# University of West Florida

## Time Series Analysis STA 6856 | Professor Dr.Tharindu De Alwis

Edwin Quijano

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**1. (10 points) Identify the following as specific ARMA models. That is, what are $p$, $q$, and what are the values of the parameters (the $\phi$ 's and $\theta$ 's)**

**(a)** $Y_t = Y_{t-1} - 0.25Y_{t-2} + e_t - 0.1e_{t-1}$

$AR(p) = $ values of $Y_t$ $MA(q) = $ White Noise $e_t$

ARMA(2,1)

$Y$ lag $= p = 2$

$e$ lag $= q = 1$

$\phi_1 = 1, \phi_2 = -0.25$

$\theta_1 = -0.1$

**(b)** $Y_t = 0.5Y_{t-1} - 0.5Y_{t-2} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$

$AR(p) = $ values of $Y_t$ $MA(q) = $ White Noise $e_t$

ARMA(2,1)

$Y$ lag $= p = 2$

$e$ lag $= q = 2$

$\phi_1 = 0.5, \phi_2 = -0.5$ $\theta_1 = -0.5.\theta_2 = 0.25$

**2. (10 points) Consider the following time series model**

$$Y_t = 0.64Y_{t-2} + e_t - 0.8e_{t-1}$$

**(a) Characterize this model as models in the ARMA(p,q) family, that is identify p and q.**

ARMA(2,1)

$Y$ lag $= p = 2$

$e$ lag $= q = 1$

$\phi_1 = 0, \phi_2 = -0.64$

$\theta_1 = -0.8$

**(b) Is this model causal? Is this model a stationary model?**

$$\phi B = 1 - \phi_1 B - \phi B_2 = 1 - 0.64B^2 = \phi_1 = 0, \phi_2 = 0.64$$

$$\theta B = 1 + \theta_1 = 1 - 0.8B = \theta_1 = 0.8$$

$$1 - 0.64z^2 = 0$$

$$z^2 = \frac{1}{0.64} = 1.56 = z = \pm 1.25$$

AR is Causal and stationary, modulos $= 1.25 > 1$

**3. (10 points)** Consider the following time series model

$$(1 - 0.8B)(1 - 1.2B)(1 - B)Y_t = e_t$$

**(a) Is this model can be characterized as models in the ARMA(p,q) family? if so, then identify p and q.**

This is an ARIMA(2,1,0) since, $(1 - 0.8B)(1 - 1.2B) = Y_t = e_t$

**(b) Is this series stationary?**

Not stationary,

$$B = \frac{1}{0.8} = 1.25$$

$$B = \frac{1}{1.2} = 0.83$$

$$B = 1$$

**4. (10 points) Suppose that** $Y_t$ **follows the ARMA(1,1) model,** $(1 - \phi B)Y_t = (1 - \theta B)e_t$, **where e_t is a white noise. Let** $X_t = (1 - \tau B)Y_t$. **What is the model for** $X_t$, **that is find** $p$ **and** $q$.

$$X_t = (1 - \phi B)X_t$$

$$(1 - \phi B)X_t = (1 - \phi B)(1 - \tau B)Y_t = (1 - \tau B)(1 - \phi B)Y_t = (1 - \tau B)(1 - \theta B)e_t$$

$$1 - \phi B)X_t = (1 - \theta B)(1 - \tau B)e_t$$

X_t=ARMA(1,2)

$p = 1, q = 2$

AR=$\phi$

MA $= \theta_1 = \theta + \tau,\ \theta_2 = -\theta\tau$

**5. (20 points) Simulate an AR(2) time series of length** $n = 72$ **with** $\phi 1 = 0.7$ **and** $\phi 2 = -0.4$

**(a) Calculate and plot the theoretical autocorrelation function for this model. Plot sufficient lags until the correlations are negligible. Hint: use ARMAacf() function.**

```
n<-75
phi<-c(0.75, -0.4)
lag.max <- 30

acf_model_1 <- ARMAacf(ar = phi, ma = numeric(0), lag.max = lag.max)

# Plot
plot(0:lag.max, acf_model_1, type = "h", lwd = 2,
     xlab = "Lag", ylab = "Theoretical ACF",
     main = "Theoretical ACF for AR(2): phi1=0.7, phi2=-0.4")
abline(h = 0, lty = 2)
```



Theoretical ACF for AR(2): phi1=0.7, phi2=−0.4
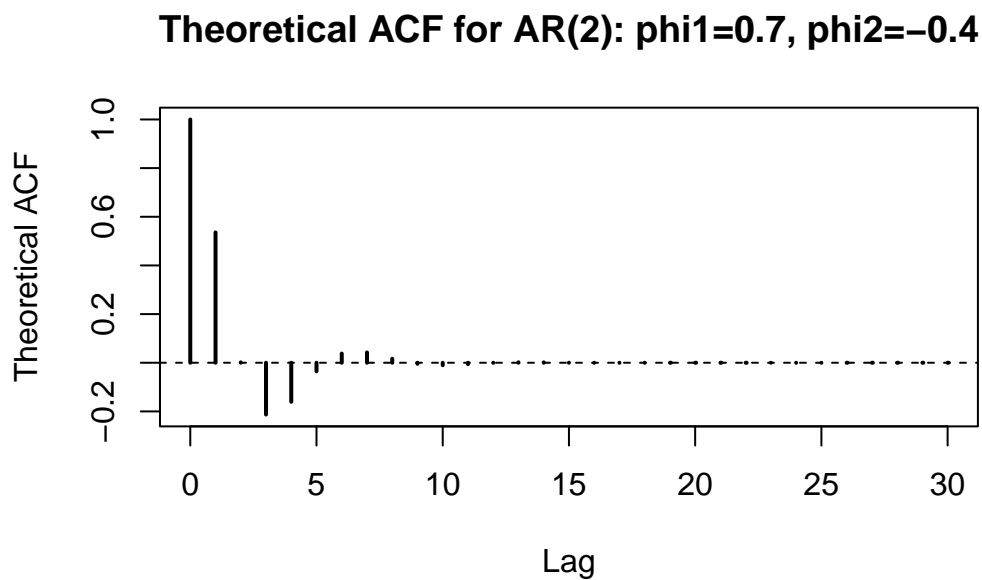
(b) Calculate and plot the sample ACF for your simulated series. How well do the values and patterns match the theoretical ACF of part (a)?

```
y     <- arima.sim(model = list(ar = phi), n = n)

acf_sample_1   <- acf(y, lag.max = lag.max, plot = FALSE)


plot(0:lag.max, acf_model_1, type = "h", lwd = 2,
     xlab = "Lag", ylab = "ACF",
     main = "AR(2): Theoretical vs Sample ACF")
points(acf_sample_1$lag[,1,1], acf_sample_1$acf[,1,1], pch = 16)
abline(h = 0, lty = 2)
```

## AR(2): Theoretical vs Sample ACF
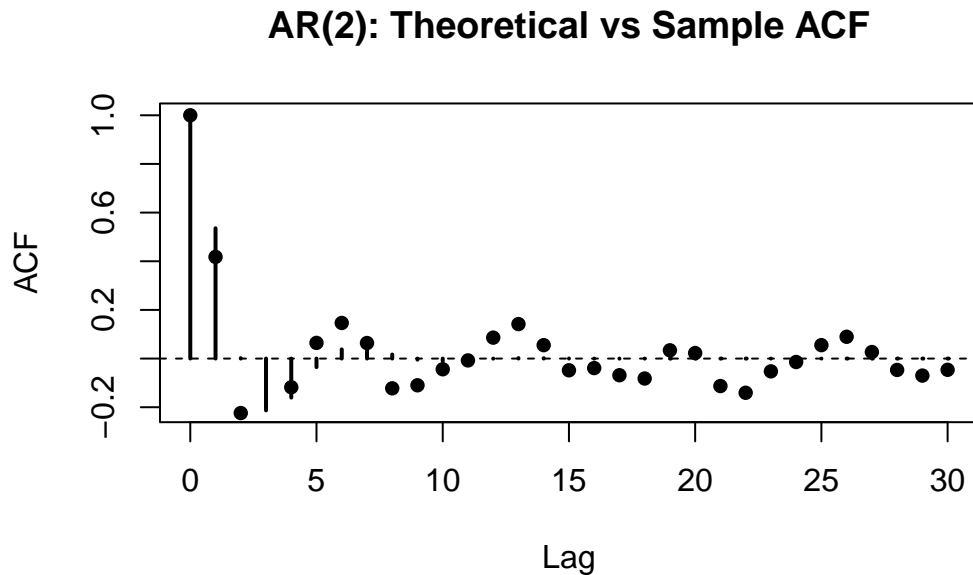


Sample ACF tracks the Theorical ACF with sampling noise.

(c) What are the theoretical partial autocorrelations for this model? Hint: use AR-MAacf(...,pacf = TRUE) function.

```
pacf_model_1 <- ARMAacf(ar = phi, lag.max = lag.max, pacf = TRUE)

pacf_model_1
```

```
 [1]  5.357143e-01 -4.000000e-01  9.268409e-17  3.259138e-17 -2.935496e-17
 [6]  2.301066e-17  2.896378e-18 -1.288888e-17  1.665417e-17 -8.689134e-18
[11]  2.099874e-18 -4.887638e-19  3.620472e-19 -3.077402e-19  2.127028e-19
[16] -9.956299e-20  1.176654e-19 -2.262795e-21  1.357677e-20  2.262795e-21
[21]  1.866806e-20 -1.654669e-20  1.131398e-20 -7.536622e-36 -2.757782e-21
[26]  3.677042e-21 -2.474932e-21  1.732453e-21 -7.955139e-22  3.005275e-22
```

```
plot(1:lag.max, pacf_model_1, type = "h", lwd = 2,
     xlab = "Lag", ylab = "Theoretical PACF",
     main = "Theoretical PACF for AR(2): phi1=0.7, phi2=-0.4")
abline(h = 0, lty = 2)
```

## Theoretical PACF for AR(2): phi1=0.7, phi2=−0.4
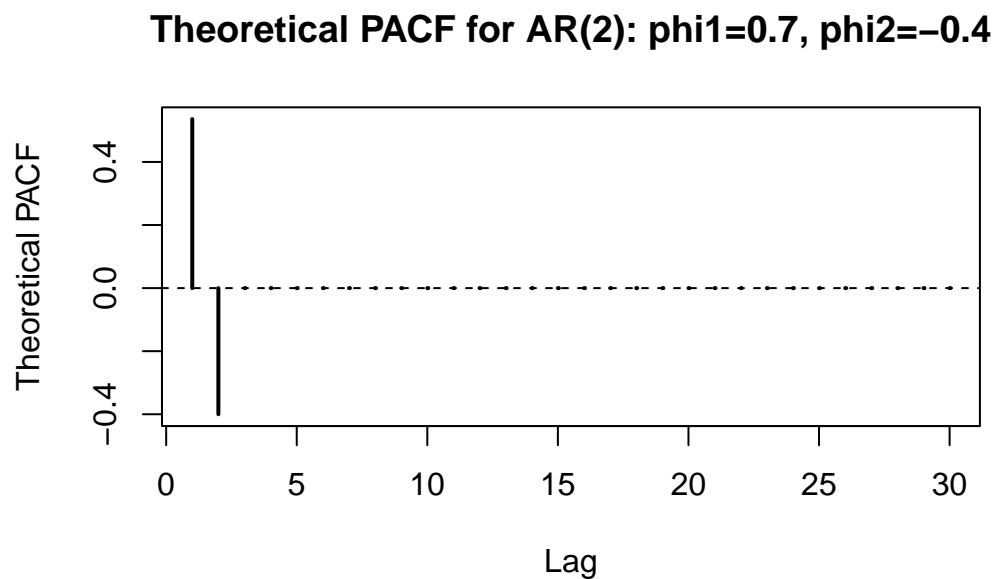
(d) Calculate and plot the sample PACF for your simulated series. How well do the values and patterns match the theoretical ACF of part (c)?

```
pacf_sample_1 <- pacf(y, lag.max = lag.max, plot = FALSE)


plot(1:lag.max, pacf_model_1, type = "h", lwd = 2,
     xlab = "Lag", ylab = "PACF",
     main = "AR(2): Theoretical vs Sample PACF (n = 72)")
points(pacf_sample_1$lag[,1,1], pacf_sample_1$acf[,1,1], pch = 16)
abline(h = 0, lty = 2)
```

## AR(2): Theoretical vs Sample PACF (n = 72)
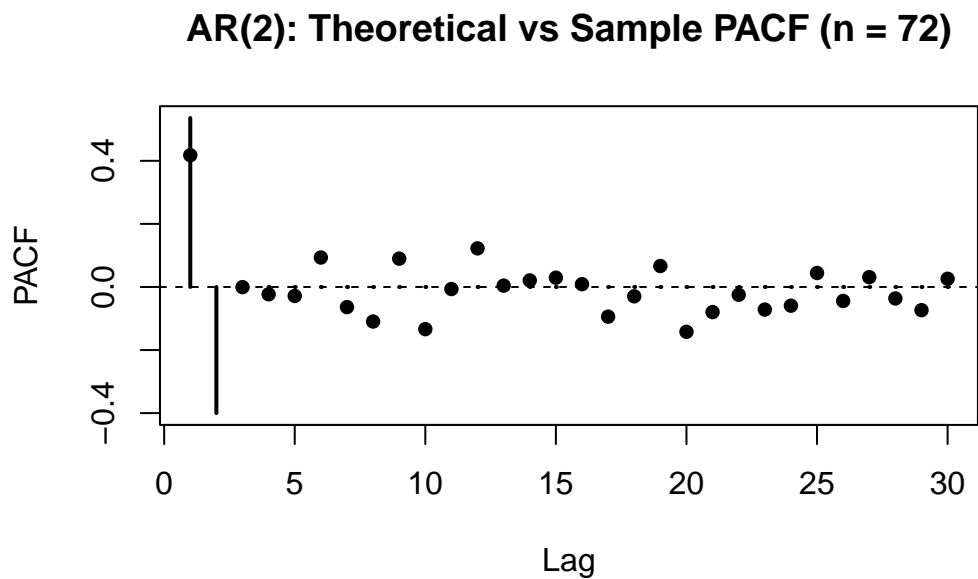


Large spikes at $Lags1 = 0.5$ and $Lag2 = -0.4$ matches the theorical PACF