Complex Analysis HW1

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Problem 1. Find the principal argument, $\arg(z)$, when

(a) $z = \frac{i}{-2-2i}$

(b) $z = (\sqrt{3} - i)^6$

Solution 1. (a) Since

$$z = \frac{i}{-2-2i} = \frac{i(-2+2i)}{4+4} = \frac{-(1+i)}{4}$$

Then the point has principal argument $\arg(z) = -\pi/3$

Problem 2. Use de Moivere's formula to derive the following trigonometric identities:

(a) $\cos(3\theta) = \cos^3(\theta) - 3\cos(\theta)\sin(\theta)$

(b) $\sin(3\theta) = 3\cos^2(\theta)\sin(\theta) - \sin^3(\theta)$

Problem 3. Find the square roots of

(a) $2i$

(b) $1 - \sqrt{3}i$

and express them in rectangular coordinates.

Problem 4. Find the four zeros of the polynomial $z^4 + 4$, one of them being $z_0 = \sqrt{2}e^{i\pi/4} = 1 + i$. Then use those zeros to factor $z^4 + 4$ into quadratic factors with real coefficients.

Problem 5. Show that if c is any nth root of unit other than unity itself, then

$$1 + c + c^2 + \cdots + c^{n-1} = 0$$

Problem 6. Sketch the following sets and determine which are domains:

(a) $\mathrm{Im}(z) > 1$

(b) $\mathrm{Im}(z) = 1$

(c) $0 \leq \arg(z) \leq \pi/4$, $(z \neq 0)$

(d) $|z-4| \geq |z|$

Problem 7. In each case, sketch the closure of the set:

(a) $|\mathrm{Re}(z)| < |z|$

(b) $\mathrm{Re}(1/z) \leq 1/2$

(c) $\mathrm{Re}(z^2) > 0$

Problem 8. Prove that a finite set of points $z_1, z_2, \ldots, z_n$ cannot have any accumulation points.