

$$\underline{x_{t+1}} = \underbrace{\begin{bmatrix} 1 & 0,1 \\ -0,1 & 0,9 \end{bmatrix}}_A \underline{x_t} + \underbrace{\begin{bmatrix} 0 \\ 0,1 \end{bmatrix}}_B u_t$$

$$x_0 = [5 \ 1]^T \quad \hat{x}_0 = [6 \ 0]^T$$

Problem 1

$$P = Q + A^T P (I + BR^{-1}B^T P)^{-1} A \quad (*)$$

$$(I + BR^{-1}B^T P)^{-1} = I^{-1} - I^{-1} B \left[(R^{-1})^T + B^T P I^{-1} B \right] B^T P I^{-1}$$

S U T V

$$= I - B(R + B^T P B)^{-1} B^T P$$

$$\begin{aligned} (*) \Rightarrow & A^T P (I - B(R + B^T P B)^{-1} B^T P) A + Q - P = 0 \\ & = \underline{\underline{A^T P A - P - A^T P B (R + B^T P B)^{-1} B^T P A + Q = 0}} \end{aligned}$$

Problem 2

$$y_t = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_C x_t$$

a) $f^\infty(z) = \frac{1}{2} \sum_{t=0}^{\infty} \hat{x}_{t+1}^T Q \hat{x}_{t+1} + u_t^T R u_t$

$$Q = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \quad R = 1$$

$$K = \underline{\underline{\begin{bmatrix} 1,0373 & 1,6498 \end{bmatrix}}}$$

$C L_{\text{eigs}} = 0,8675 \pm 0,0531j$
 → closed-loop eigenvalues.

b) Poles used: $0,5 \pm 0,03j$

The state estimate follows poorly in the start, but after about $t=10$ the state and estimate follows each other perfectly.

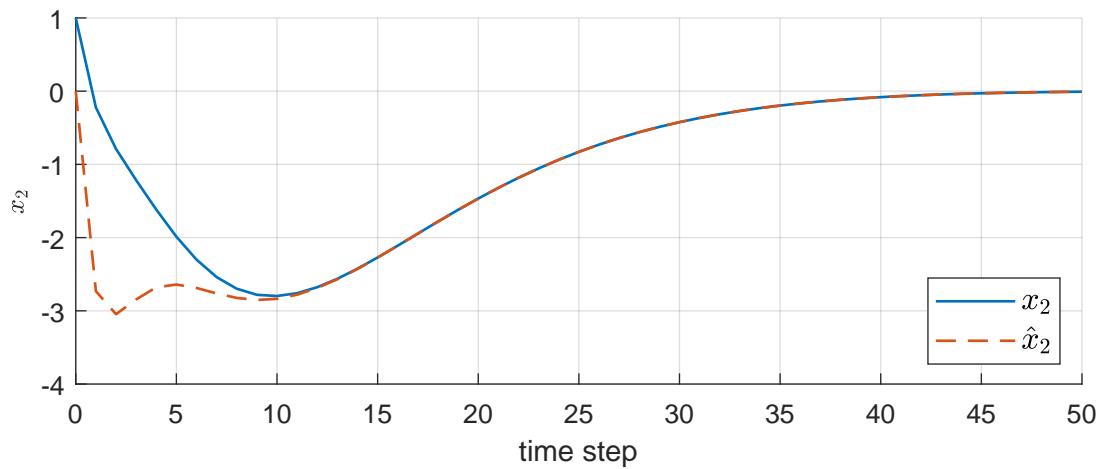
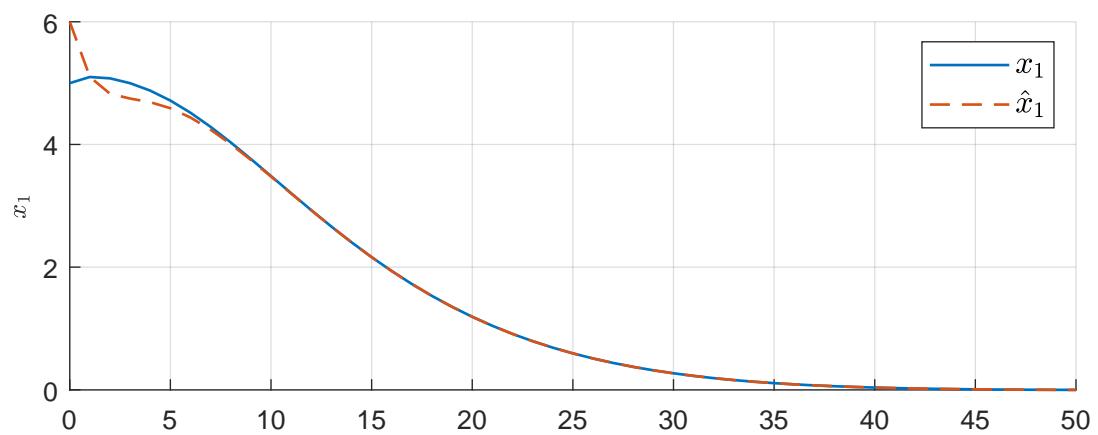
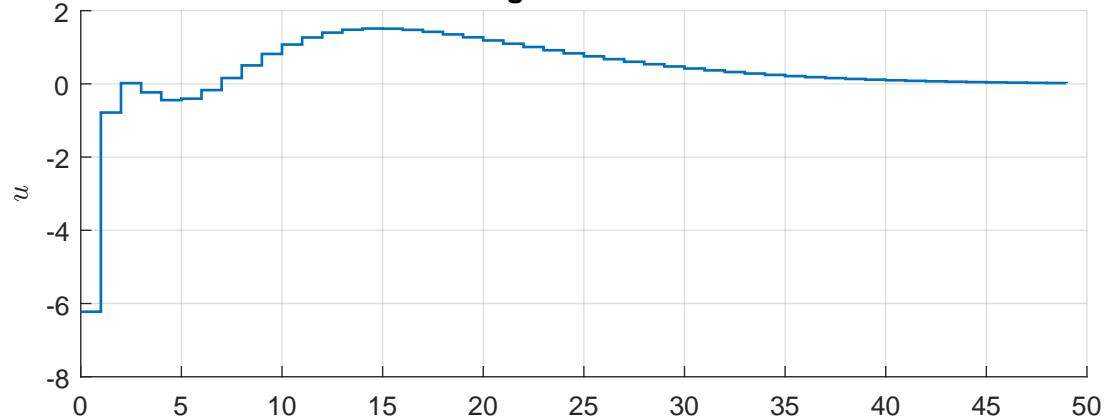
c) $\tilde{x}_{t+1} = \begin{bmatrix} x_{t+1} \\ \hat{x}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 - BK & BK \\ 0 & A - K_f C \end{bmatrix}}_{\Phi} \tilde{x}_t$

$$\tilde{x}_t = x_t - \hat{x}_t$$

$$\Phi = \underline{\underline{\begin{bmatrix} 1 & 0,1 & 0 & 0 \\ -0,1204 & 0,735 & 0,104 & 0,165 \\ 0 & 0 & 0,1 & 0,1 \\ 0 & 0 & -1,609 & 0,9 \end{bmatrix}}}$$

The eigenvalues of Φ are the eigenvalues of $A - BK$ and $A - K_f C$.

TTK4135 Assignment 7 problem 2b
Feedback using LQR and state-observer



Problem 3

$$-4 \leq u_t \leq 4 \quad t = 1, \dots, M-1$$

a) $\mathbf{z} = [\hat{x}_1^T \dots \hat{x}_M^T \quad u_0^T \dots u_{M-1}^T]^T$

$$\mathbf{H} = \begin{bmatrix} Q & & \\ \underbrace{\dots}_{M} & R & \\ & \underbrace{\dots}_{M} & R \end{bmatrix}$$

$$M = 10$$

↳ time horizon

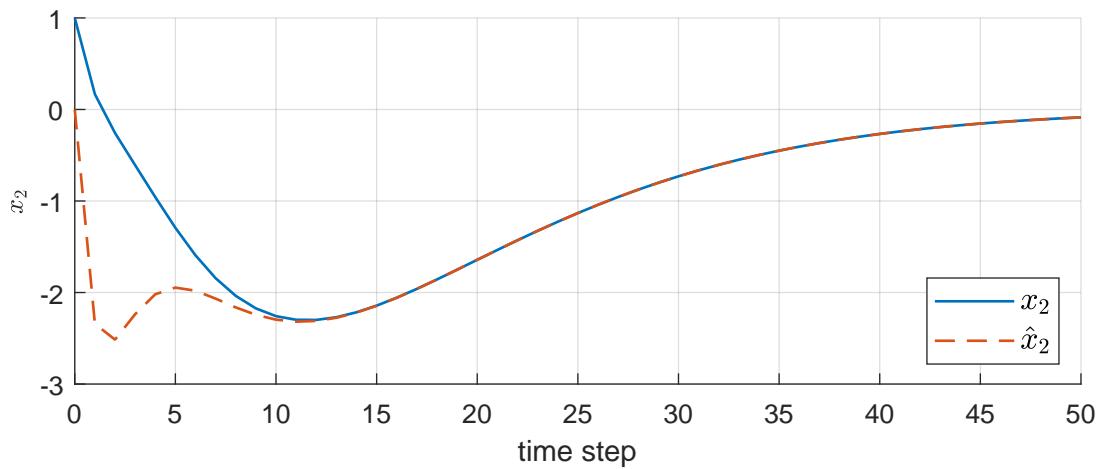
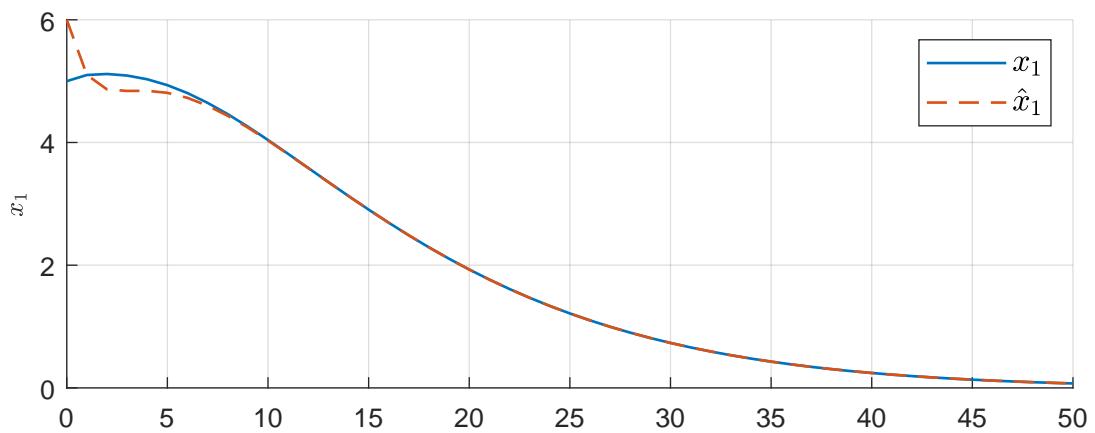
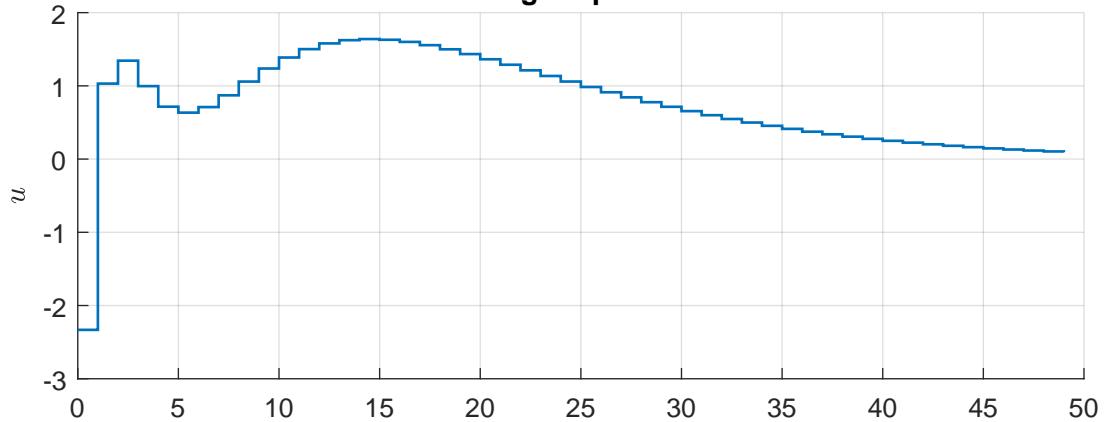
$$A_{eq} = \begin{bmatrix} I & & & & -B \\ -A & I & & & -B \\ -A & I & I & & -B \\ & & & I & -B \\ & & & & -A & I & & -B \\ & & & & & & I & \\ & & & & & & & -B \end{bmatrix}$$

$$b_{eq} = [(A\hat{x}_0)^T \quad 0 \quad 0 \quad \dots \quad 0]^T$$

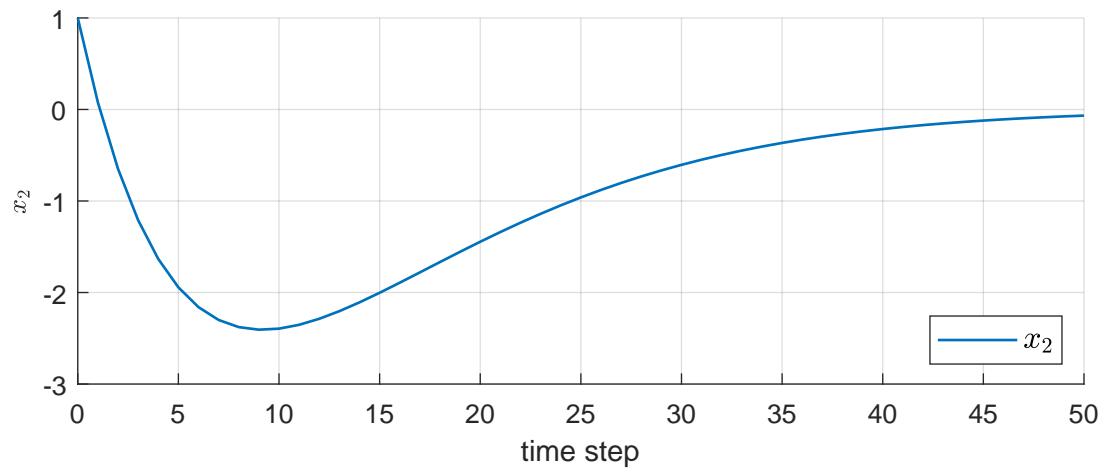
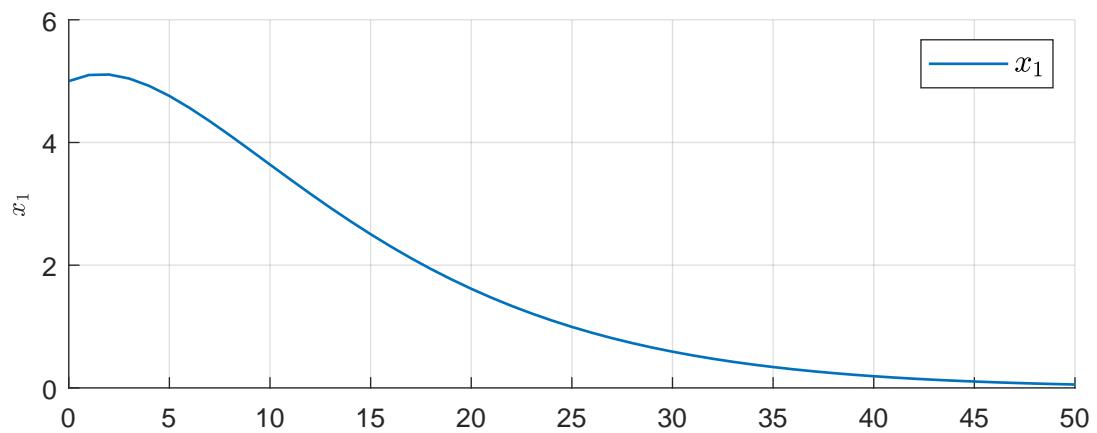
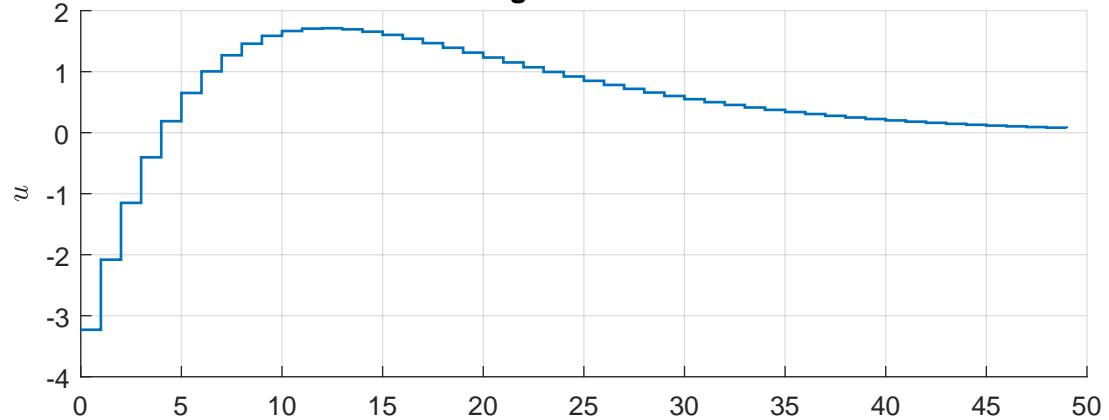
The same Q and R from problem 2 were used.

- b) Both states are similar in shape, although with full state feedback the response is somewhat faster. This is more visible in x_2 where the "dip" comes before $t=10$ with full state feedback whereas it comes after $t=10$ with state observer. The input is however less steep in the beginning which is interesting.

TTK4135 Assignment 7 problem 3a
MPC using output-feedback



TTK4135 Assignment 7 problem 3b
MPC using full state feedback



Problem 4

a) To calculate P , the following function was called

$$[K, \underline{P}] = \text{dlqr}(A, B, Q, R, [J]).$$

This gave

$$P = \begin{bmatrix} 55,03 & 14,54 \\ 14,54 & 20,47 \end{bmatrix}$$

$$b) z = \underbrace{[x_1^T \dots x_{M-1}^T]}_{M-1} \underbrace{\dots}_{M-1} \underbrace{Q_{M-1}}_{M-1} \underbrace{u_0^T \dots u_{M-1}^T}_{M-1}^T$$

M : time horizon

$$G = \begin{bmatrix} Q \\ \vdots \\ \overbrace{Q}^{M-1} P \\ \vdots \\ R \end{bmatrix}$$

Infinite horizon response is slightly faster, but reaches higher amplitude in x_2 . The input is only active at the start.

Decreasing N has little effect on the infinite horizon, but affects the finite horizon more.

TTK4135 Assignment 7 problem 4b
Infinite horizon MPC with full state feedback
vs finite horizon (problem 3b). N = 10

