University of West Florida

Time Series Analysis STA 6856 | Professor Dr.Tharindu De Alwis

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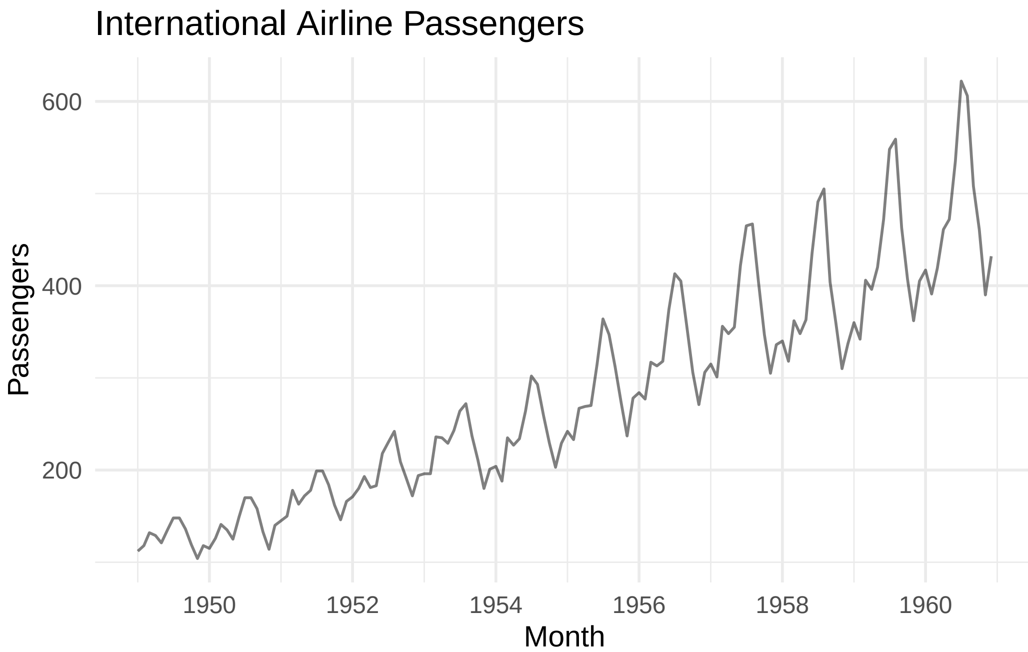
1. (10 points) dataset - Air Passenger Data: AirPassengers.csv.

(a) Import your chosen dataset into R.

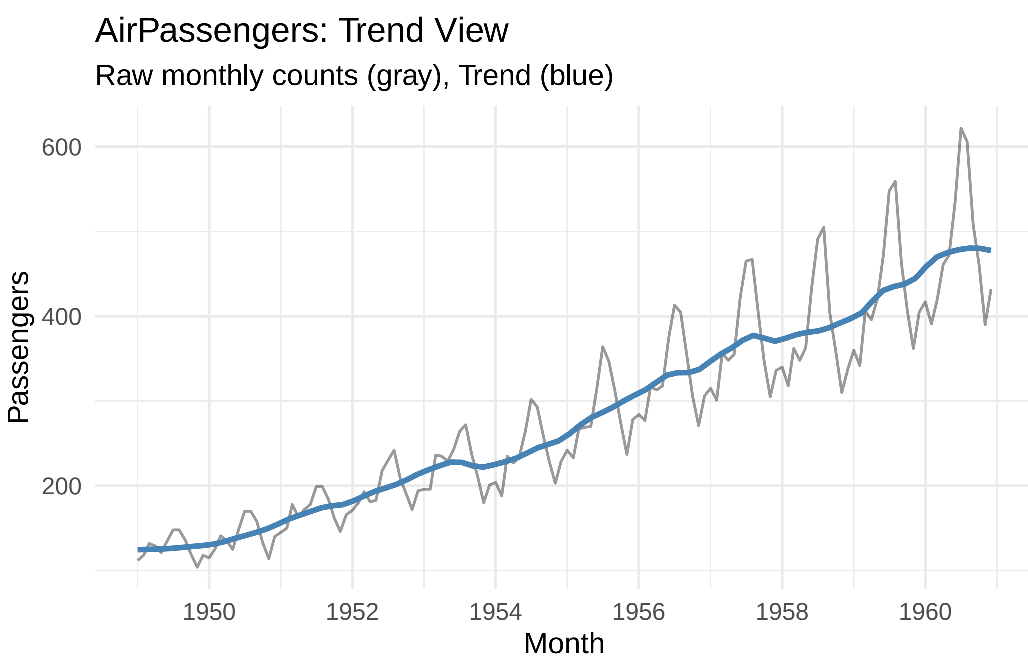
```r
dataset <- read.csv("AirPassengers.csv")

head(dataset,10)
```

```
     Month Passengers
1  1949-01        112
2  1949-02        118
3  1949-03        132
4  1949-04        129
5  1949-05        121
6  1949-06        135
7  1949-07        148
8  1949-08        148
9  1949-09        136
10 1949-10        119
```

(b) Construct the time series plot and interpret it.



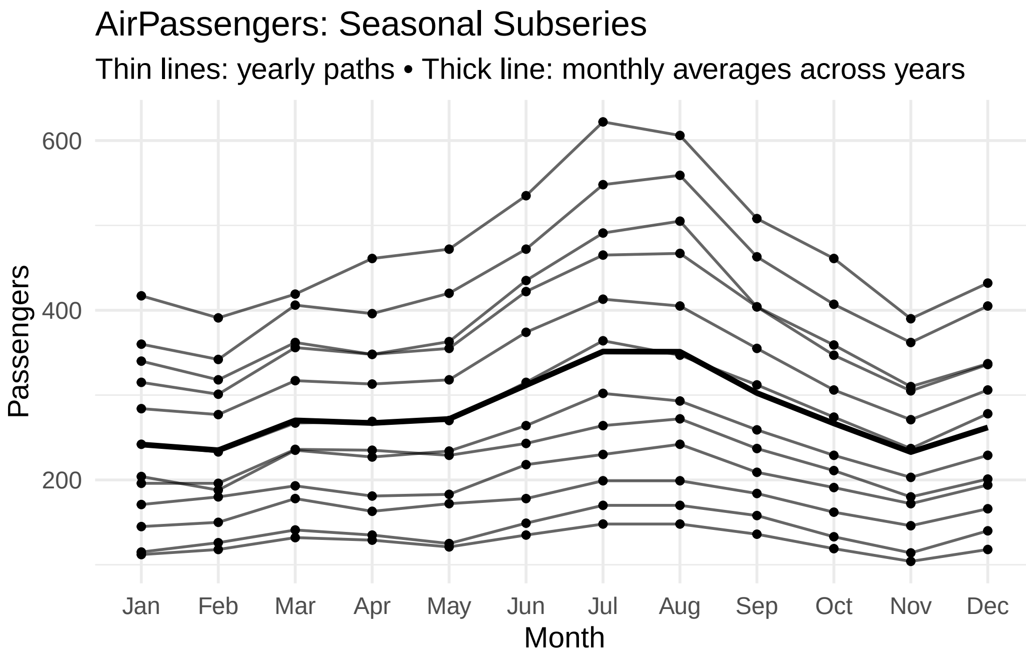
Trend



Air travel expanded rapidly through the period, with predictable, growing seasonal surges—

biggest in summer—suggesting demand was rising and becoming more volatile in absolute terms as the industry scaled.

Seasonality



The series is non-stationary: it has a strong upward trend, clear annual seasonality, and increasing variance; a log transform plus first and seasonal differencing would be needed to approximate stationarity.

2. (10 points) If $Var(X) = Var(Y)$, find $Cov(X+Y, X-Y)$

$$Cov(X+Y, X-Y) = Cov(X,X) - Cov(X,Y) - cov(Y,Y) = Var(X) - Var(Y) = 0$$

$$\text{then, } Cov(X+Y, X-Y) = 0.$$

3. (15 points) Recall: To show that a process Yt is not stationary, try to show that $E(Yt)$ depends on t. If this fails, try to show that $Var(Yt)$ depends on t. If this fails, show that $\gamma k = Cov(Yt, Yt-k)$ depends on t

(a) Let $Yt = \sum_{i=1}^{t} e_t$ where the e_t are iid with $\Xi(e_t) = \mu > 0$ and $Var(e_t) = \sigma^2$. Show that Yt is not stationary.

$$E(e_t) = \mu > 0$$

$$E[Y_t] = E(\sum_{i-1}^{t} e_t) = \sum_{i-1}^{t}(e_t) = \sum_{i-1}^{t} \mu = t\mu$$

Condition fails, $t\mu$ changes with t, therefore Y_t **is not stationary**

(b) Let $Yt = \sum_{i=1}^{t} e_t$ where the e_t are iid with $E(e_t) = 0$ and $Var(e_t) = \sigma_2$. Show that Yt is not stationary.

$$E(e_t) = \mu = 0$$
$$E[Y_t] = \sum_{i-1}^{t} E(e_t) = \sum_{i-1}^{t} 0 = 0$$

Mean is constant, does not depend on t

$$Var(Y_t) = (\sum_{i=1}^{t} e_i) = \sum_{i=1}^{t} Var(e_t) = \sum_{i-1}^{t} \sigma^2 = t\sigma^2$$

Condition fails, $Var(Y_t)$ is dependent on t, Y_t **is not stationary**

4.(10 points) Suppose Yt = X for all t where E(X) = μ and Var(X) = 2.

(a) Show that {Yt} is stationary.

$$E(Y_t) = E(X) = \mu \text{ not dependendt on t}$$
$$Var(Y_t) = Var(X) = \sigma^2 \text{ not dependend on } t$$

$$Cov(Y_t, Y_{t-k}) = Cov(X, X) = Var(X) = \sigma^2$$
$$(Y_t) \text{ is stationary}$$

(b) Find the autocorrelation function k for Y_t.

$$\gamma_k = Cov(Y_t, Y_t - k) = \sigma^2, \text{ for all } \sigma \geq 0$$
$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{\sigma^2}{\sigma^2} = 1 \text{ for all k } \geq 0$$

Every pair is correlated.

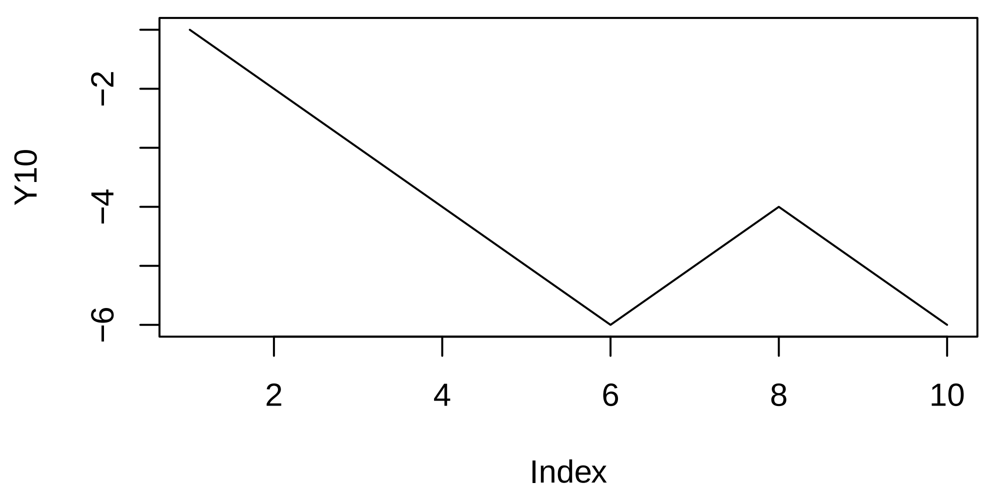
5.(15 points) The R output below displays the results of 10 coin flips, where 0 represents Tails and 1 represents Heads.

0 0 0 0 0 0 1 1 0 0

Let the et be iid where et = 1 for Heads and et = −1 for Tails (so change the 0 to a −1 from the given output. That is, e1 = e2 = e3 = e4 = e5 = e6 = −1; e7 = e8 = 1 and e9 = e10 = −1.) Let Yt = Pt t=1 et.

(a) Plot Yt on the vertical axis versus time t on the horizontal axis.

```
flips10 <- c(0,0,0,0,0,0,1,1,0,0)
e10 <- ifelse(flips10 == 1, 1, -1)
Y10 <- cumsum(e10)

plot(Y10, type = "l")
```



(b) The process {Yt} is a random walk with E(Yt) = 0, as shown in class. Does there seems to be a trend in the plot in (a) or are the Yt scattering about the horizontal axis in a roughly even band?

Drifts downward, That apparent trend is just random fluctuation; with only 10 flips, long runs of Tails can create a strong-looking drift.

(c) Repeat parts (a) and (b) for a 100 flips and provide the plot of Yt versus t.

```r
## (c) Repeat for 100 flips
set.seed(42)
flips100 <- rbinom(100, 1, 0.5)
e100 <- ifelse(flips100 == 1, 1, -1)
Y100 <- cumsum(e100)

plot(Y100, type = "l")
```