Failure Inference for Shortening Traffic Detours

Appendix

Lemma 1. *In networks with symmetric link weights, given any detour from a link to a destination, we can always find a graceful detour that is not longer than the given detour.*

Proof. Suppose the given detour is p_1, the failed link is a-b, the destination is d. If and only if there is only one cross link on p_1, p_1 must be a graceful detour for the destination. Thus, we only need to give a method, which can convert the detour p_1 to be a detour that has only one cross link. We now give the conversion method. Suppose that there are n cross links on the detour p_1. The number n must be equal to or greater than one. If $n = 1$, the detour p_1 is a graceful detour for the destination. If $n > 1$, we use $c_1, c_2, ..., c_n$ to denote the n cross links. Suppose that the cross link c_1 is denote by x-y. We first look at the situation 1. If the failed link a-b is on the routing path from y to d, the reverse path of the routing path from y to a plus the path from y to d on p_1 form a new detour towards d. Obviously, the number of cross links on the new detour is n-1. If the link weights of the network are symmetric, the length of the reverse path of the routing path from y to a must be less than or equal to the length of the path from a to y on p_1. Therefore, if the link weights of the network are symmetric, the length of the new detour is equal to or less than the length of p_1. We use the same method to convert the new detour. We now look at the situation 2. If the failed link a-b is not on the routing path from y to d, obviously, the length of the routing path from y to d is not longer than the length of the path from y to d on the detour p_1. Thus, the path from a to x on p_1, plus the cross link c_1, and plus the routing path from y to d forms a new detour against the failed link a-b towards d. The new detour has only one cross link, so the new detour is a graceful detour. In addition, the length of the new detour is equal to or less than the length of p_1. Because p_1 is detour from the failed link a-b to d, suppose c_n is denoted by u-v, the failed link a-b must be not on the routing path from v to d. Therefore, the situation must be able to be satisfied under cross link c_n. This lemma is proven. □

Theorem 1. *In networks with symmetric link weights, given any link and any destination, the shortest graceful detours for the given link and the given destination are also shortest detours for the given link and the given destination.*

Proof. Suppose that the shortest graceful detour is p_1. If p_1 is not a shortest detour, there is another detour p_2, whose length is less than p_1. According to Lemma 1, p_2 can be converted a graceful detour p_3, whose length is less than or equal to p_2. Therefore, the length of p_3 is less than the length of p_1. Contradiction occurs. Therefore, p_1 is a shortest detour.

□

Theorem 2. *Given a shortest graceful detour from a failed link to a destination d, if a router s is on the reversely-forwarding path of the detour, the path from s to d on the detour must be a shortest graceful path from s to d without traversing the failed link.*

Proof. According to the definition of the graceful detour, the path from s to d must be a graceful path without traversing the failed link. We now prove that the graceful path is a shortest graceful path from s to d without traversing the failed link. Suppose that the path from the failed link to s is p_1, the path from s to d is p_2. If there is another graceful path p_3 from s to d without traversing the failed link and the length of p_3 is less than of the length of p_2, the path p_1 plus the path p_3 must be a graceful detour from the failed link to the destination, which is shorter than the given detour. Contradiction occurs. This theorem is proven. □

Theorem 3. *Given a router s, a destination d and a cross link x-y of d, the cross link x-y can generate exactly one graceful path from s to d if and only if in the routing tree of d the LCA of x and y is an ancestor of s and x is s or a descendant of s. The graceful path generated by the cross link x-y is the reverse path of the routing path from x to s, plus the cross link x-y and plus the routing path from y to d.*

Proof. According to the definition of the routing tree, the reverse path of the routing path from x to s is a reversely-forwarding path for d, and the routing path from y to d is a normally-forwarding path for d. According to the definition of the routing tree, if the LCA of x and y is an ancestor of s, and y is an ancestor of s and x is s or a descendant of s, one node appears at most one time on the path constructed by the reverse path of the routing path from x to s, plus the cross link x-y and plus the routing path from y to d. According to the definition of the graceful path, this theorem is proven.

□

Theorem 4. *Given a router s, a destination d and a cross link x-y that can generate a graceful path from s to d, the graceful path generated by the cross link x-y from s to d must not traverse the links on the routing path from s to LCA(x,y), where LCA(x,y) is the least common ancestor of x and y in the routing tree of d.*

Proof. If the graceful path generated by the cross link x-y from s to d traverses a link that is on the routing path from s to $LCA(x, y)$, there must exist a node that is an ancestor of x and y and that has the greater depth than $LCA(x, y)$ in the routing tree of the destination. Thus, $LCA(x, y)$ is not the LCA of x and y. Contradiction occurs. This theorem is proven. □

Theorem 5. *In any network, where Assumption 1, Assumption 2 and Assumption 3 hold, FITD-basic guarantees recovery from any single-link failure only if a detour for the failed link exists. In particular, if the link weights of the network are symmetric, FITD-basic guarantees to generate the shortest detour from any failed link to any destination.*

Proof. Suppose that the failed link is a-b and the destination is d. We first consider the network with symmetric link weights. According to Theorem 1 and Theorem 2, only if a detour from the failed link to the destination exists, FITD-basic can always find a detour from the failed link to the destination and the found deour is the shortest if the link weights of the network are symmetric. We now consider the network with asymmetric link weights. We only need to prove: if there is detour from the failed link to the destination, the detour must be able to be converted to a graceful detour. We can prove that by using the conversion method of Lemma 1. The only difference is as follows. If the link weights of the network are asymmetric, the length of the graceful detour that is got by using the conversion method of Lemma 1 is not always shorter than the original detour. This theorem is proven.

□

Theorem 6. *In any network, where Assumption 1 holds, FITD-extended guarantees recovery from any single-link failure only if a detour for the failed link exists. In particular, if the link weights of the network are symmetric, FITD-extended guarantees to generate the shortest detour from any failed link to any destination.*

Proof. Suppose that the failed link is a-b and the destination is d, if b is not the virtual primary next hop of a towards d, according to the forwarding logic of FITD-extended, the router a needs to directly forward the traffic towards d via its virtual primary next hop towards d. This is because the routing path from the virtual primary next hop to d must not traverse the failed link a-b and is the shortest path from a to d. We only need to prove this theorem in the situation, where b is the virtual primary next hop of a towards d. Under this situation, the affected traffic is forwarded to the backup next hop. Suppose that the detour generated by FITD-extended based on the virtual routing is a∼x-y-z∼d, where a∼x is a virtual reversely-forwarding path, x-y is a virtual cross link and y-z∼d is a virtual routing path based on the virtual routing. According to Theorem 5, a∼x-y-z∼d is the shortest if the link weights of the network are symmetric. We only need to prove that the affected traffic can be forwarded along the detour a∼x-y-z∼d. For FITD-extended, if a router receives a packet from its virtual primary next hop, the router can forward the packet to the correct backup next hop, so the reversely-forwarding path a x can be achieved by FITD-extended. On the router y, y receives the packet from a virtual cross link, according to the forwarding logic of FITD-extended, the router y will forwards the packet to its virtual primary next hop, which is z. From the router z, the packet can be normally forwarded. We now prove that the normally-forwarding paths from z to d must not traverse the failed link a-b. If the normally-forwarding paths from z to d traverse the failed link a-b, there must exist another virtual cross link denoted by c_2 making $corssLen[c_2]$ be less than $corssLen[x-y]$ and the virtual detour generated by c_2 must can also protect the link a-b. Contradiction occurs. This theorem is proven.

□

Proof for the implicit thorem in Subsection B in Section 3. We first prove that the number of traversed hops from the failed link to the remote router is $l - v \bmod l$. We let $v_1' = v_1 - v_1 \bmod l$. Obviously, v_1' is divisible by l. Suppose that the number of traversed hops from the failed link to the remote router is h. Obviously, $v_1' - v_2 = h$. Because l is the maximum routing-tree depth, we can get $1 \leq v_1' - v_2 = h < l$. Therefore, $l - v_2 \bmod l = l - (v_1' - h) \bmod l = l - (l - h) = h$. This theorem is proven. We now prove the reset TTL value $v_1 - v_1 \bmod l$ must be less than the original TTL value of the packet. Suppose that the original TTL value of the packet is v_0. Because the operation of reducing the TTL value by one is before the operation of resetting the TTL value, $v_1 = v_0 - 1$. Obviously, $v_1 \bmod l \geq 0$. Therefore, $v_1 - v_1 \bmod l \leq v_1 = v_0 - 1 < v_0$. This theorem is proven.

□

Proof for the implicit thorem in Subsection C in Section 3. We first prove if the color of x is 0 and the color of y is not 0, the LCA of x and y must be $Father_d^k[s]$, x must be s or a descendant of s, and the LCA of x and y must be an ancestor of s. Obviously, if the color of x is 0, x must be s or a descendant of s. $Father_d^k[s]$ is a common ancestor of x and y. If $Father_d^z[s]$ is the LCA of x and y, where $z < k$, the color of y will be z instead of k. Contradiction occurs. Therefore, the LCA of x and y must be $Father_d^k[s]$, which is an ancestor of s. We now prove if the LCA of x and y is an ancestor of s and x is s or a descendant of s, the color of x is 0 and the color of y is not 0. If x is s or a descendant of s, the color of x must be 0. Further, if the color of y is 0, the LCA of x and y must be s or a descendant of s. Contradiction occurs. This theorem is proven.

□