Modsim Assignment 11 Eirik Falck

Problem 1

a) The ball hits the surface and bounces back up when the balls position equals that of the floor, i.e. when $p_2(t) = 0$. The ball also has to be travelling downwards, hence $\dot{p}_2 < 0$.

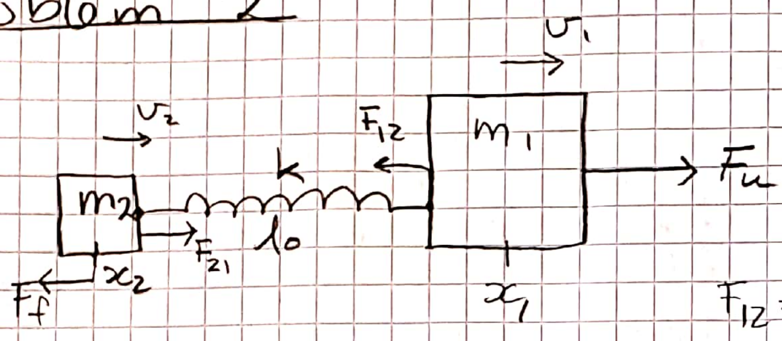
When the ball hits the floor, it's modelled as instantly changing its y-velocity, i.e. $\dot{p}_2 \leftarrow -\dot{p}_2$. Its x-velocity doesn't change. The matrix product in (3) ensures this.

b) See figure below.

c) See code below.

d) The naive approach never yields satisfying results. This is because it's more than likely will overshoot $p_{k,2} = 0$ and thus have a greater speed and "bounce" higher than it started.
The event detector restarts when $p_{k,2} = 0$ thus ensuring a nice path.

Problem 2



$F_{12} = F_{21} = F_b = k(x_1 - x_2 - l_0)$

a) $x = [x_1 \;\; x_2 \;\; v_1 \;\; v_2]^T$

Stiction model: $F_f = \begin{cases} F_s & v_2 = 0 \\ F_c \, \mathrm{sgn}(v_2) & v_2 \neq 0 \end{cases}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ F_u - F_b \\ F_b - F_f \end{bmatrix}$$

b) ODE45 has more oscillations on the speed, The positions are quite similar, though the ODE45 solution is still a bit more oscillatory.

c) Event slip: detects when the second mass m_2 has zero velocity, $v_2 = 0$.

Event stick: detects when the spring force becomes larger than F_s causing the m_2 mass to slip and move.

Since the code integrates first the sticking part then the slipping part, it must know when to terminate each integration.

d) We can see from the error that the approximation is quite similar to the exact model except at $v=0$ where the approximation has smooth edges while the exact model have sharp corners.

e) The state-space model is equal to the one in a), but the friction force F_f is given by (5.10).