TTK4130 Assignment 8 Eirik Falck

## Problem 1

a) See code below

b) $f(x) = \begin{bmatrix} x_1 \cdot x_2 - 2 \\ \frac{x_1^4}{4} + \frac{x_2^3}{3} - 1 \end{bmatrix} = 0$

The infinity norm increases from the initial value and then steadily fall to almost 0.
This convergence is (close to) quadratic.

c) The initial value starts close to a stationary point and jumps around a lot before settling on a solution.

The middle subplot shows the "jumpy" behaviour of the $x$ value.

To improve Newtons method to circumvent this, we can reduce its step length.

d) We see a very quick convergece in the first iteration, before it becomes slower.

At $[1 \ \pi]^T$ the Jacobian becomes rank defficient and the method becomes unreliable. The end point $[1 \ 3.14]^T$ is close to this point.

Newtons method can improved by approximating the gradient so it's always full rank.?

e) $x^* = [1 \ \ 1]^T$

How do I calculate the convergence order? I've looked too long at $\|e_{n+1}\| = \mu \|e_n\|^p$ and taken the logarithm to try to figure something out. The only thing I find is the slope in the logarithmic plot, which is $-0.1761$. Is this related to the order somehow?

## Problem 2

a) Implicit euler:

$$x_{k+1} = x_k + \Delta t \, f(t_k, x_{k+1})$$

$$(1) \quad r(x_{k+1}) = x_k + \Delta t \, f(t_k, x_{k+1}) - x_{k+1}$$

$$(2) \quad \frac{\partial r}{\partial x_{k+1}} = \Delta t \frac{\partial f}{\partial x_{k+1}} - I$$

Solve (1) using Newtons method at each time step (this requires (2)).

b) As we can see, the implicit simulation is lagging somewhat behind the true solution, but seems to settle well towards the end since the error gets smaller.