University of West Florida

Time Series Analysis STA 6856 | Professor Dr.Tharindu De Alwis

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1. (10 points) Consider the following time series model

$$Y_t = 0.25Y_{t-1} - 0.25Y_{t-12} + 0.0625Y_{t-13} + e_t - 0.1e_{t-1} + 0.1e_{t-12} - 0.01e_{t-13}$$

$$Y_t - 0.25Y_{t-1} + 0.25Y_{t-12} - 0.0625Y_{t-13} = e_t - 0.1e_{t-1} + 0.1e_{t-12} - 0.01e_{t-13}$$

AR Side

$$Y_t - 0.25Y_{t-1} + 0.25Y_{t-12} - 0.0625Y_{t-13} = (1 - 0.25B + 0.25B^{12} - 0.0625B^{13})Y_t$$

MA Side

$$e_t - 0.1e_{t-1} + 0.1e_{t-12} - 0.01e_{t-13} = (1 - 0.1B + 0.1B^{12} - 0.01B^{13})e_t$$

Model

$$(1 - 0.25B + 0.25B^{12} - 0.0625B^{13})Y_t = (1 - 0.1B + 0.1B^{12} - 0.01B^{13})e_t$$

(a) Recognize the model as ARIM A(p, d, q) × (P, D, Q) model. That is, what are the values for p, d, q, P, D, and Q.

$$ARIMA(1, 0, 1) \times (1, 0, 1)_{12}$$

(b) Write down all the coefficient values using the standard notation. That is, Φ's, Θ's, ϕ's, and θ's.

$\Phi = 0.25$
$\Theta = -0.25$
$\phi = -0.1$
$\theta = 0.1$

2. (10 points) An AR model has AR characteristic polynomial

$$(1 - 1.6x + 0.7x^2)(1 - 0.8x^{12})$$

(a) Is th model stationary?

$$(1 - 1.6x + 0.7x^2)(1 - 0.8x^{12}) = 0$$

$$0.7X^2 - 1.6X + 1 = 0$$

$$X = \frac{1.6 \pm \sqrt{-1.6^2 - 4(0.7)(1)}}{2(0.7)} = \frac{1.6 \pm \sqrt{2.56 - 2.8}}{1.4} = \frac{1.6 \pm \sqrt{-0.24}}{1.4} = \frac{1.6 \pm 0.49i}{1.4} \approx 1.143 \pm 0.35i$$

Modulus

$$|x| = \sqrt{1.143^2 + 0.35^2} \approx \sqrt{1.306 + 0.123} \approx \sqrt{1.43} \approx 1.20 > 1$$

$$1 - 0.8x^{12} = 0$$

$$0.8x12 = 1$$

$$x12 = \frac{1}{0.8} = 1.25$$

$$|x| = 1.25^{\frac{1}{12}} \approx 1.02 > 1$$

All roots have moudlus tha lie outside the unit circle, therefore the the model is stationary

(b) Identify the model as a certain seasonal ARIMA model

$$(1 - 1.6B + 0.7B^2)(1 - 0.8B^{12}).$$

ARIMA $(2,0.0) \times (1,0,0)\_{12}$
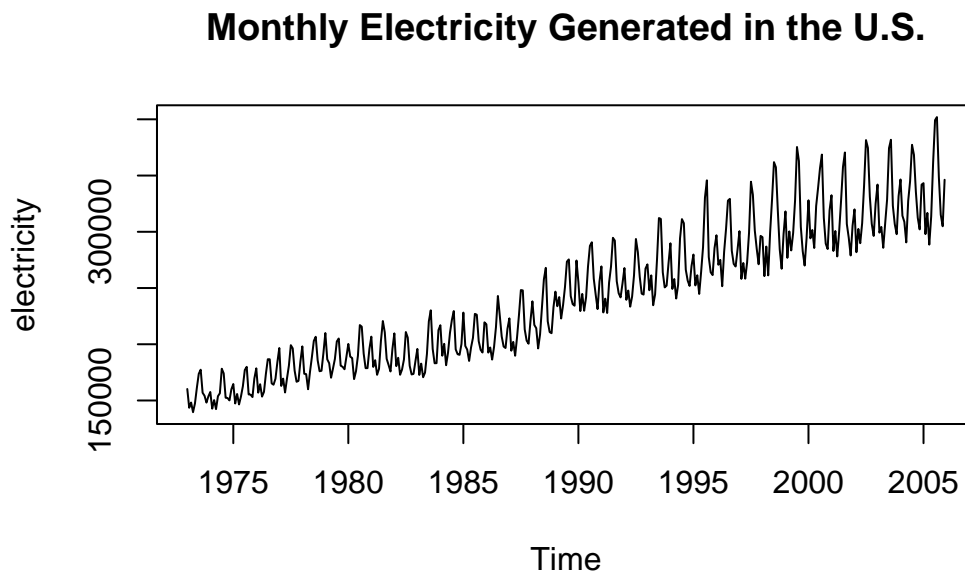
$\phi_1 = 1.6$
$\phi_2 = -0.7$
$\Phi = -0.8$

3. (20 points) Consider electricity dataset in the TSA R package that contains the monthly electricity generated in the United States.

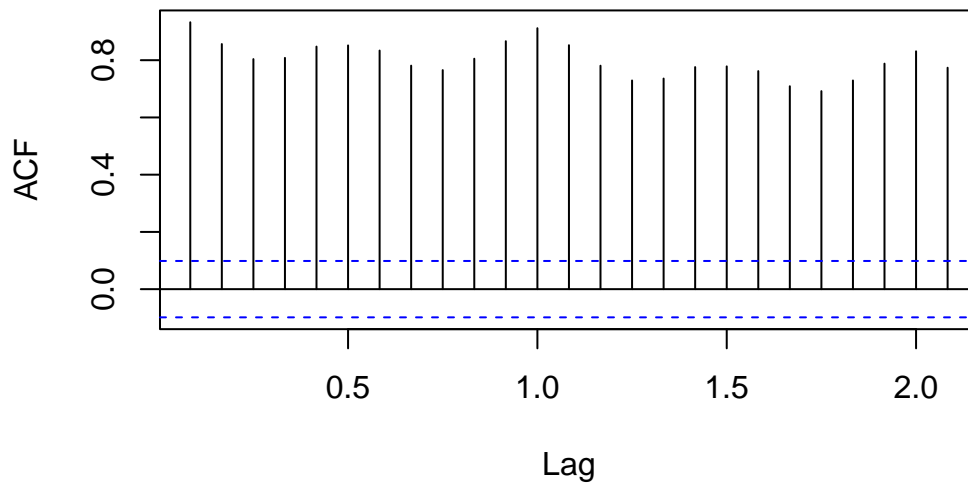(a) Construct the sample ACF of the data, is the data stationary?

```
library(TSA)
data(electricity)
```

```
plot(electricity, main = "Monthly Electricity Generated in the U.S.")
```

Monthly Electricity Generated in the U.S.


```
acf(electricity, main = "ACF of Electricity Series")
```

ACF of Electricity Series

The raw electricity series is nonstationary, the time series plot shows:

- upward trend over time

- changing variance

- strong repeating seasonal cycles

- Shows large spikes at lag 12, 24 indicating yearly seasonality

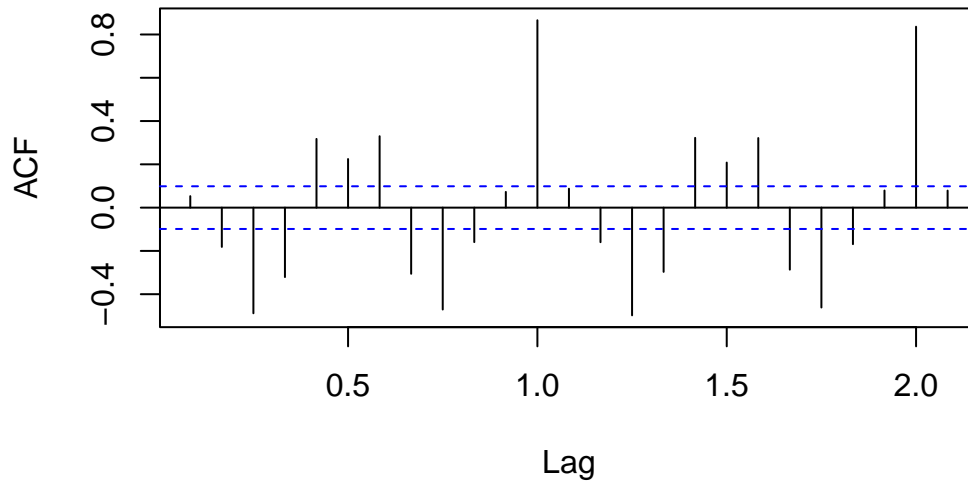- It contains trend and seasonality.

Therefore, the data is not stationary.

(b) Calculate the sample ACF of the first difference of the logged transformed series. Is the seasonality visible in this display? If so, what is your seasonal component?

```
log.elec <- log(electricity)
d.log.elec <- diff(log.elec)
```

```
acf(d.log.elec, main = "Electricity ACF of First-Differenced Log ")
```

Electricity ACF of First–Differenced Log

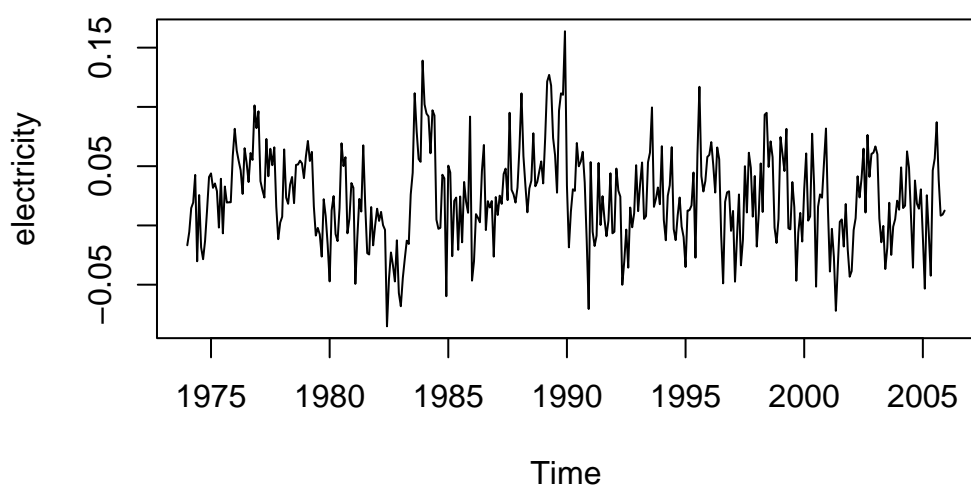The ACF now shows reduction in long-lag autocorrelation bUT still large spikes at lag 12, 24

The seasonality is still visible

Seasonal componenet is $=12$ (monthly data $\rightarrow$ yearly seasonal cycle)

(c) Plot the time series of seasonal difference, use s = 12 or the s value you have suggested in part (b), and first difference of the logged series. Does a stationary model seem appropriate now?
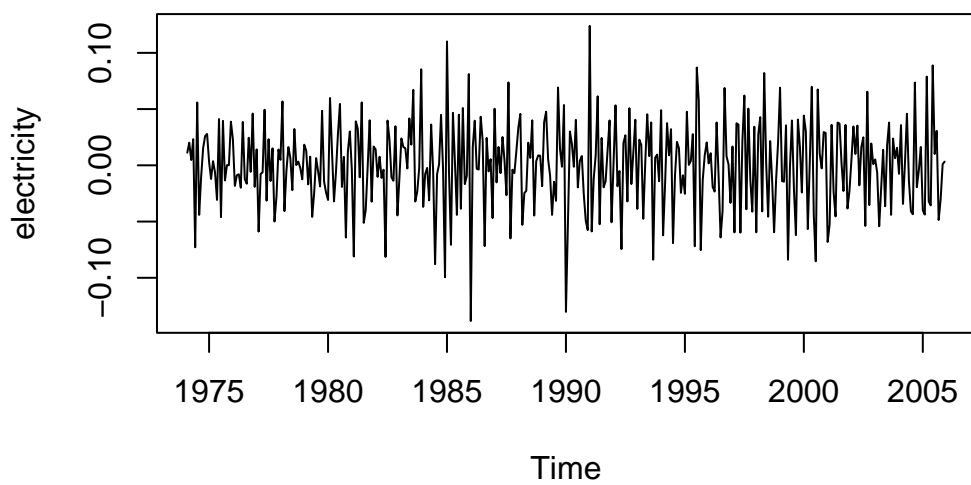
```
seas.diff <- diff(log.elec, lag = 12)
plot(seas.diff, main = "Electricity Seasonally Differenced Log ")
```

Electricity Seasonally Differenced Log


```
double.diff <- diff(seas.diff)
plot(double.diff, main = "Electricity First + Seasonal Differenced Log ")
```

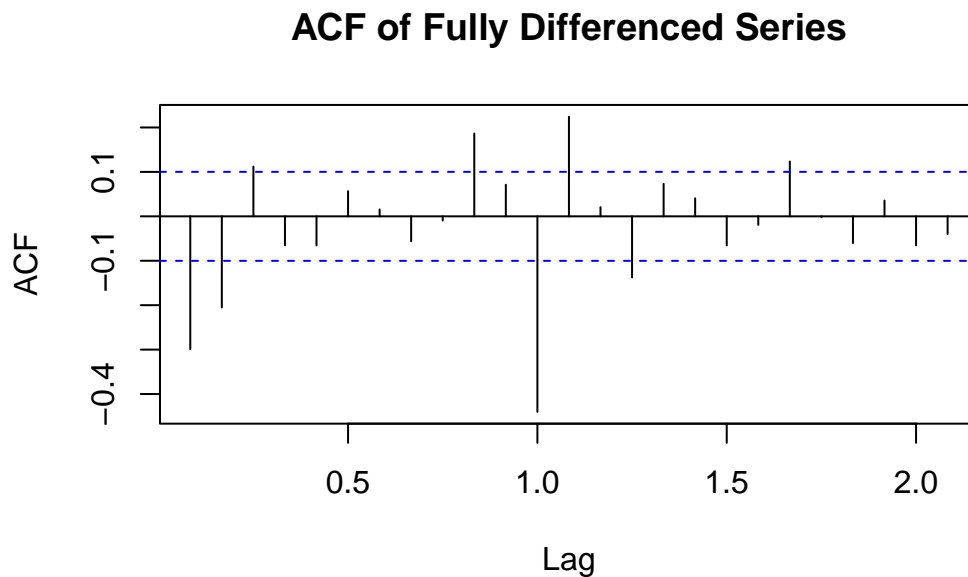## Electricity First + Seasonal Differenced Log

Yes, a stationary model seems appropriate after log transform, seasonal difference and first difference This means the needed ARIMA model is likely:
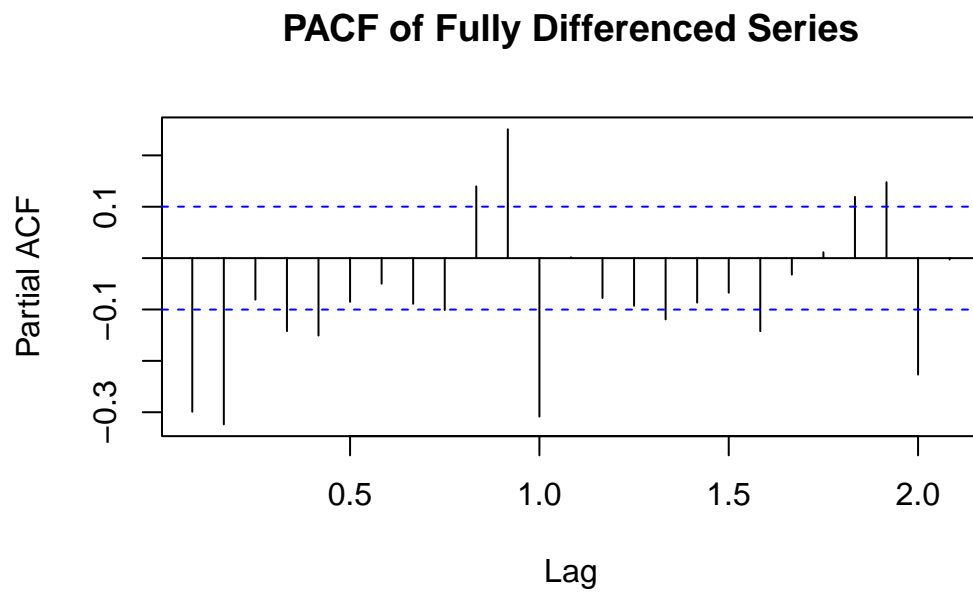
$\text{ARIMA}(p, 1, q) \times (P, 1, Q)_{12}$

(d) Display the sample ACF of the series after a seasonal difference and a first difference have been taken of the logged series in part (d). What model(s) might you consider for the electricity series?

```
acf(double.diff, main="ACF of Fully Differenced Series")
```


ACF of Fully Differenced Series

```
pacf(double.diff, main="PACF of Fully Differenced Series")
```
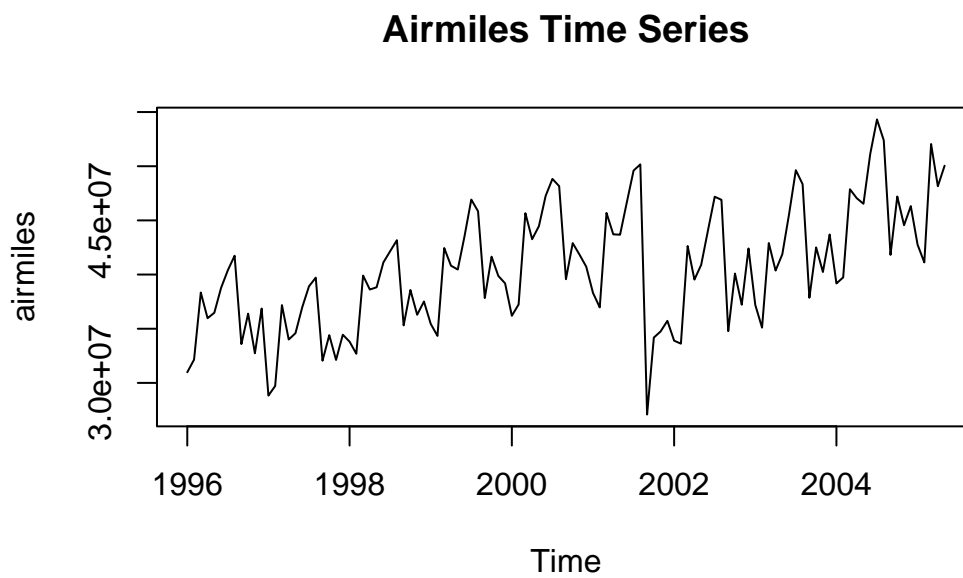
PACF of Fully Differenced Series

Possible models to consider SARIMA $(0, 1, 1) \times (0, 1, 1)_{12}$

4. (20 points) Consider the air passenger miles time series in TSA R package. The file is named air miles.
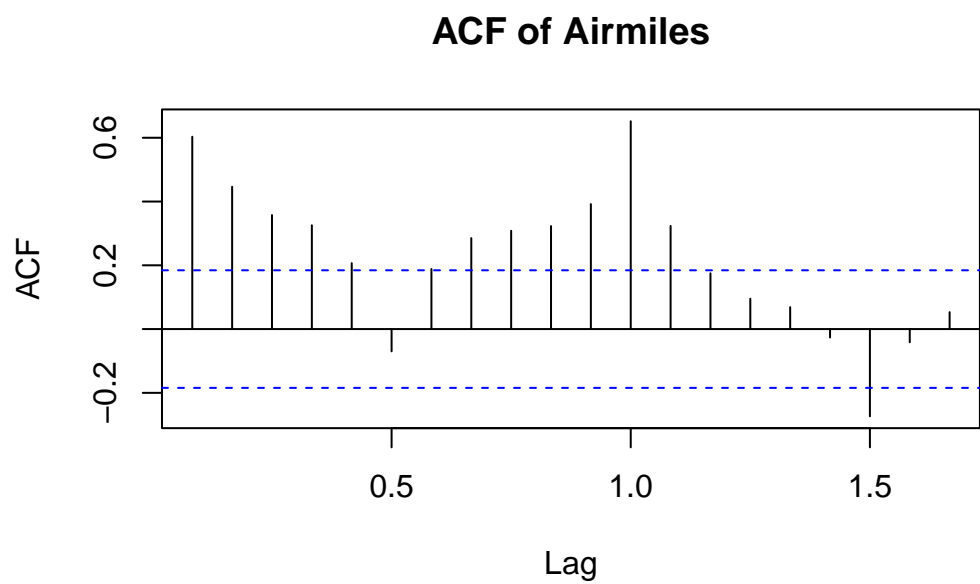
(a) Create the sample ACF and PACF for the airmiles data. Suggest the candidate values for p, d, q, P, D, Q, and s.

```r
library(TSA)

data(airmiles)
plot(airmiles, main="Airmiles Time Series")
```
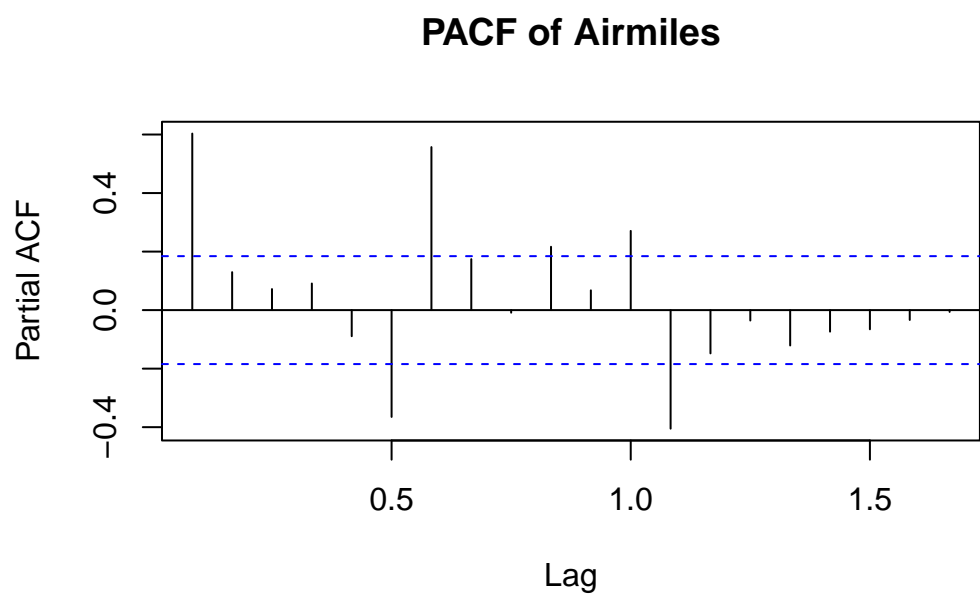

Airmiles Time Series

```r
acf(airmiles, main="ACF of Airmiles")
```

ACF of Airmiles


```
pacf(airmiles, main="PACF of Airmiles")
```

PACF of Airmiles

The airmiles series Clearly shows:

nonstationary (upward trend)

No obvious seasonality (annual data, not monthly)

So s=1 (no seasonal cycle)

ACF:

spike at lag 1, MA(1)

PACF:

spike at lag $1 \rightarrow$ AR(1)

Candidate values:

$$p = 0 \text{ or } 1, d = 1, q = 1,$$
$$P = 0, D = 0, Q = 0,$$
$$s = 1(\text{no seasonality})$$

(b) Fit ARIM A(0, 1, 1) × (0, 1, 0)12 and assess its adequacy.

```
model_b <- arima(airmiles, order = c(0,1,1),
                 seasonal = list(order=c(0,1,0), period=12))
```

```
model_b
```

```
Call:
arima(x = airmiles, order = c(0, 1, 1), seasonal = list(order = c(0, 1, 0),
    period = 12))

Coefficients:
          ma1
      -0.4104
s.e.   0.0987

sigma^2 estimated as 5.057e+12:  log likelihood = -1604.57,  aic = 3211.15

tsdiag(model_b)
```

Model fits, but the adequacy of the model is limited by since the series is annual and therefore does not contain a meaningful seasonal component at period 12. As a result, the imposed seasonal differencing does not correspond to godd structure in the data and may lead to unnecessary overdifferencing. Residuals show some patterns thus the model is not optimal.

(c) Use auto.arima{forecast} to select the parameters p, d, q, P, D, Q, and s.

```
library(forecast)

auto_model <- auto.arima(airmiles)
auto_model
```

```
Series: airmiles
ARIMA(1,0,1)(0,1,1)[12] with drift

Coefficients:
          ar1      ma1      sma1      drift
       0.9021  -0.3742  -0.6902  108191.18
s.e.   0.0575   0.1226   0.1207   37540.33

sigma^2 = 3.588e+12:  log likelihood = -1605.31
AIC=3220.62   AICc=3221.25   BIC=3233.69
```

12

Nonseasonal part ARIMA(1,0,1):

p=1: AR(1) term Coefficient: 0.9021, showing strong year-to-year persistence.

d=0: No nonseasonal differencing was applied.

q=1: MA(1) term Coefficient: –0.3742

Seasonal part (0,1,1)[12]:

P=0: no seasonal AR component

D=1: seasonal differencing

Q=1: seasonal MA(1)

Coefficient: –0.6902

s=12: season length = 12

(d) Refit the selected model from part(c) using arima{stats} function and assess its adequacy.

```
model_d <- arima(airmiles, order=c(1,0,1))
```

```
model_d
```

```
Call:
arima(x = airmiles, order = c(1, 0, 1))

Coefficients:
         ar1      ma1  intercept
      0.8048  -0.2917   40513691
s.e.  0.0910   0.1434    1421540

sigma^2 estimated as 1.862e+13:  log likelihood = -1887.01,  aic = 3780.02
```
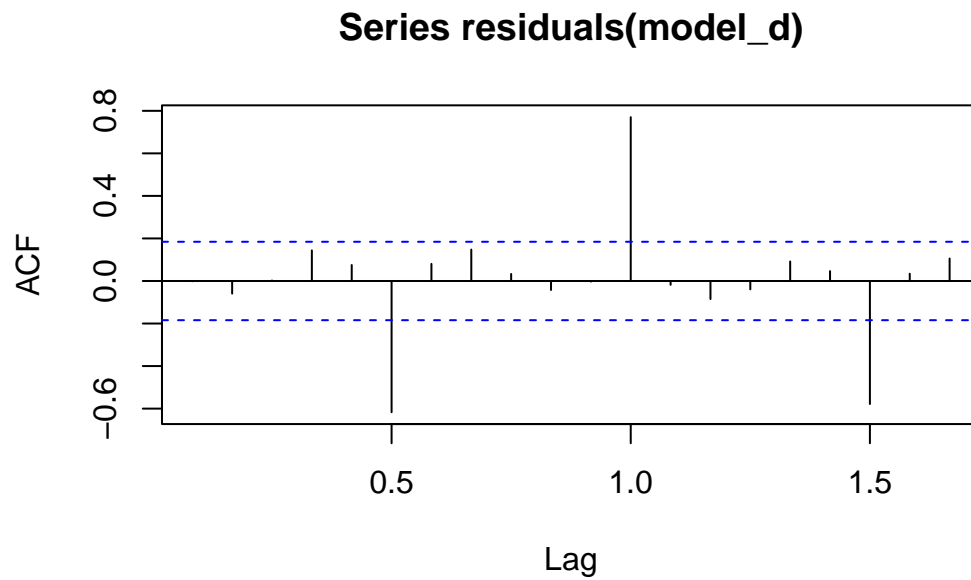
```
tsdiag(model_d)
```

```
Box.test(residuals(model_d), lag=24, type="Ljung")
```

```
acf(residuals(model_d))
```

Series residuals(model_d)

all coefficients statistically significant, residuals behave like white noise and no remaining autocorrelation in the residuals Ljung–Box p-values > 0.05

ACF/PACF of residuals show no pattern

The model is adequate and provides good fit for the airmiles data.