



### Problem 1 (25 %) Second-Order Necessary Conditions

a

b

c

### Problem 2 (40 %) The Newton Direction

- a The Newton direction minimizes the model function  $m_k$ .
- b You need to find the rate of change in  $f$  in the direction  $p_k^N$  (if/when  $p_k^N$  is defined, and then determine whether this rate of change is positive or negative.
- c If you find an expression for the minimizer for this unconstrained problem and then find the Newton direction, you can show that the iteration sequence converges to the minimum regardless of  $x_k$ .
- d First, determine if the domain of the function is convex. Then, use the definition of a convex function. There will be a lot of algebra, so you can apply the function the left-hand side of the definition first, and then to the right-hand side. You will eventually arrive at an expression that makes it clear that  $f(x)$  is convex.

### Problem 3 (35 %) The Rosenbrock Function

When you show that the Hessian is positive definite it is probably easiest to show that all the leading principal minors are positive (sometimes called Sylvester's criterion, see a book on linear algebra or linear system theory). You can also find the eigenvalues (use MATLAB if you want, but you should be able to check for positive definiteness by hand on an exam).

The Rosenbrock function is plotted in Figure 1. The flatness and curviness of the valley makes this function a challenge for many algorithms.

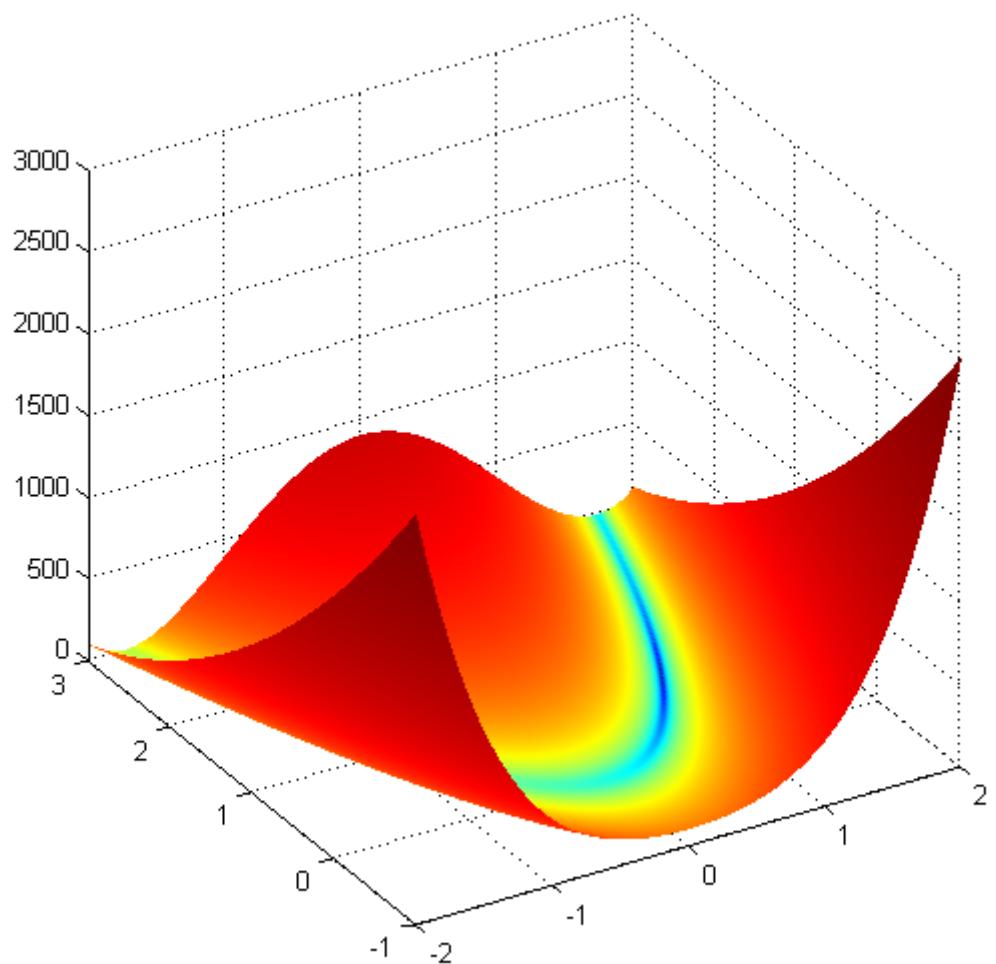


Figure 1: Surface plot of the Rosenbrock function.