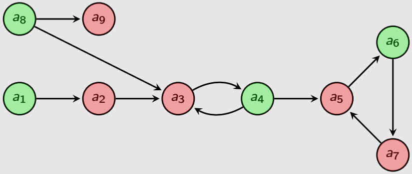

Well-founded AFs

Definition [Dung 1995]

An AF $F = \langle A, R \rangle$ is well-founded iff $\nexists a_0, a_1, \ldots$ an infinite sequence of arguments s.t. $\forall i \in \mathbb{N}^+$, $(a_{i+1}, a_i) \in R$

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- If A is finite, then well-founded is equivalent to acyclic

Theorem [Dung 1995]

For every well-founded AF F, $\text{gr}(F) = \text{st}(F) = \text{pr}(F) = \text{co}(F)$

- So it might be useful to check whether the graph is acyclic before computing extensions

Stable Semantics and SAT

Intuition:

- Encoding arguments' acceptance in Boolean variables
- Define a formula such that each model corresponds to an extension

Logical Encoding of Stable Semantics [Besnard and Doutre 2004]

For $F = \langle A, R \rangle$, $S \subseteq A$ is a stable extension of F iff S is a model of

$$\phi_{st}(F) = \bigwedge_{a \in A} (a \leftrightarrow \bigwedge_{(b,a) \in R} \neg b)$$

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- Stable extensions corresponds to kernels of the graph (see the Algorithmic Complexity lessons)

Admissible Sets and SAT

Logical Encoding of Conflict-freeness [Besnard and Doutre 2004]

For $F = \langle A, R \rangle$, $S \subseteq A$ is conflict-free in F iff S is a model of

$$\phi_{cf}(F) = \bigwedge_{(a,b) \in R} (\neg a \vee \neg b)$$

Logical Encoding of Defense [Besnard and Doutre 2004]

For $F = \langle A, R \rangle$, $S \subseteq A$ defends itself in F iff S is a model of

$$\phi_{def}(F) = \bigwedge_{a \in A} (a \rightarrow \bigwedge_{(b,a) \in R} (\bigvee_{(c,b) \in R} c))$$

Logical Encoding of Admissibility [Besnard and Doutre 2004]

For $F = \langle A, R \rangle$, $S \subseteq A$ is admissible in F iff S is a model of
$\phi_{ad}(F) = \phi_{cf}(F) \wedge \phi_{def}(F)$

Complete Extensions and SAT

- Complete extensions contain everything that they defend
- Modify $\phi_{def}(F)$:

$$\phi'_{def}(F) = \bigwedge_{a \in A} (a \leftrightarrow \bigwedge_{(b,a) \in R} (\bigvee_{(c,b) \in R} c))$$

Logical Encoding of Complete Semantics [Besnard and Doutre 2004]

For $F = \langle A, R \rangle$, $S \subseteq A$ is a complete extension of F iff S is a model of

$$\phi_{co}(F) = \phi_{cf}(F) \wedge \phi'_{def}(F)$$

- For grounded semantics, just apply unit propagation to $\phi_{co}(F)$

Preferred Extensions and SAT

- Preferred extensions cannot be directly encoded to SAT, since skeptical acceptance is Π_2^P-complete (except if the polynomial hierarchy collapses)
- An algorithm for using SAT at the second level of polynomial hierarchy: CEGAR [Dvorak *et al* 2012]
- Counter Example Guided Abstraction Refinement
- Principle of the algorithm:
  - Find an abstraction of the solution (*e.g.* a complete extension S instead of a preferred extension)
  - If S is preferred, then it's ok
  - Otherwise, modify the logical encoding to forbid S, and try again

A Competition of Solvers

- Since 2015, ICCMA (International Competition of Computational Models of Argumentation) evaluates te best algorithms for reasoning with AFs
- The best solvers are usually based on SAT
  - CoQuiAAS [Lagniez *et al* 2015]
  - ArgSemSAT [Cerruti *et al* 2014]
  - Cegartix [Dvorak *et al* 2012]
  - μ-Toksia (`https://www.cs.helsinki.fi/u/andreasn/`)
- Next competition: in 2023
  - `http://argumentationcompetition.org`

More Semantics

Other **extension-based** semantics:

- naive
- ideal
- stage
- semi-stable
- eager
- ...

See [Baroni *et al* 2018]

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More Semantics

Other types of semantics:

- **Labelling-based** semantics
  - Every argument is assigned a label: **in**, **out** or **undec**.
  - a is labelled **in** iff $\forall b$ s.t. $(b, a) \in R$, b is labelled out
  - a is labelled **out** iff $\exists b$ s.t. $(b, a) \in R$ and b is labelled in
  - a is labelled **undec** iff a is neither labelled in nor out
  - Several possible labellings can result.
  - Correspondence shown between labellings and extensions

$\Rightarrow$ [Caminada 2006]

More Semantics

Other types of semantics:

- **Ranking-based** semantics
  - A **pre-order** on arguments is defined, instead of sets of collectively acceptable arguments.
  - The pre-order compares the acceptability of arguments.
  - The comparison may be based on the number of attacking and defending arguments, for example.

$\Rightarrow$ See [Bonzon, Delobelle, Konieczny, Maudet 2016] for a comparative study of ranking-based semantics

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Outline

Generalization of Dung's Framework

There are many ways to **extend** the expressiveness of abstract argumentation, *e.g.*:

- Preferences
- Uncertainty in the graph
- Support relation
- Abstract dialectical frameworks

Preference-based Argumentation

- besides conflicts between arguments, the agent has some **preferences** between arguments [Amgoud and Cayrol 2002]
- if a_1 is preferred to a_2, then no attack from a_2 to a_1 can succeed

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$PrF = \langle A, R, \leq \rangle$ with

- $A = \{a_1, a_2\}$
- $R = \{(a_1, a_2), (a_2, a_1)\}$
- $a_2 \leq a_1$ (*i.e.* a_1 is "better than" a_2)

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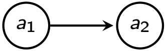

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  - Other definitions exist for the defeat relation [Amgoud and Vesic 2014, Kaci *et al* 2018]

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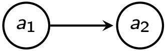

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Uncertainty in Abstract Argumentation

- Uncertainty is omnipresent in reel world, and should be taken into account in AI
- Two possibilities:
  - Qualitative uncertainty: "I'm not sure whether this is true or false"
  - Quantitative uncertainty: "I believe that this is true at some degree"
    $\rightarrow$ probabilities, possibilities,...

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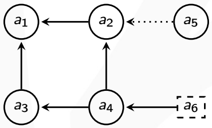
  - Qualitative uncertainty: "I'm not sure whether this is true or false"
  - Quantitative uncertainty: "I believe that this is true at some degree"
    $\rightarrow$ probabilities, possibilities,...
- Both qualitative and quantitative approaches are considered in argumentation:
  - Probabilistic argumentation
    [Li *et al* 2011, Thimm 2012, Hunter 2014, Gaignier *et al* 2021]
  - Partial/Incomplete argumentation frameworks
    [Coste-Marquis *et al* 2007, Baumeister *et al* 2018, Dimopoulos *et al* 2018]

Incomplete AFs [Baumeister *et al* 2018]



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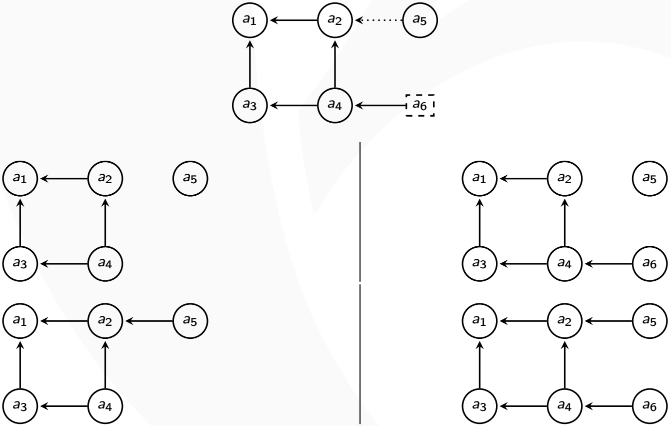

Bipolar Argumentation

- besides conflicts between arguments, there are **supports** between arguments
- pure support graphs also exist (*e.g.* recommendation letter, job application,. . .)

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$$a_1 \longrightarrow a_2 \dashrightarrow a_3$$

- Concepts like conflict-freeness and admissibility are generalized for this setting, then semantics can also be defined (see [Amgoud *et al* 2008] for technical details)

Abstract Dialectical Frameworks

- Proposed by [Brewka *et al* 2013]
- Abstract entities are called **statements**
- Statements can be linked together
- Each statement s_i is associated with a propositional formula built on other statements s_j s.t. there is a **link** from s_j to s_i
- s_i is accepted if its formula is true
- acceptance formulas can express (collective) attacks, (collective) supports or any complex relation

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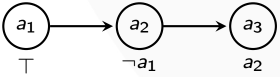

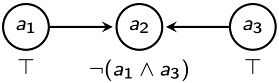

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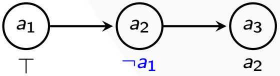

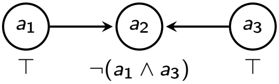

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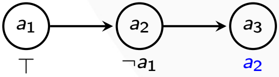

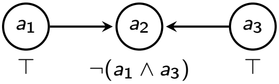

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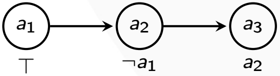
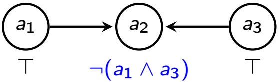
- acceptance formulas can express (collective) attacks, (collective) supports or any complex relation





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