# Numerical Analysis HW10
# Ch21 - 1,2,4 (pg540)
# Ch22 - 1,8,9 (pg583)

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## 21

**Problem 21.1.** Compute forward and backward difference approximations of $O(h)$ and $O(h^2)$, and central difference approximations of $O(h^2)$ and $O(h^4)$ for the first derivative of $\cos x$ at $x = \pi/4$ using a value of $h = \pi/12$. Estimate the true percent relative error $\varepsilon_s$ for each approximation.

**Solution 21.1.a.** Forward difference $O(h)$: $\boxed{-0.7911}$, $|\varepsilon_r| = \boxed{211\%}$

**Solution 21.1.b.** Forward difference $O(h^2)$: $\boxed{-0.7260}$ $|\varepsilon_r| = \boxed{203\%}$

**Solution 21.1.c.** Backward difference $O(h)$: $\boxed{-0.6070}$, $|\varepsilon_r| = \boxed{186\%}$

**Solution 21.1.d.** Backward difference $O(h^2)$: $\boxed{-0.7197}$, $|\varepsilon_r| = \boxed{202\%}$

**Solution 21.1.e.** Central difference $O(h^2)$:$\boxed{-0.6991}$, $|\varepsilon_r| = \boxed{198\%}$

**Solution 21.1.f.** Central difference $O(h^4)$:$\boxed{-0.7070}$, $|\varepsilon_r| = \boxed{200\%}$

**Problem 21.2.** Use centered difference approximations to estimate the first and second derivatives of $y = e^x$ at $x = 2$ for $h = 0.1$. Employ both $O(h^2)$ and $O(h^4)$ formulas for your estimates.

**Solution 21.2.a.** First Derivative $O(h^2)$: $\boxed{7.4014}$, $|\varepsilon_r| = \boxed{0.0017\%}$

**Solution 21.2.b.** First Derivative $O(h^4)$: $\boxed{7.3890}$, $|\varepsilon_r| = \boxed{0.000\%}$

**Solution 21.2.c.** Second Derivative $O(h^2)$: $\boxed{7.3952}$, $|\varepsilon_r| = \boxed{0.008\%}$

**Solution 21.2.d.** Second Derivative $O(h^4)$: $\boxed{7.3890}$, $|\varepsilon_r| = \boxed{0.000\%}$

**Problem 21.4.** Use Richardson extrapolation to estimate the first derivative of $y = \cos x$ at $x = \pi/4$ using step sizes of $h_1 = \pi/3$ and $h_2 = \pi/6$. Employ centered differences of $O(h^2)$ for the initial estimates.

# 22

**Problem 22.1.** Solve the following initial value problem over the interval from $t = 0$ to 2 where $y(0) = 1$. Display all your results on the same graph.

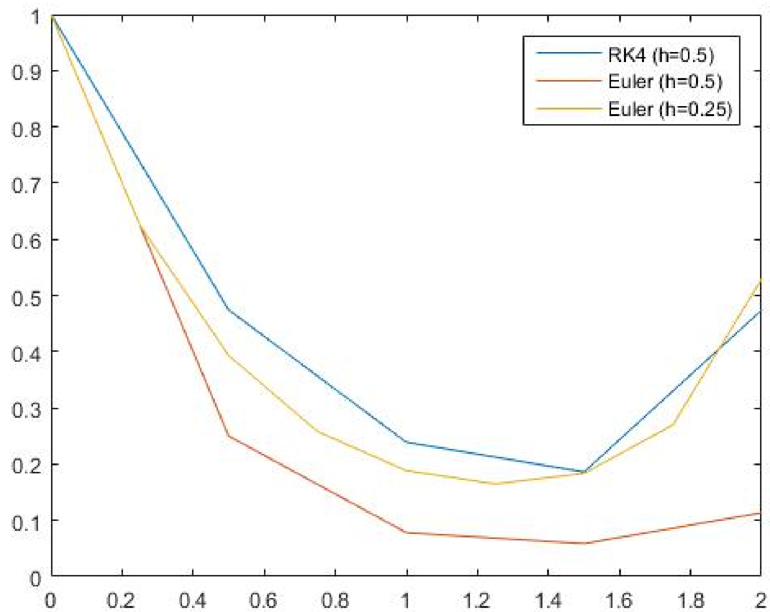
$$\frac{dy}{dt} = yt^3 - 1.5y$$

(a) Analytically

(b) Using Euler's method with $h = 0.5$ and 0.25.

(c) Using the midpoint method with $h = 0.5$.

(d) Using the fourth-order RK method with $h = 0.5$.




**Solution 22.1.a.** Analytically:

**Solution 22.1.b.** Using Euler's method with $h = 0.5$ and 0.25:

**Solution 22.1.c.** Using the midpoint method with $h = 0.5$:

**Solution 22.1.d.** Using the fourth-order RK method with $h = 0.5$:

**Problem 22.8.** The *van der Pol equation* is a model of an electric circuit that arose back in the days of vacuum tubes:

$$\frac{d^2y}{dt^2} - (1 - y^2)\frac{dy}{dt} + y = 0$$

Given initial conditions, $y(0) = y'(0) = 1$, solve this equation from $t = 0$ to 10 using Euler's method with a step size of (a)0.25 and (b) 0.125. Plot both solutions on the same graph.

HW10-ODE2.jpg

**Problem 22.9.** Given the initial conditions, $y(0) = 1$ and $y'(0) = 0$, solve the following initial-value problem from $t = 0$ to 4:

$$\frac{d^2y}{dt^2} + 4y = 0$$

Obtain your solutions with (a) Euler's method and (b) the fourth-order RK method. In both cases, use a step size of 0.1. Plot both solutions on the same graph along with the exact solution $y = \cos 2t$.

HW10-ODE3.jpg