# University of West Florida

**Time Series Analysis STA 6856 | Professor Dr.Tharindu De Alwis**

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**1. (5 points) Suppose $Y_t$ follows ARIMA(3,2,4) model. What ARIMA(p,d,q) mode is followed by the second differences $Z_t = \nabla 2 Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$?**

ARIMA (3,2,4) by definition:

$$\phi(B)(1-B)^2 Y_t = \theta(B)\varepsilon_t$$

$$\text{Order 3}: \theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3$$
$$\text{Order 4}: \phi(B) = 1 + \phi_1 B + \phi_2 B^2 + \phi_3 B^3 + \phi_4 B^4$$
$$\phi(B) Z_t = \theta(B)\varepsilon_t$$

$$Z_t = \text{ARIMA}(3,0,4)$$
$$\text{therfore this is an ARMA}(3,4) \text{ model}$$

**2. (10 points) Consider the oil filter sales data in the TSA R package. The data are in the file named oilfilters.**

a. Fit an AR(1) model to this series. Is the estimate of the   parameter significantly different from zero statistically?

*Oil Filters Dataset*

```
library(TSA)
data(oilfilters)
oilfilters
```

```
      Jan  Feb  Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec
1983                               2385 3302 3958 3302 2441 3107
1984 5862 4536 4625 4492 4486 4005 3744 2546 1954 2285 1778 3222
1985 5472 5310 1965 3791 3622 3726 3370 2535 1572 2146 2249 1721
1986 5357 5811 2436 4608 2871 3349 2909 2324 1603 2148 2245 1586
1987 5332 5787 2886 5475 3843 2537
```

```
model <- arima(oilfilters, order = c(1,0,0))
(model)
```

```
Call:
arima(x = oilfilters, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.3115  3370.6744
s.e.  0.1368   253.1499

sigma^2 estimated as 1482802:  log likelihood = -409.19,  aic = 822.37
```

$H_0 : \phi = 0$ Not significantly different from zero

$H_1 : \phi \neq 0$ Significantly different from zero

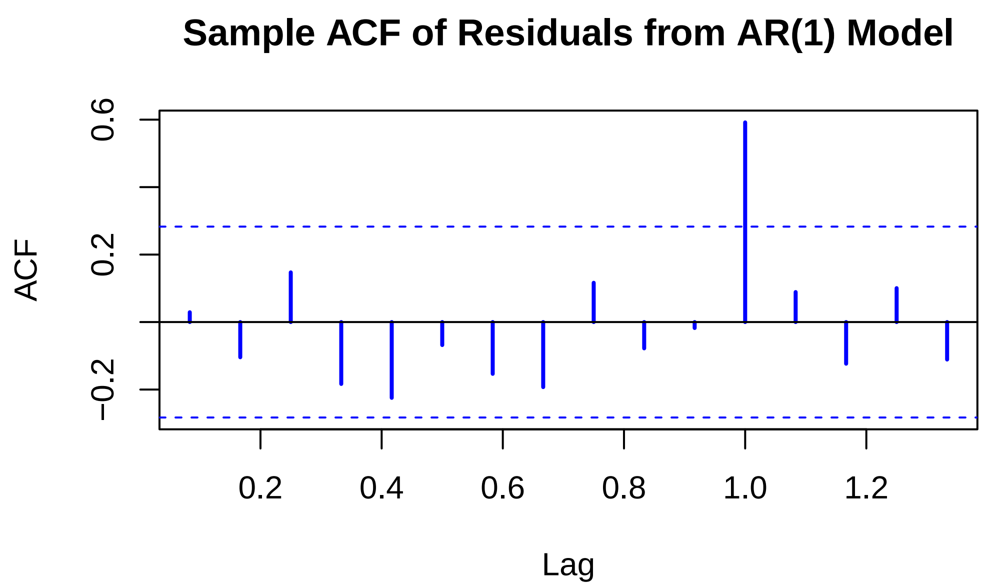
$$t = \frac{\phi}{SE\ \phi} = t = \frac{0.3115}{0.1368} = 2.28$$

Therfore, $(t = 2.28 > 1.96)$ we reject $H_0$

**AR(1) coefficient ($\phi = 0.3115$) is statistically significantly different from zero at the 5% level.**

b. Display the sample ACF of the residuals from the AR(1) fitted model. Comment on the display.

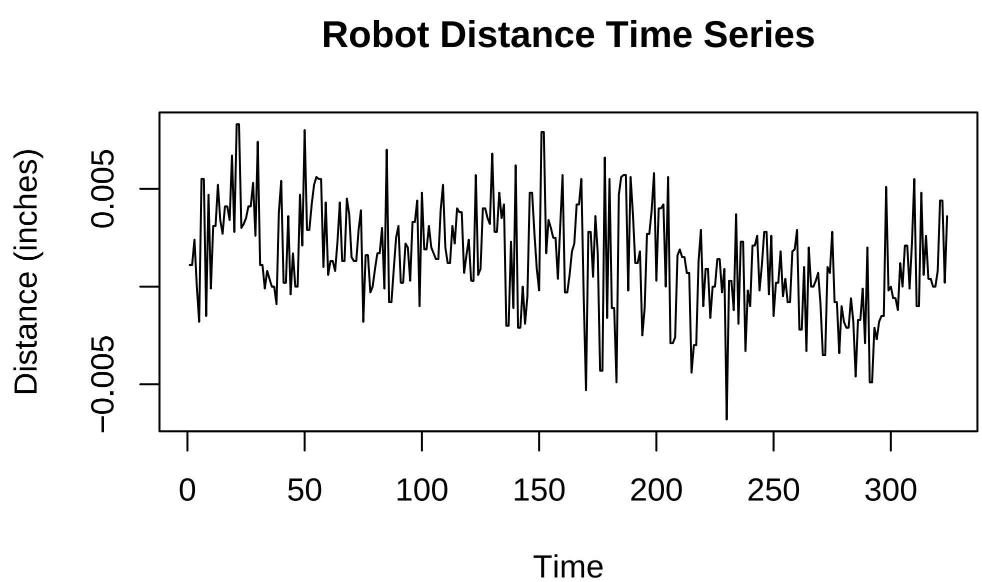
```r
res <- residuals(model)

acf(res, main = "Sample ACF of Residuals from AR(1) Model",
    col = "blue", lwd = 2, )
```




**The AR(1) coefficient $\phi = 0.3115$ is significant, and the residuals show no strong correlation, confirming the model is statistically appropriate.**

**3. (20 points) The data file named robot contains a time series obtained from an industrial robot. The robot was put through a sequence of maneuvers, and the distance from a desired ending point was recorded in inches. This was repeated 324 times to form the time series. Compare the fits of an AR(1) model and an ARIMA(0,1,1) model for these data in terms of the diagnostic tests. Note: The data set is available in the TSA R package.**

```
library(TSA)
data(robot)
plot(robot, main = "Robot Distance Time Series", ylab = "Distance (inches)")
```



**Model 1 AR(1)**

```r
model_ar1 <- arima(robot, order = c(1,0,0))
(model_ar1)
```

```

Call:
arima(x = robot, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.3074     0.0015
s.e.  0.0528     0.0002

sigma^2 estimated as 6.482e-06:  log likelihood = 1475.54,  aic = -2947.08
```

**Model 2 ARIMA(0,1,1)**

```r
model_ma1 <- arima(robot, order = c(0,1,1))
(model_ma1)
```

```

Call:
arima(x = robot, order = c(0, 1, 1))

Coefficients:
          ma1
      -0.8713
s.e.   0.0389

sigma^2 estimated as 6.069e-06:  log likelihood = 1480.95,  aic = -2959.9
```

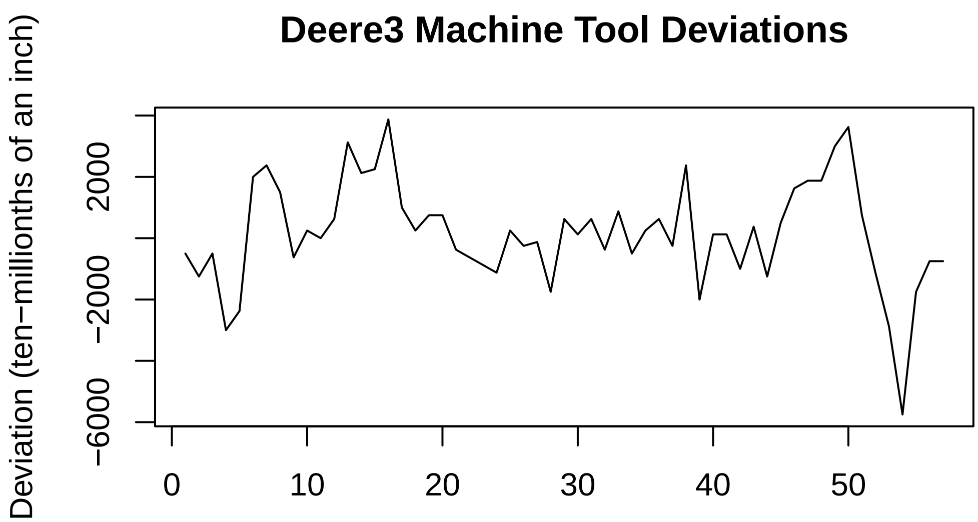
| Model | logLik | AIC | $^2$ (Residual variance) | Interpretation |
|---|---|---|---|---|
| **AR(1)** | 1475.54 | **−2947.08** | 6.482e−06 | Good fit |
| **ARIMA(0,1,1)** | **1480.95** | **−2959.90** | **6.069e−06** | Better fit |

- AIC is lower for ARIMA(0,1,1) (−2959.9 vs. −2947.08) → better for model.
- Log likelihood is higher for ARIMA(0,1,1) better explanation of data.
- Residual variance ( $^2$) is smaller for ARIMA(0,1,1) → more precise model.

**Conclusion: ARIMA(0,1,1) fits the data better statistically.**

**4. (10 points) The data file named deere3 contains 57 consecutive values from a complex machine tool process at Deere & Co. The values given are deviations from a target value in units of ten millionths of an inch. The process employs a control mechanism that resets some of the parameters of the machine tool depending on the magnitude of deviation from target of the last item produced.**

```
library(TSA)
data(deere3)

plot(deere3, main = "Deere3 Machine Tool Deviations",
     ylab = "Deviation (ten-millionths of an inch)", xlab = "Observation")
```




**a. Using an AR(1) model for this series, forecast the next ten values.**

**AR(1) model**

```
model_ar1 <- arima(deere3, order = c(1, 0, 0))
(model_ar1)
```

```
Call:
arima(x = deere3, order = c(1, 0, 0))
```

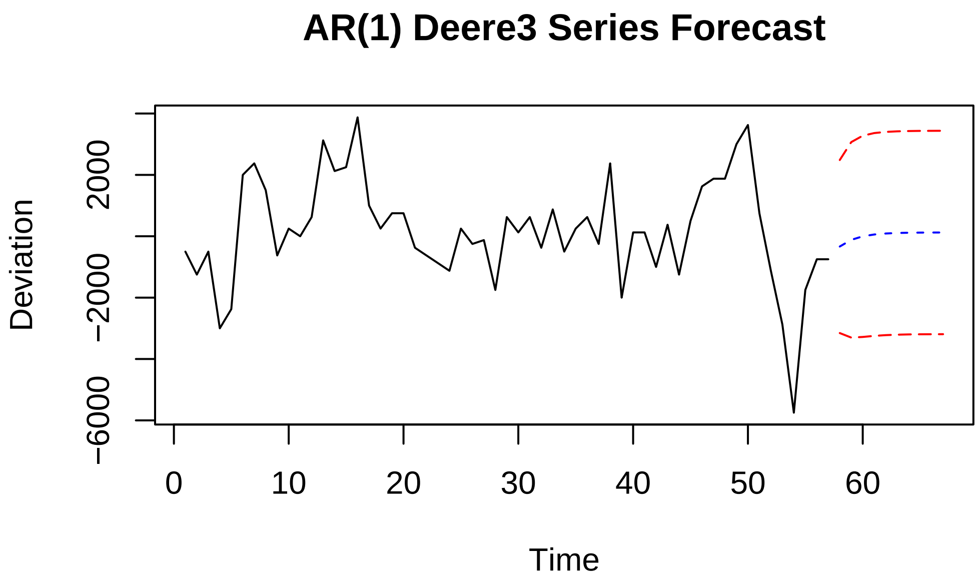
```
Coefficients:
         ar1  intercept
      0.5255   124.3832
s.e.  0.1108   394.2067

sigma^2 estimated as 2069355:  log likelihood = -495.51,  aic = 995.02
```

```r
forecast_values <- predict(model_ar1, n.ahead = 10)
forecast_values
```

```
$pred
Time Series:
Start = 58
End = 67
Frequency = 1
 [1] -335.145928 -117.120772   -2.538388   57.679997   89.327566  105.959839
 [7]  114.700873  119.294695  121.708962  122.977772

$se
Time Series:
Start = 58
End = 67
Frequency = 1
 [1] 1438.525 1625.088 1672.953 1685.934 1689.502 1690.486 1690.758 1690.833
 [9] 1690.854 1690.859
```

**b. Plot the series, the forecasts, and 95% forecast limits, and interpret the results. Note: The data set is available in the TSA R package.**

```r
ts.plot(deere3, forecast_values$pred,
        col = c("black", "blue"), lty = c(1,2),
        main = "AR(1) Deere3 Series Forecast",
        ylab = "Deviation")

upper <- forecast_values$pred + 1.96 * forecast_values$se
lower <- forecast_values$pred - 1.96 * forecast_values$se
lines(58:67, upper, col = "red", lty = 5)
lines(58:67, lower, col = "red", lty = 5)
```




**The 95% forecast interval widens slightly as the horizon increases, representing growing uncertainty. Since deviations are small and mean-centered, the process appears stable and well-regulated. Overall, the results indicate that the machine tool process is well-controlled and self-correcting. Short-term deviations occur but are quickly minimize, and the process consistently returns to its target value over time.**