Complex Analysis HW3

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Problem 1. Verify the mapping of the region and boundary shown in Fig. 7 of Appendix 2, where the transformation is $w = e^z$.

Problem 2. Find the image of the semi-infinite strip $x \geq 0, 0 \leq y \leq \pi$ under the transformation $w = e^z$, and label corresponding portions of the boundaries.

Problem 3. Use definition (2), Sec. 15, to prove that

a) $\lim_{z \to z_0} \mathrm{Re}(z) = \mathrm{Re}(z_0)$ b) $\lim_{z \to z_0} \bar{z} = \bar{z_0}$ c) $\lim_{z \to z_0} (\bar{z}^2/z) = 0$

Problem 4. Let a, b, c denote complex constants. Then use definition (2), Sec. 14, to show that

a) $\lim_{z \to z_0} (az + b) = az_0 + b$ b) $\lim_{z \to z_0} (z^2 + c) = z_0^2 + c$

Problem 5. Show that the limit of the function

$$f(z) = \left(\frac{z}{\bar{z}}\right)^2$$

as z tends to 0 does not exist.

Problem 6. Show that

$$\lim_{z \to z_0} f(z)g(z) = 0 \text{ if } \lim_{z \to z_0} f(z) = 0$$

and if there exists a positive number M such that $|g(z) \leq M$ for all z in some neighborhood of z_0

Problem 7. Use the theorem in Sec. 17 to show that

a) b) c)

$$\lim_{z \to z_0} \frac{4z^2}{(z-1)^2} = 4 \qquad \lim_{z \to z_0} \frac{1}{(z-1)^3} = \infty \qquad \lim_{z \to z_0} \frac{z^2+1}{(z-1)^3} = \infty$$