 TTK4135 Assignment 0 Eirik Falck

Problem 1

a) For a function $f: \mathbb{R}^n \rightarrow \mathbb{R}$, the <u>gradient</u> is

$$\nabla f(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix} = \left[\frac{\partial f}{\partial x}\right]^T$$

The direction of the gradient vector can be interpreted as pointing in the direction of steepest ascend.

b) For a function $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$, the <u>Jacobian</u> is

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

c) $f(x): \mathbb{R}^n \rightarrow \mathbb{R} \Leftrightarrow x \in \mathbb{R}^n, f \in \mathbb{R}$

Size of ∇f: $\underline{\underline{n \times 1}}$

d) $f(x): \mathbb{R}^n \rightarrow \mathbb{R}^m \Leftrightarrow x \in \mathbb{R}^n, f \in \mathbb{R}^m$

Size of $\frac{\partial f}{\partial x}$: $\underline{\underline{m \times n}}$

Problem 2

$$f(x) = Ax = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a)
$$\frac{df}{dx} = \begin{bmatrix} \frac{df_1}{dx_1} & \frac{df_1}{dx_2} \\ \frac{df_2}{dx_1} & \frac{df_2}{dx_2} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \underline{\underline{A}}$$

This is the Jacobian

b)
$$\frac{d(Ax)}{dx} = A\frac{dx}{dx} = \underline{\underline{A}}$$

move constants out

Problem 3

$$f(x,y) = x^T G y$$

$$\underset{2\times1}{x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} \qquad \underset{2\times3}{G = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \end{bmatrix}} \qquad \underset{3\times1}{y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}$$

a) $f(x,y)$: $(2\times1)^T \cdot 2\times3 \cdot 3\times1 = \underline{\underline{1\times1}} \Leftrightarrow \underline{\text{scalar}}$

$\nabla_x f(x,y)$: gradient wrt. x

$$= \left[\frac{df}{dx}\right]^T$$

No, $\nabla_x f(x,y) = \left[\frac{df(x,y)}{dx}\right]^T$

b) $f(x,y) = [x_1 \quad x_2]\, G \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = [x_1 \quad x_2] \begin{bmatrix} g_{11}y_1 + g_{12}y_2 + g_{13}y_3 \\ g_{21}y_1 + g_{22}y_2 + g_{23}y_3 \end{bmatrix}$

$= x_1(g_{11}y_1 + g_{12}y_2 + g_{13}y_3) + x_2(g_{21}y_1 + g_{22}y_2 + g_{23}y_3)$

$\nabla_x f(x,y) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} g_{11}y_1 + g_{12}y_2 + g_{13}y_3 \\ g_{21}y_1 + g_{22}y_2 + g_{23}y_3 \end{bmatrix} = \underline{\underline{Gy}}$

c) $f(x,y) = y_1(x_1g_{11} + x_2g_{21}) + y_2(x_1g_{12} + x_2g_{22}) + y_3(x_1g_{13} + x_2g_{23})$

$\nabla_y f(x,y) = \begin{bmatrix} \frac{\partial f}{\partial y_1} \\ \frac{\partial f}{\partial y_2} \\ \frac{\partial f}{\partial y_3} \end{bmatrix} = \begin{bmatrix} x_1g_{11} + x_2g_{21} \\ x_1g_{12} + x_2g_{22} \\ x_1g_{13} + x_2g_{23} \end{bmatrix} = \underline{\underline{x^TG}} = \underline{\underline{G^Tx}}$

d) $f(x) = x^T H x \qquad x \in \mathbb{R}^n \qquad H \in \mathbb{R}^{n \times n}$

$\nabla f(x) = \nabla_{x^T} f(x) + \nabla_x f(x) = Hx + H^Tx = \underline{\underline{(H+H^T)x}}$

If H is symmetric: $H = H^T$

$\Rightarrow \nabla f(x,y) = \underline{\underline{2Hx}}$

Problem 4

- $\mathcal{L}(\ldots) \in \mathbb{R}$
- $G = G^T$ (symmetric)
- $x \in \mathbb{R}^n$: $n \times 1$

$$\mathcal{L}(x, \lambda, \mu) = x^T G x + \lambda^T (Cx - d) + \mu^T (Ex - h)$$
$$= x^T G x + \lambda^T C x - \lambda^T d + \mu^T E x - \mu^T h$$

a) $\nabla_x \mathcal{L}(x, \lambda, \mu) = \underline{\underline{2Gx + \lambda^T C + \mu^T E}}$

b) $\nabla_\lambda \mathcal{L}(x, \lambda, \mu) = \underline{\underline{Cx - d}}$

c) $\nabla_\mu \mathcal{L}(x, \lambda, \mu) = \underline{\underline{Ex - h}}$