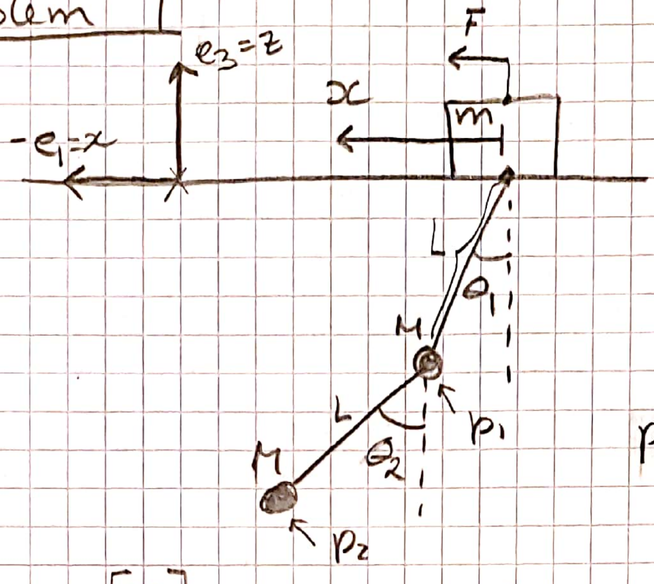
TTK4130 Assignment 4 Eirik Falck

Problem 1



$m = 1\,kg$

$M = 1\,kg$

$L = 1\,m$

$$p_1 = \begin{bmatrix} x + L\sin\theta_1 \\ -L\cos\theta_1 \end{bmatrix}$$

a) $q = \begin{bmatrix} x \\ \theta_1 \\ \theta_2 \end{bmatrix}$ $\quad p_0 = \begin{bmatrix} x \\ 0 \end{bmatrix}$ $\quad p_2 = \begin{bmatrix} x + L\sin\theta_1 + L\sin\theta_2 \\ -L\cos\theta_1 - L\cos\theta_2 \end{bmatrix}$

$$T(q,\dot{q}) = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} M \dot{p}_1^T \dot{p}_1 + \frac{1}{2} M \dot{p}_2^T \dot{p}_2$$

$$V(q) = M g\, p_{1,2} + M g\, p_{2,2}$$

second element, i.e. height

$$\mathcal{L}(q,\dot{q}) = T(q,\dot{q}) - V(q)$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = Q$$

$$\frac{d}{dt} p_1(q) = \frac{\partial p_1}{\partial q} \cdot \dot{q} \quad , \quad \frac{d}{dt} p_2(q) = \frac{\partial p_2}{\partial q} \cdot \dot{q}$$

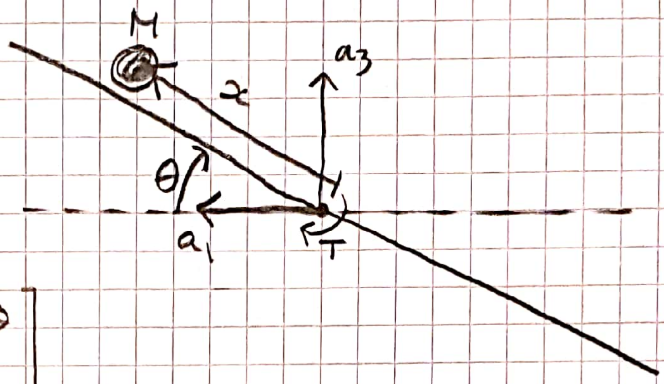
b) $F = -10x - \dot{x} \quad \rightarrow \quad Q = \begin{bmatrix} -10x - \dot{x} \\ 0 \\ 0 \end{bmatrix}$

The simulation shows very sensible results. The pendulums swing in the manner one would expect, and the box glides back and fourth on the board.

Problem 2

$J = 1\,kg\,m^2$, $M = 10\,kg$, $R = 0{,}25\,m$

$q = \begin{bmatrix} x \\ \theta \end{bmatrix}$



$p = \begin{bmatrix} x\cos\theta \\ 0 \\ x\sin\theta \end{bmatrix}$

Rotation velocity of ball

$T_{beam} = \frac{1}{2} J \dot{\theta}^2$ $\qquad \omega = \dot{\theta} + \frac{\dot{x}}{R}$, $J_{ball} = \frac{2}{5} M R^2$

$T_{ball,x} = \frac{1}{2} M \dot{x}^2$ $\qquad T_{ball,\omega} = \frac{1}{2}\frac{2}{5} M R^2 \left(\dot{\theta} + \frac{\dot{x}}{R}\right)^2$

$$\Rightarrow \quad T(q,\dot{q}) = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} J \dot{\theta}^2 + \frac{1}{2}\frac{2}{5} M R^2 \left(\dot{\theta} + \frac{\dot{x}}{R}\right)^2$$

$V(q) = Mg\,p_3 = Mg\,x\sin\theta$

$\Rightarrow \mathcal{L} = T - V$

$\text{Torque} = 200(x - \theta) + 70(\dot{x} - \dot{\theta})$

$$\Rightarrow \quad Q = \begin{bmatrix} 0 \\ \text{Torque} \end{bmatrix}$$

The simulation shows how the PD-controller tries to keep the ball on the beam by rotating the beam.

The results are reasonable. See plot for coordinates vs. time.