Out: February 3, 2020
Due: February 16, 2020

NTNU – Trondheim
Norwegian University of
Science and Technology

TTK4130 Modeling and Simulation

Assignment 4

Introduction

In this assignment we will study the modeling of complex mechanical systems based on the Lagrange approach. Since the calculations of the partial derivatives of the Lagrangian can be involved, we will outsource this task by using the Matlab Symbolic Math Toolbox™.

Problem 1 (Double pendulum on a cart)

In this task, we consider the double pendulum depicted in Figure 1. We assume that the system is frictionless. The cart (grey cube in the figure) is of mass $m = 1\,\text{kg}$ and the two black spheres are of mass $M = 1\,\text{kg}$. We assume the rest of the system as massless. The position of the cart on the rail is x and the position of the pendulums are described by the angles $\theta_1, \theta_2 \in \mathbb{R}$, which give the deviation from the vertical. The length of the pendulum arms are unitary (=1 m). A force F is applied laterally on the cart to move it along the rail.

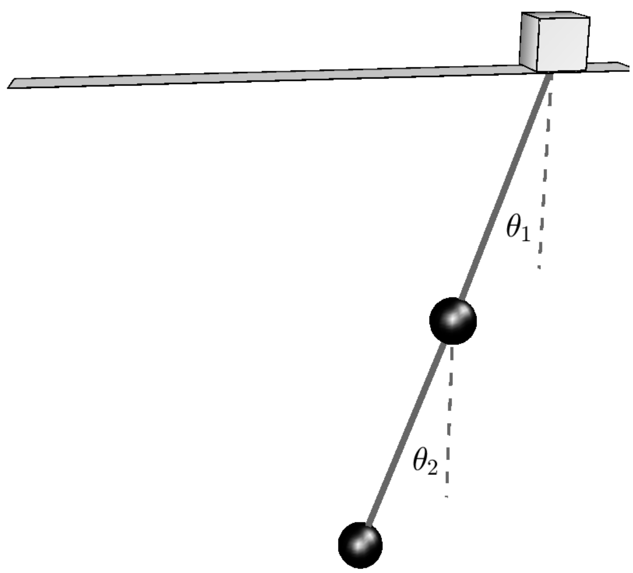

Figure 1: Schematic of the double pendulum.

(a) Build the Lagrange equations describing the system.
You are encouraged to compute the Lagrange equations using the Matlab's symbolic toolbox.
If you do so, add the relevant code to your answer.

(b) Export your Lagrange model in order to run a simulation of the resulting equations.
Assume that the external force F is given by the PD control law:

$$F = -10x - \dot{x} \tag{1}$$

Simulate your system using e.g. $x(0) = 0$, $\theta_1(0) = \frac{\pi}{4}$, $\theta_2(0) = \frac{\pi}{2}$ as initial conditions with initial velocities at rest.
Build an animated graphic representation of your system allowing you to observe the result of your simulation.
What do you observe? Are these results reasonable? Explain.

Hint: Build your model symbolically. As shown in the course, your ODE will read as

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix}, \qquad \dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ \frac{\partial^2 \mathcal{L}}{\partial \dot{\boldsymbol{q}}^2}^{-1} \left(\boldsymbol{Q} + \frac{\partial \mathcal{L}}{\partial \boldsymbol{q}} - \frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{q}} \partial \boldsymbol{q}} \dot{\boldsymbol{q}} \right) \end{bmatrix}. \tag{2}$$

The term $\frac{\partial^2 \mathcal{L}}{\partial \dot{\boldsymbol{q}}^2}^{-1}$ *is fairly complex. Equation* (2) *is best built by creating the matrix* $\frac{\partial^2 \mathcal{L}}{\partial \dot{\boldsymbol{q}}^2}$ *and the vector* $\boldsymbol{Q} + \frac{\partial \mathcal{L}}{\partial \boldsymbol{q}} - \frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{q}} \partial \boldsymbol{q}} \dot{\boldsymbol{q}}$ *symbolically and exporting them as a Matlab function, and then evaluating the second term in* (2) *numerically during the simulation.*

Problem 2 (Ball on a beam)

We consider here a ball on a beam system as depicted in Figure 2. The ball rolls without friction on a beam that is articulated without friction in the middle. A torque T acts on the beam joint. We will use the following numerical values: The rotational inertia of the rail around its joint is $J = 1\,\mathrm{kg\,m}^2$, the mass of the ball is $M = 10\,\mathrm{kg}$ and its radius is $R = 0.25\,\mathrm{m}$. The position of the ball respect to the joint of the rail will be labelled by x. A positive rotation $\theta > 0$ obeys the right-hand rule for reference frames $\vec{a}$ and $\vec{b}$, and a positive displacement $x > 0$ moves the ball in the direction $\boldsymbol{b}_1$ (see Figure 2).

(a) Build the Lagrange equations describing the system.

You are encouraged to compute the Lagrange equations using the Matlab's symbolic toolbox.

If you do so, add the relevant code to your answer.

(b) Export your Lagrange model in order to run a simulation of the resulting equations.

Assume that the external torque T is given by the PD control law:

$$T = 200(x - \theta) + 70(\dot{x} - \dot{\theta}). \tag{3}$$

Simulate your system using e.g. $x(0) = 1$, $\theta(0) = 0$ as initial conditions with initial velocities at rest. Build an animated graphic representation of your system allowing you to observe the result of your simulation.

What do you observe? Are these results reasonable? Explain.

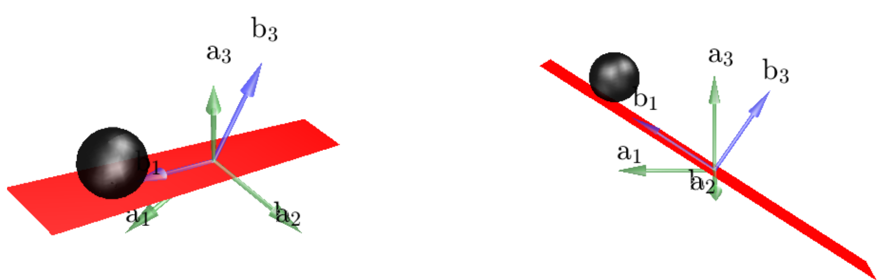

Figure 2: Schematic of the ball on a beam.