Modsim Assignment 1! Erik Falck

Problem 1

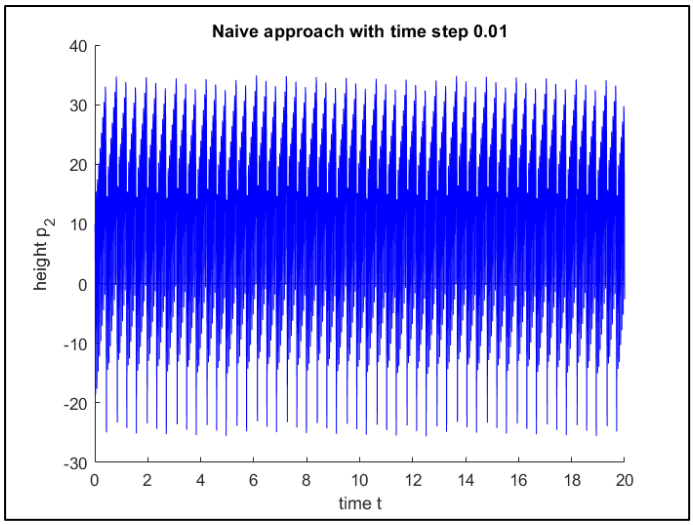
a) The ball hits the surface and bounces back up when the balls position equals that of the floor, i.e. when $p_2(t) = 0$. The ball also has to be travelling downwards, hence $\dot{p}_2 < 0$.

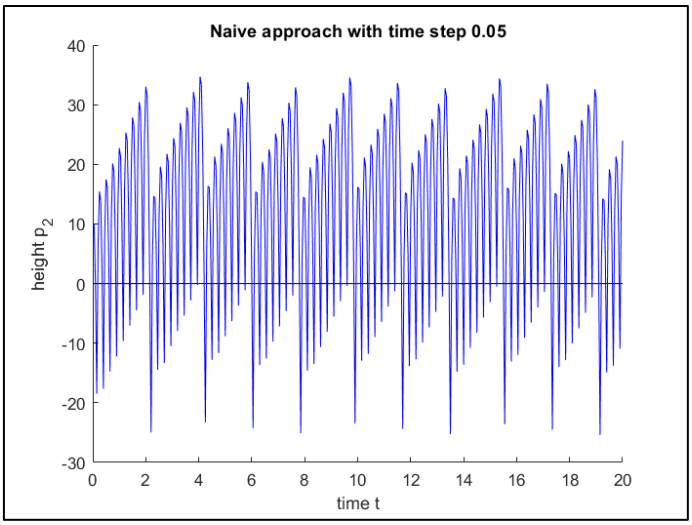
When the ball hits the floor, it's modelled as instantly changing its y-velocity, ie. $\dot{p}_2 \leftarrow -\dot{p}_2$. Its x-velocity doesn't change. The matrix product in (3) ensures this.

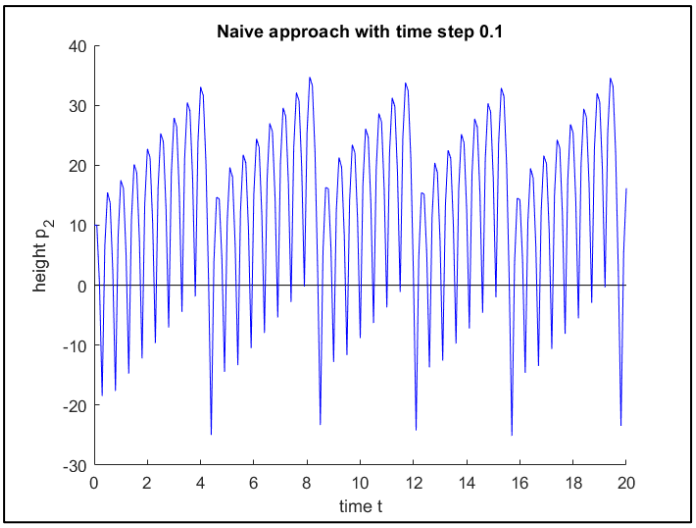
b) See figure below.

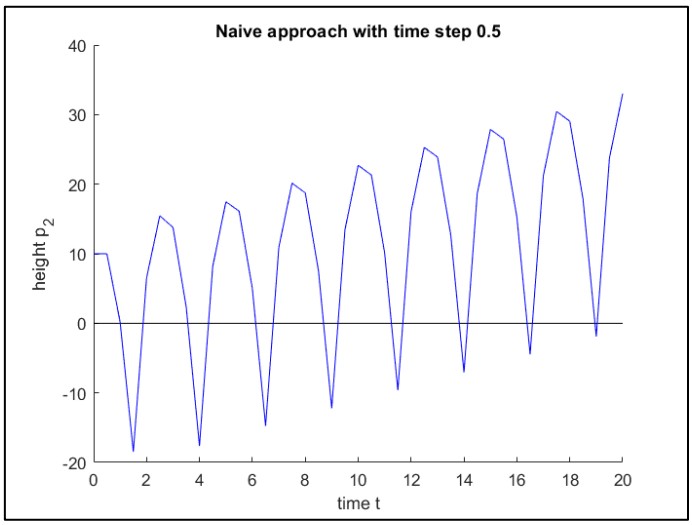
c) See code below.

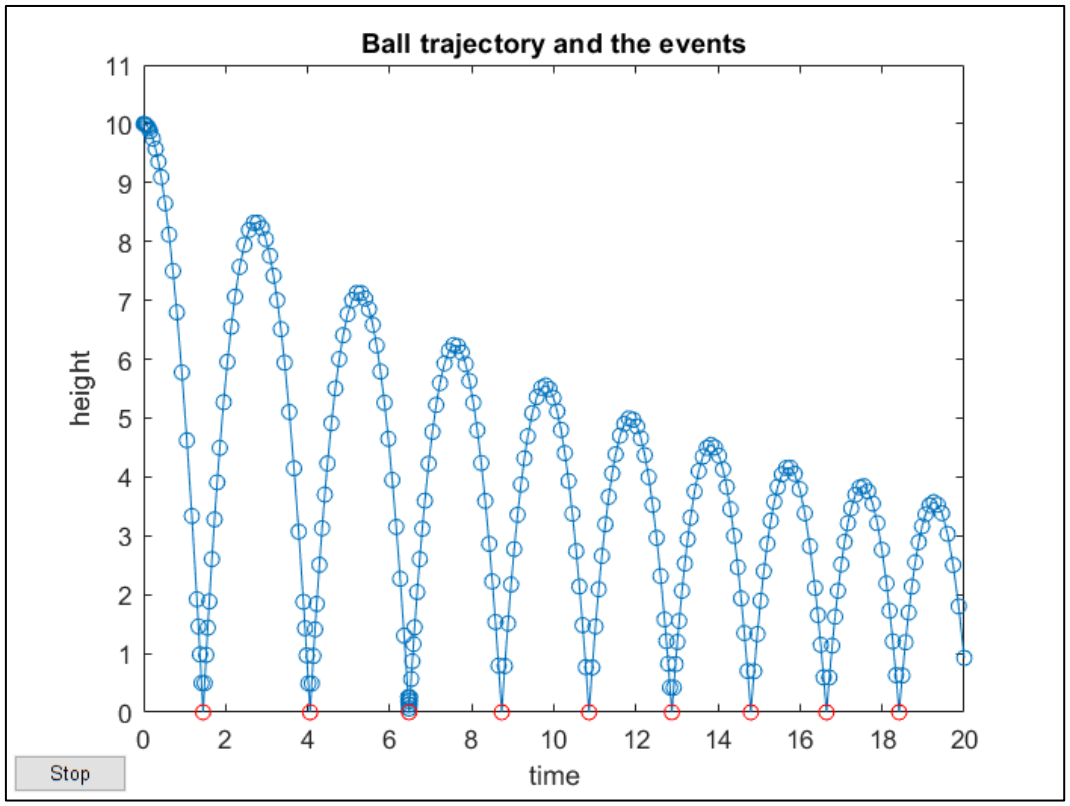
d) The naive approach never yields satisfying results. This is because it's more than likely will overshoot $p_{k,2} = 0$ and thus have a greater speed and "bounce" higher than it started.
The event detector restarts when $p_{k,2} = 0$ thus ensureing a nice path.


Naive approach with time step 0.01
height p_2
time t

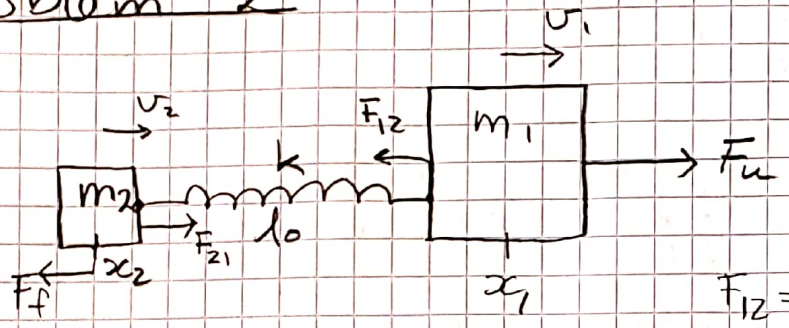

Naive approach with time step 0.05
height p_2
time t


Naive approach with time step 0.1
height p_2
time t


Naive approach with time step 0.5
height p_2
time t


Ball trajectory and the events
height
time
Stop

Problem 2



$$F_{12} = F_{21} + F_b = k(x_1 - x_2 + l_0)$$

a) $x = [x_1 \quad x_2 \quad v_1 \quad v_2]^T$

Stiction model: $F_f = \begin{cases} F_s & v_2 = 0 \\ F_c \, \mathrm{sgn}(v_2) & v_2 \neq 0 \end{cases}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ F_u - F_b \\ F_b - F_f \end{bmatrix}$$

b) ODE45 has more oscillations on the speed. The positions are quite similar, though the ODE45 solution is still a bit more oscillatory.

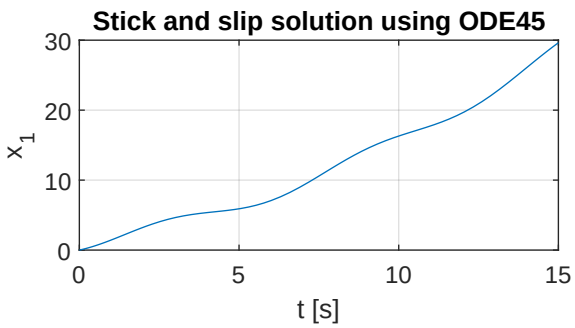
c) EventSlip: detects when the second mass m_2 has zero velocity, $v_2 = 0$.

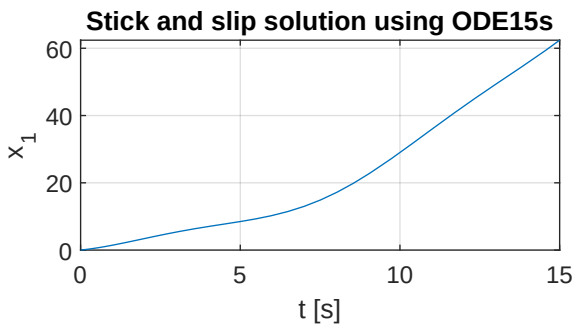
EventStick: detects when the spring force becomes larger than F_s causing the m_2 mass to slip and move.

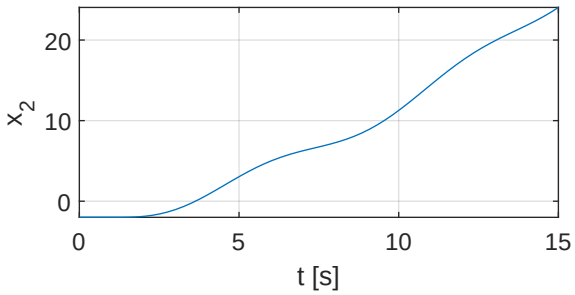
Since the code integrates first the sticking part then the slipping part, it must know when to terminate each integration.

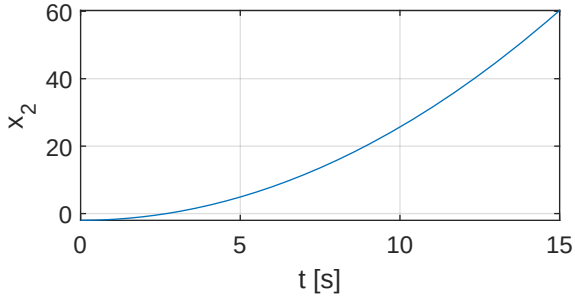
d) We can see from the error that the approximation is quite similar to the exact model except at $v=0$ where the approximation has smooth edges while the exact model have sharp corners.

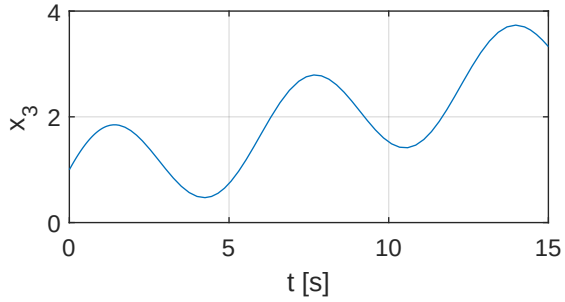
e) The state-space model is equal to the one in a), but the friction force F_F is given by (5.10).

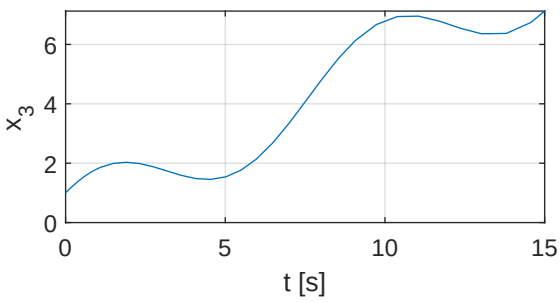

Stick and slip solution using ODE45
x1
t [s]

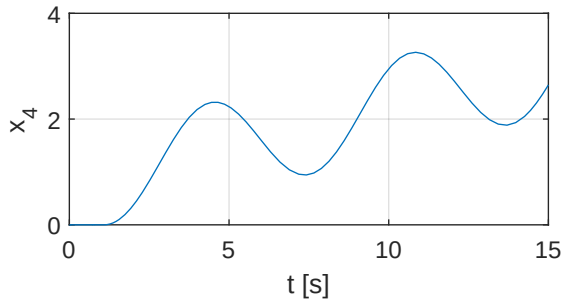

Stick and slip solution using ODE15s
x1
t [s]

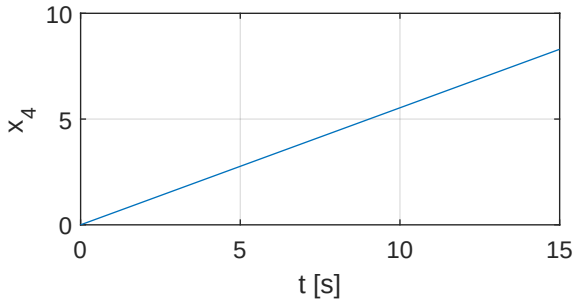

x2
t [s]

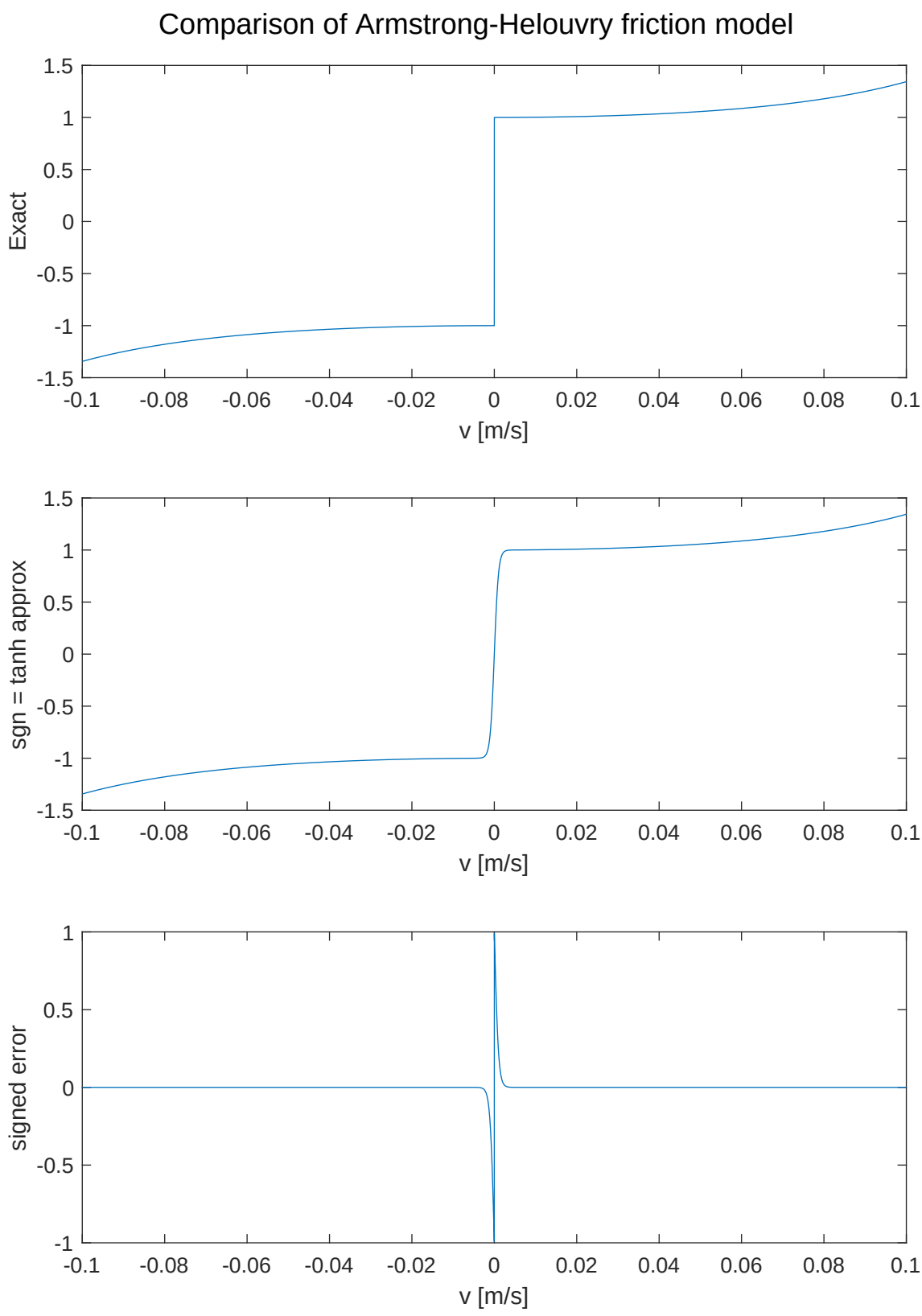

x2
t [s]


x3
t [s]


x3
t [s]


x4
t [s]


x4
t [s]


Comparison of Armstrong-Helouvry friction model
Exact
sgn = tanh approx
signed error
v [m/s]
1.5
1
0.5
0
-0.5
-1
-1.5
-0.1
-0.08
-0.06
-0.04
-0.02
0.02
0.04
0.06
0.08
0.1