Recurrence in the Left Shift

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1 Notation

Consider some arbitrary system $\mathcal{A}^{\mathbb{Z}}$ with a dynamic $T : \mathcal{A} \to \mathcal{A}$ that causes $\mathcal{A}$ to take on a finite number of states $\mathcal{A} = a_0, \cdots, a_n$. We call these states our alphabet, and say the dynamic represents "motion" of the system from one state to another under this dynamic: $T(a^{\{k\}}) = a^{\{k+1\}}$.

Definition 1.1. Alphabet of a system $\mathcal{A}^{\mathbb{Z}}$

Let $\mathcal{A}^{\mathbb{Z}}$ have a range $\mathcal{A}$ that takes on finitely many states. We then call the collection of these states the system's alphabet.

Thus if we know the state of the system at some point $k \in \mathbb{Z}, a^{\{k\}} \in \mathcal{A}$, we can mark prior and subsequent applications of the dynamic in a sequence $a \in \mathcal{A}^{\mathbb{Z}}$

$$a = \{\cdots, a^{k-1}, a^k, a^{k+1}, \cdots\}$$

I call this the state sequence of a at k.

Definition 1.2. State Sequence of an element in a dynamic system at some point
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