NTNU – Trondheim
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Science and Technology

TTK4130 Modeling and Simulation
Assignment 3

Introduction

The objectives of this assignment are:

- To understand the Newton-Euler equations, and apply them to simple mechanical systems.
- To learn how to select convenient SO(3) representations and reference frames in order to simplify the associated Newton-Euler equations.
- To understand and apply the parallel axis theorem, also known as Huygens–Steiner theorem.

Problem 1 (Satellite)

In this task, we will consider a satellite orbiting Earth. We define an inertial reference frame with its origin at Earth's center and with an arbitrary and fixed orientation.

We will consider two cases:

(a) The satellite is a cube of uniform, unitary density, having an edge of 50cm.

(b) The satellite is the cube mentioned above, with the addition of a punctual mass of $m_0 = 0.1$kg placed at one of the cube's corners.

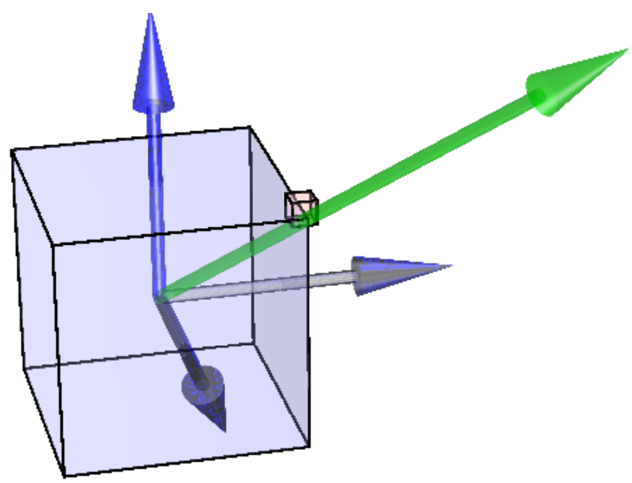

Figure 1: Schematic of the satellite.

We will assume that the force of gravity is given by Newton's law of universal gravitation:

$$\vec{F} = -\frac{G\, m_{\mathrm{T}}\, m}{\|\vec{r}_{\mathrm{c}}\|^2} \cdot \frac{\vec{r}_{\mathrm{c}}}{\|\vec{r}_{\mathrm{c}}\|} \tag{1}$$

The inertia matrix in the reference frame attached to the cube with its origin at the cube's center of mass and with the axes going through the center of the cube's faces is given by $\frac{1}{6}ml^2 I$, where m is the mass, l is the length of the sides and I is the 3-by-3 identity matrix.

For both cases, select a frame for the satellite and a representation of the SO(3) Lie group (orientation). Then apply correctly the Newton-Euler equations to describe the satellite's motion (position and orientation).

What is your resulting state-space model?

Complete the function `SatelliteDynamics.m` accordingly, and add it to your answer.

Simulate your equations for both cases using the Matlab ODE integration function `ode45` (which we will discuss later in the course). An example code is provided on Blackboard. You will find in the code means to show a 3D animation of your simulation.

What do you observe? Are the results reasonable? What is the main difference between both cases? Explain.

Hints:

- *Note that in case (b), the added punctual mass will shift the position of the center of mass of the satellite from the center of the cube. You may need the parallel axes theorem.*
- *The selection of a convenient body frame and* SO(3) *representation will make the problem much easier.*
- *You will find code templates / examples on Blackboard to help you get started. See* `Satellite3DTemplate.m` *for a template on how to build the simulation, and* `Satellite3DExample.m` *for tools to do 3D animations. These animations will allow you to assess your simulations.*
- *For parameter and initial values that are not given, you are free to choose reasonable numerical values. For example, Earth's radius is* 6356 km *and its orbital height is* 36 Mm.

Problem 2 (Spinner)

In this task, we will consider a "spinner", i.e. a disk of uniform density, radius $R = 1\,\mathrm{m}$ and mass $m = 1\,\mathrm{kg}$ mounted on a massless rod of length $L = 2\,\mathrm{m}$ connected to a free joint.

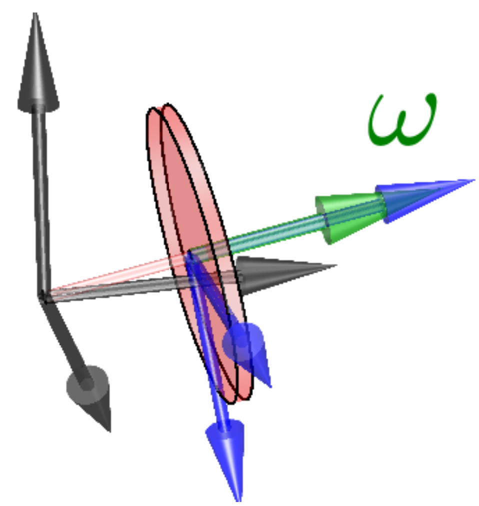

Figure 2: Schematic of the spinner.

We assume that the thickness of the disk is zero. Hence, the inertia matrix of the disk taken in a frame attached at its center with the radial symmetry axis as the third axis, is given by:

$$M_c^b = \frac{mR^2}{4}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{2}$$

Select a frame for the spinner and a representation of the SO(3) Lie group (orientation of the spinner). Then apply correctly the Newton-Euler equations to describe the motion of the spinner.

We will consider that the force of gravity is given by:

$$\vec{F} = -m\vec{g} \tag{3}$$

What is your resulting state-space model?

Implement your model and simulate it for different angular velocities $\boldsymbol{\omega}$, where

$$\boldsymbol{\omega}_{ab}^{b} = \omega \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tag{4}$$

i.e. $\boldsymbol{\omega}$ is aligned with the radial symmetry axis of the disk. Test e.g. $\omega = \pi, 2\pi, 4\pi, 6\pi \,\mathrm{rad\,s^{-1}}$.

What do you observe? Are the results reasonable? Explain.

Hints:

- *The selection of a convenient body frame and* SO(3) *representation will make the problem much easier.*
- *You may need the parallel axes theorem.*
- *You will find code templates / examples on Blackboard to help you. See* `Gyropscope3DExample.m` *and previously delivered codes for tools to do 3D animations. These animations will allow you to assess your simulations.*