Norwegian University of Science and Technology
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Exercise 10
NLP and SQP

Problem 1 (30 %) The QP approximation

When developing a local SQP method, we approximate the NLP

$$\min_x \quad f(x) \tag{1a}$$
$$\text{s.t.} \quad c(x) = 0 \tag{1b}$$

($f : \mathbb{R}^n \to \mathbb{R}$ and $c : \mathbb{R}^n \to \mathbb{R}^m$ are smooth functions) as the QP

$$\min_p \quad f_k + \nabla f_k^\top p + \frac{1}{2} p^\top \nabla^2_{xx} \mathcal{L}_k p \tag{2a}$$
$$\text{s.t.} \quad A_k p + c_k = 0 \tag{2b}$$

at the iterate (x_k, λ_k). See Section 18.1 in the textbook.

a Explain how we arrive at the approximation and derive both the objective function and the constraint in the quadratic program (2).

b State the KKT conditions for the quadratic program (2) as a matrix equation.

Problem 2 (40 %) Merit functions

In SQP we use a merit function to assess the quality of a step, combining the objective with a measure of constraint violation (see Section 15.4 in the textbook).

a Where in the SQP algorithm (Algorithm 18.3 in the textbook) do we use the merit function?

b Write down the merit functions ϕ_1, ϕ_2, and ϕ_F (Section 15.4).

c What is the purpose of the parameter μ? Does it change during the course of the SQP algorithm?

d Explain the concept *exact merit function*. Which of the three merit functions are exact?

e What is the Maratos effect? Which of the three merit functions mentioned above suffer from the Maratos effect?

f Will the merit function generally decrease from one iteration to the next? Explain.

g Will the objective function generally decrease from one iteration to the next? Explain. Is this different from or the same as what we are used to from unconstrained problems, LP problems, and QP problems?

Problem 3 (30 %) Feasibility and local solutions

Assume that Algorithm 18.3 is used to solve an NLP problem of the form

$$\min_{x} \quad f(x) \tag{3a}$$

$$\text{s.t.} \quad c_i(x) = 0, \quad i \in \mathcal{E} \tag{3b}$$

$$c_i(x) \geq 0, \quad i \in \mathcal{I} \tag{3c}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ and $c_i : \mathbb{R}^n \to \mathbb{R}$ are smooth functions.

a The user has to supply a starting point (x_0, λ_0). Is it important that x_0 is feasible? Explain. Is this different from or the same as what we are used to from LP problems and QP problems?

b Are all iterates (x_k, λ_k) feasible? Explain. Is this different from or the same as what we are used to from LP problems and QP problems?

c Under which conditions can the NLP problem (3) have multiple local solutions?

d Assume that we have a problem of the form (3) that we have solved with SQP. The solution x^* returned by the SQP algorithm does not impress you and you know the problem may have multiple local solutions. How would you try to find a better solution? Suggest a very simple method.