In Shuffling Cards, 7 Is Winning Number


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By Gina Kolata

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IT takes just seven ordinary, imperfect shuffles to mix a deck of cards thoroughly, researchers have found. Fewer are not enough and more do not significantly improve the mixing.

The mathematical proof, discovered after studies of results from elaborate computer calculations and careful observation of card games, confirms the intuition of many gamblers, bridge enthusiasts and casual players that most shuffling is inadequate.

The finding has implications for everyone who plays cards and everyone, from casino operators to magicians, who has a stake in knowing whether a shuffle is random.

The mathematical problem was complicated because of the immense number of possible ways the cards in a deck can be arranged; any of 52 could be first in the deck, any of 51 could be second, 50 could be third and so on. Multiplied out, the number of possible permutations, 52 factorial, or 52;51;50, etc. is 1063 or 10 with 62 zeros after it.

No one expected that the shuffling problem would have a simple answer, said Dr. Dave Bayer, a mathematician and computer scientist at Columbia who is a co-author of the recent discovery. Other problems in statistics, like analyzing speech patterns to identify speakers, might be amenable to similar approaches, he said.

The new result "definitely solves the problem," said Dr. David Aldous, a statistician at the University of California at Berkeley. "All their calculations are right. It's a fascinating result." Dr. Persi Diaconis, a mathematician and statistician at Harvard University who is the other author of the discovery, said the methods used are already helping mathematicians analyze problems in abstract mathematics that have nothing to do with shuffling or with any known real-world phenomena.

Dr. Diaconis, who is also a magician, has invented numerous card tricks and has been carefully watching casino dealers and casual card players shuffle for the past 20 years. The usual shuffling produces a card order that "is far from random," Dr. Diaconis said. "Most people shuffle cards three

or four times. Five times is considered excessive."

The realization that most shuffled decks are not actually random allows gamblers to improve their odds of winning. "There are people who go to casinos and make money on this," Dr. Diaconis said. "I know people who are out there doing that now."

How Casinos Do It

In Las Vegas, cards are shuffled from four to seven times, at the discretion of the casino owners, said Richard Ingram, a Las Vegas enforcement agent for the state gambling control board. Dr. Diaconis said he almost never sees a dealer shuffle seven times. He said his research also shows that when dealers shuffle several decks at once, they need to shuffle more. Two decks should be shuffled nine times, he said, and six decks should be shuffled 12 times, which is unheard of in the casinos.

At Trump Plaza in Atlantic City, blackjack dealers shuffle eight decks twice at the beginning of each game, said Howard Dreitzer, who is senior vice president of casino operations. "We've tested these shuffles and feel that they are random," he said, adding that "no one has ever complained."

Bridge players usually shuffle about four times, except in some tournaments when a computer randomly mixes the cards, said Edgar Kaplan, who is editor and publisher of Bridge World magazine. Asked whether he expected bridge players to change their shuffling habits, Mr. Kaplan replied, "There will be a few who will be affected and will doggedly shuffle seven times to the irritation of everyone else." As for himself, Mr. Kaplan said, "I probably will move up from four to five" shuffles, a decision which, the research shows, will not appreciably improve the randomness of the shuffled cards.

Dr. Diaconis has found that many bridge players take advantage of the non-randomness of seemingly shuffled cards. He said a bridge club in New York State once consulted him, as a magician, to find out whether several players were cheating. After watching play "and doing a little thinking in between," Dr. Diaconis knew what was going on. These players had figured out that the cards were not being randomly shuffled, and that they could predict the distributions of cards by knowing what the deck looked like at the end of the previous hand.

A Punishment of Sorts

The players "admitted to it readily," Dr. Diaconis said. "But they didn't think they were doing anything wrong. After all, they were just thinking." The club asked those players not to play together for a year.

When computers were introduced into tournament bridge about 18 years ago, some players were puzzled and others outraged by the random hands the computer dealt and complained that the computers were not working right.

At about the same time, a bridge encyclopedia was published. The encyclopedia "used a computer to figure out odds," Dr. Diaconis said. "For example, given that between my opponents there are seven hearts, what's the chances that one has four hearts and the other has three? Some of these odds were at variance with expert play. The experts had intuited - correctly - the actual ways the cards were shuffled. People thought the encyclopedia was wrong."

By saying that the deck is completely mixed after seven shuffles, Dr. Diaconis and Dr. Bayer mean that every arrangement of the 52 cards is equally likely or that any card is as likely to be in one place as in another.

The cards do get more and more randomly mixed if a person keeps on shuffling more than seven times, but seven shuffles is a transition point, the first time that randomness is close. Additional shuffles do not appreciably alter things.

Grist for Magicians

Magicicans have long taken advantage of the nonrandomnesss of most card shuffling, Dr. Diaconis said. In fact, he said, Charles T. Jordan, a magician, chicken farmer and professional contest entrant from Petaluma, Calif., made a fair amount of money around the turn of the century by selling a card trick exploiting the fact Dr. Diaconis said he first began to think about the shuffling problem 20 years ago after a visit to A.T.&T Bell Laboratories in Murray Hill, N.J. Mathematicians there told him about the problem but said they had given up trying to solve it in 1955 because there were so many ways to arrange a deck.

Dr. Diaconis began working with Dr. Jim Reeds at Bell Laboratories and showed that a deck is perfectly mixed if it is shuffled between 5 and 20 times.

Next, Dr. Diaconis worked with Dr. Aldous and showed that it takes 5 to 12 shuffles to perfectly mix a deck. But, said Dr. Diaconis, "nobody in practice shuffles 12 times," adding, "We needed some new ideas."

In the meantime, he also worked on "perfect shuffles," those that exactly interlace the cards. Almost no one except a magician can do perfect shuffles every time. But Dr. Diaconis showed several years ago that if a person actually does perfect shuffles, the cards would never be thoroughly mixed. He derived a mathematical proof showing that if a deck is perfectly shuffled eight times, the cards will be in the same order as they were before the shuffling.

To find out how many ordinary shuffles were necessary to mix a deck, Dr. Diaconis and Dr. Bayer watched players shuffle. He also watched Las Vegas dealers to see how perfectly they would interlace the cards they shuffled.

Observations During Poker

Dr. Bayer said he seized every opportunity to get data. "I asked everyone in my poker game, once they dropped out of a hand, to shuffle for me," he said.

Then the researchers did extensive simulations of shuffling on a computer. To get the proof, the researchers looked at a lot of shuffles, guessed that the answer is seven, and finally proved it by finding an abstract way to describe what happens when cards are shuffled.

"When you take an honest description of something realistic and try to write it out in mathematics, usually it's a mess," Dr. Diaconis said. "We were lucky that the formula fit the real problem. That is just miraculous, somehow."

Correction: Jan. 17, 1990

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An article in Science Times on Jan. 9 about card shuffling misstated the value of 52 factorial, or 52 X 51 X 50, etc. It is approximately 0.8 X 10 to the 68th power.

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