Numerical Analysis HW5
Ch6 - 1,3,9 (pg178)
Ch7 - 1,3,5 (pg199)

Neal D. Nesbitt

March 2, 2016

Problem 1. Employ fixed-point iteration to locate the root of

$$f(x) = \sin\left(\sqrt{x}\right) - x$$

Use an initial guess of $x_0 = 0.5$ and iterate until $\epsilon_a \leq 0.01\%$.
Verify that the process is linearly convergent as described at the end of Sec 6.1.

If we can arrange a function such that $f(x) =$, simple fixed point iteration is taken by letting $x_{i+1} = f(x_i) + x_i$. Since f given above is already in this form, we can step through this method where

$$x_{i+1} = f(x_i) + x_i = \sin\left(\sqrt{x_i}\right)$$

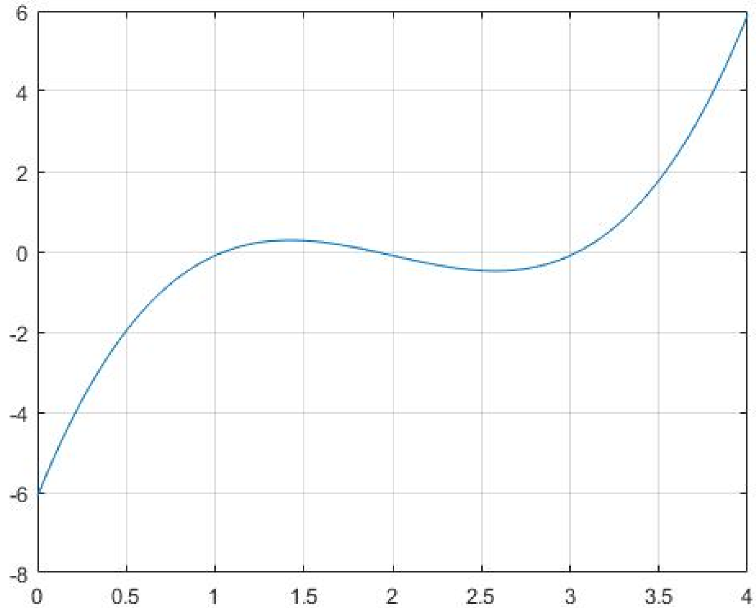
i	x_i	$\lvert\epsilon_a\rvert_i$	$\lvert\epsilon_a\rvert_i$	$\lvert\epsilon_t\rvert_i \,/\, \lvert\epsilon_t\rvert_{i-1}$
0	0.5000		34.95%	0.4429
1	0.6496	23.03%	15.48%	0.3960
2	0.7215	9.97%	6.13%	0.3752
3	0.7509	3.92%	2.30%	0.3696
4	0.7621	1.47%	0.85%	0.3696
5	0.7678	0.21%	0.10%	0.1176
6	0.7683	0.07%	0.04%	0.4000
7	0.7685	0.03%	0.01%	0.2500
8	0.7686	0.01%	0.00%	0.0000

Problem 3. Determine the highest real root of

$$f(x) = x^3 - 6x^2 + 11x - 6.1$$

- Graphically
- Using the Newton-Raphson method (three iterations, $x_0 = 3.5$)
- Using the secant method (three iterations, $x_{i-1} = 2.5$ and $x_i = 3.5$)
- Using the modified secant method (three iterations, $x_i = 3.5$, $\delta = 0.01$)
- Determine all the roots with MATLAB

Graphically



The graph suggests roots just above $x = 1$, just below $x = 2$, and just above $x = 3$, making the root at $x \approx 3$ the largest.

Newton-Raphson

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

i	x_i	$\lvert\epsilon_a\rvert_i$	$f(x_i)$	$f'(x_i)$
0	3.5000		1.7750	5.7500
1	3.1913	9.6730%	0.3994	3.2576
2	3.0687	3.9954%	0.0519	2.4264
3	3.0473	0.7017%	0.0015	2.2906

Secant

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i))}$$

i	x_i	$\lvert\epsilon_a\rvert_i$	$f(x_i)$
-1	2.5000		-0.4750
0	3.5000	28.57%	1.7750
1	2.7111	29.0984%	-0.4515
2	2.8711	5.5721%	-0.3101
3	3.2219	10.8889%	0.5024

Modified Secant ($\delta = 0.01$)

$$x_{i+1} = x_i - \frac{\delta x_i f(x_i)}{f(x_i + \delta) - f(x_i))}$$

i	x_i	$\lvert\epsilon_a\rvert_i$	$f(x_i)$	$f(x_i + \delta x_i)$
0	3.5000		1.7750	1.9818
1	3.1996	9.3888%	0.4267	0.5365
2	3.0753	4.0410%	0.0681	0.1471
3	3.0488	0.8694%	0.0049	0.0780

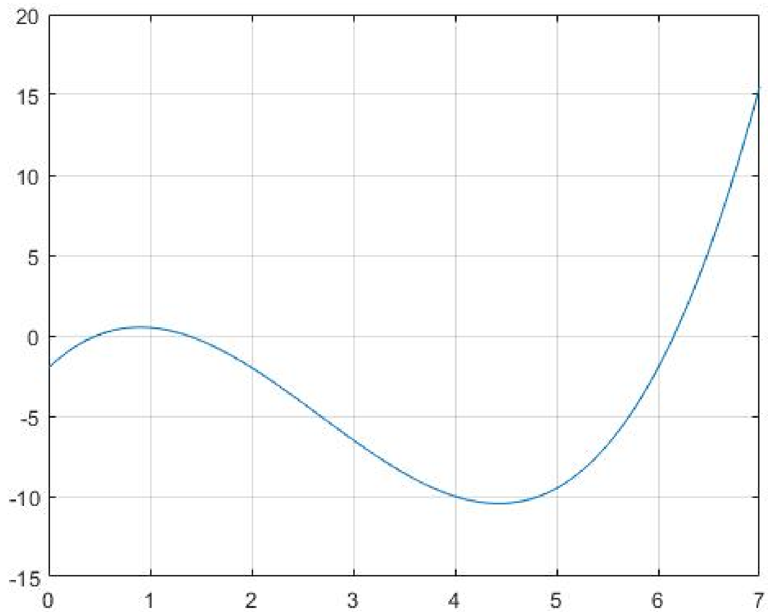
MATLAB

MATLAB's `roots()` command tells us the 3 real roots are at $x = 1.0544, 1.8990, 3.0467$.

Problem 9. Employ the Newton-Raphson method to determine a real root for

$$f(x) = -2 + 6x - 4x^2 + 0.5x^3$$

using an initial guess of 4.5 and 4.43. Discuss and use graphical and analytical methods to explain any peculiarities in your results.



Notice that we are very close to a local minimum for $x = 4.5, 4.43$. Then since the Newton-Raphson method uses the x intersection of the tangent line to pick its next point this throws us either wildly to the left or right depending on how close to this minimum we are. The closer we get, the farther it throws us away.

$$f'(x_0) = 0 \text{ and } f''(x_0) > 0 \implies \lim_{(x\to x_0)^-} \frac{1}{f'(x)} = -\infty \text{ and } \lim_{(x\to x_0)^+} \frac{1}{f'(x)} = +\infty$$

So if we look at the first few terms of the method to see this in action:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

i	x_i	$\lvert\epsilon_a\rvert_i$	$f(x_i)$	$f'(x_i)$
0	4.5		-10.4375	0.3750
1	32	86%	12912	1315
2	22.5	43.6%	3814.1	586.5
3	16	40.6%	1121.9	262.6

The first few terms of starting at $x = 4.43$ have a order in the range of 10^{10} and don't even show up on the readout.

All together, the $x = 4.5$ gives the greatest root at a value of $x_r = 6.1563$ after 11 iterations, and the starting point at $x = 4.43$ shoots to the left and ends up giving back the least root of $x_r = 0.4746$ after 25 iterations.

Problem 1. Perform three iterations of the Newton-Raphson method to determine the root of Eq. (E7.1.1):

$$\frac{dz}{dt} = v_0 e^{-(c/m)t} - \frac{mg}{c}\left(1 - e^{-(c/m)t}\right)$$

Use the parameter values from Example 7.1 ($g = 9.81$ m/s^2, $z_0 = 100$ m, $v_0 = 55$ m/s, $m = 80$ kg, and $c = 15$ kg/s) along with an initial guess of $t = 3$ s.

i	t_i	$\lvert\epsilon_a\rvert_i$	$f(t_i)$	$f'(t_i)$
0	3		8.8291	-11.4655
1	3.7701	20.4257%	0.6078	-9.9240
2	3.8313	1.5986%	0.0035	-9.8107
3	3.8317	0.0092%	0.0000	-9.8100

Problem 3. Consider the following function:

$$f(x) = 3 + 6x + 5x^2 + 3x^3 + 4x^4$$

Locate the minimum by finding the root of the derivative of this function. Use bisection with initial guesses of $x_l = -2$ and $x_u = 1$.

Begin by taking the derivative.

$$f'(x) = 6 + 5x + 9x^2 + 16x^3$$

Then using the bisect algorithm in MATLAB with the parameters given, we find the minimum to be at $x = -0.7791$ after 15 iterations with an absolute error of ± 0.0118.

Problem 5. Solve for the value of x that maximizes $f(x)$ in Prob. 7.4

$$f(x) = -1.5x^6 - 2x^4 + 12x$$

using the golden-section search. Employ initial guesses of $x_l = 0$ and $x_u = 2$ and perform three iterations.

i	x_l	x_u	x_1	x_2	$f(x_1)$	$f(x_2)$	$\lvert\epsilon_a\rvert$
0	0	2	1.2361	0.7639	4.8142	8.1879	61.8034%
1	0	1.2361	0.7639	0.4721	8.1879	5.5496	38.1966%
2	0.4721	1.2361	0.9443	0.7639	8.6778	8.1879	19.0983%
3	0.7639	1.2361	1.0557	0.9443	8.1074	8.6778	11.8034%