TTK4135 Assignment 5 Eirik Falck

Problem 1

$$x_{t+1} = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0{,}1 & -0{,}79 & 1{,}78 \end{bmatrix}}_{A} x_t + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0{,}1 \end{bmatrix}}_{B} u_t$$

$$y_t = \underbrace{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}}_{C} x_t \qquad t = 0, 1, \ldots, N-1$$

$$N = 30, \quad x_0 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

a) $$|\lambda I - A| = \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & -1 \\ -0{,}1 & 0{,}79 & \lambda - 1{,}78 \end{vmatrix} = \lambda(\lambda(\lambda - 1{,}78) + 0{,}79)$$

$$= \lambda(\lambda^2 - 1{,}78\lambda + 0{,}79) = 0$$

$$\lambda_1 = 0, \quad \lambda_2 = 0{,}936, \quad \lambda_3 = 0{,}844$$

All $\lambda_i < 1 \Rightarrow$ Stable

b) $x_t : 3 \times 1$ $\qquad u_t : 1 \times 1$ (scalar)

$$z = [x_1^T \cdots x_N^T, u_0^T \cdots u_{N-1}^T]^T$$

$$f(y_1, \ldots, y_N, u_0, \ldots, u_{N-1})$$

$$= \sum_{t=0}^{N-1} y_{t+1}^2 + r u_t^2 = \longrightarrow$$

$$= \frac{1}{2} \sum_{t=0}^{N-1} 2 y_{t+1}^T y_{t+1} + 2r \, u_t^T u_t$$

$$= \frac{1}{2} \sum_t 2 x_{t+1}^T C^T C x_{t+1} + u_t^T 2r \, u_t$$

$$= \frac{1}{2} \sum_{t=0}^{N-1} x_{t+1}^T Q x_{t+1} + u_t^T R u_t$$

where $Q = 2C^TC = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

and $R = 2r = 2 \quad (r=1)$

c) The problem is convex because $Q \succeq 0$ and $R > 0$ while also having linear constraints.

Hence, convexity depends on Q, R, A, B, and specific to this case, also C because $Q = 2C^TC$.

d) $\min_z \frac{1}{2} z^T G z \quad$ s.t. $\quad A_{eq} z = b_{eq}$.

$x_1 = Ax_0 + Bu_0 \rightarrow x_1 \quad - Bu_0 = Ax_0$

$x_2 = Ax_1 + Bu_1 \rightarrow -Ax_1 + x_2 \quad -Bu_1 = 0$

$$\Rightarrow A_{eq} = \left[\underbrace{\begin{matrix} I & 0 & \cdots & 0 & 0 \\ -A & I & \cdots & 0 & 0 \\ \vdots & & & & \\ \vdots & & & & \\ 0 & 0 & \cdots & -A & I \end{matrix}}_{\text{block: } N \times N} \; \vdots \; \underbrace{\begin{matrix} -B & 0 & \cdots & 0 \\ 0 & -B & \cdots & 0 \\ & & & \\ & & & \\ 0 & 0 & \cdots & -B \end{matrix}}_{\text{block } N \times N} \right], \quad b_{eq} = \left.\begin{bmatrix} Ax_0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right\} N \times 1$$

$$2f(z) = x_1^T Q x_1 + u_0^T R u_0 + x_2^T Q x_2 + u_1^T R u_1 + \cdots$$

$$= \begin{bmatrix} x_1^T & \cdots & x_N^T & u_0^T & \cdots & u_{N-1}^T \end{bmatrix} \begin{bmatrix} Q & & & & & \\ & \ddots & & & 0 & \\ & & Q & & & \\ & & & R & & \\ & 0 & & & \ddots & \\ & & & & & R \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \\ u_0 \\ \vdots \\ u_{N-1} \end{bmatrix}$$

(16.4):

$$\begin{bmatrix} G & -A_{eq}^T \\ A_{eq} & 0 \end{bmatrix} \begin{bmatrix} z^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} 0 \\ b_{eq} \end{bmatrix}$$

The optimal control sequence is open-loop because it only uses the last calculated state of the system to find the next state. Closed-loop would mean it uses the last measured state of the system. The advantages of feedback is that the controller can better adapt to the realities of the system and not solely rely on an approximated model.

e) A higher value for r gives a slower response/change between time steps in the input, and thus a slower response on y.

Lower value of r gives faster response to y, but also introduces some oscillations.

The optimization with quadprog only uses 1 iteration as we can expect from an equality constrained QP.

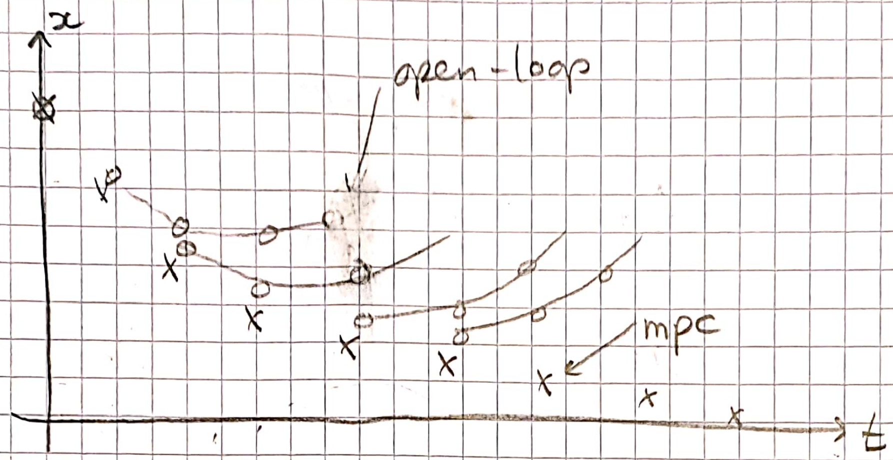
f) As seen in the figures below, the input is not free to be as large as it want, and the response is slower in all cases.

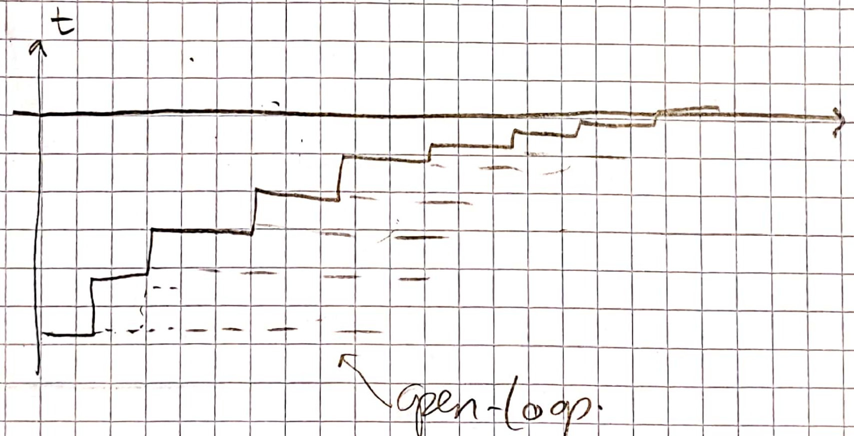
The algorithm also uses more iterations because this is now an inequality constrained QP.

Problem 2

at every timestep

a) The MPC principle states that one should calculate the open-loop optimization problem over a finite time horizon, using the current measured state as the initial condition. Then, the first control input in the optimal control sequence is applied to the system.





b) See below.

c) We see the imperfect model deviating some from the perfect model, but it manages to get close to zero by the end.