TTK4135 Assignment 7 Eirik Falck

$$x_{t+1} = \underbrace{\begin{bmatrix} 1 & 0{,}1 \\ -0{,}1 & 0{,}9 \end{bmatrix}}_{A} x_t + \underbrace{\begin{bmatrix} 0 \\ 0{,}1 \end{bmatrix}}_{B} u_t$$

$$x_0 = [5 \quad 1]^T \qquad \hat{x}_0 = [6 \quad 0]^T$$

Problem 1

$$P = Q + A^T P (I + BR^{-1}B^T P)^{-1} A \qquad (*)$$

$$(\underset{S}{I} + \underset{U}{B}\underset{T}{R^{-1}}\underset{V}{B^T P})^{-1} = I^{-1} - I^{-1}B\left[(R^{-1})^{-1} + B^T P I^{-1} B\right] B^T P I^{-1}$$

$$= I - B(R + B^T P B)^{-1} B^T P$$

$$\overset{(*)}{\Rightarrow} A^T P (I - B(R + B^T P B)^{-1} B^T P) A + Q - P = 0$$

$$= \underline{\underline{A^T P A - P - A^T P B (R + B^T P B)^{-1} B^T P A + Q = 0}}$$

Problem 2

$$y_t = \underbrace{[1 \quad 0]}_{C} x_t$$

a) $$f^{\infty}(z) = \frac{1}{2} \sum_{t=0}^{\infty} \hat{x}_{t+1}^T Q \hat{x}_{t+1} + u_t^T R u_t$$

$$Q = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \qquad R = 1$$

$$K = [1{,}0373 \qquad 1{,}6498]$$

CLeigs = 0,8675 ± 0,0531j

→ closed-loop eigenvalues.

b) Poles used: $0{,}5 \pm 0{,}03j$

The state estimate follows poorly in the start, but after about $t=10$ the state and estimate follows eachother perfectly.

c) $$\xi_{t+1} = \begin{bmatrix} x_{t+1} \\ \tilde{x}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A-BK & BK \\ 0 & A-K_f C \end{bmatrix}}_{\Phi} \xi_t$$

$$\tilde{x}_t = x_t - \hat{x}_t$$

$$\Phi = \begin{bmatrix} 1 & 0{,}1 & 0 & 0 \\ -0{,}204 & 0{,}735 & 0{,}104 & 0{,}165 \\ 0 & 0 & 0{,}1 & 0{,}1 \\ 0 & 0 & -1{,}609 & 0{,}9 \end{bmatrix}$$

The eigenvalues of Φ are the eigenvalues of $A-BK$ and $A-K_f C$.

Problem 3

$-4 \le u_t \le 4 \qquad t = 1, \dots, M-1$

a) $z = [\hat{x}_1^T \cdots \hat{x}_M^T \; u_0^T \cdots u_{M-1}^T]^T$

$$H = \begin{bmatrix} \underbrace{\begin{matrix} Q & & \\ & \ddots & \\ & & Q \end{matrix}}_{M} & \\ & \underbrace{\begin{matrix} R & & \\ & \ddots & \\ & & R \end{matrix}}_{M} \end{bmatrix}$$

$M = 10$
↳ time horizon

$$A_{eq} = \left[\begin{array}{c|c} \underbrace{\begin{matrix} I & & & & \\ -A & I & & & \\ & -A & I & & \\ & & \ddots & \ddots & \\ & & & -A & I \end{matrix}}_{M} & \underbrace{\begin{matrix} -B & & & \\ & -B & & \\ & & -B & \\ & & & \ddots \\ & & & & -B \end{matrix}}_{M} \end{array} \right]$$

$b_{eq} = [(A\hat{x}_0)^T \; 0 \; 0 \cdots 0]^T$

The same Q and R from problem 2 were used.

b) Both states are similar in shape, although with full state feedback the response is somewhat faster. This is more visible in x_2 where the "dip" comes before $t=10$ with full state feedback whereas it comes after $t=10$ with state observer.
The input is however less steep in the beginning which is interesting.

Problem 4

a) To calculate P, the following function was called

$$[k, \underline{\underline{P}}] = \mathrm{dlqr}(A, B, Q, R, [\,])$$

This gave

$$P = \begin{bmatrix} 55{,}03 & 14{,}54 \\ 14{,}54 & 20{,}47 \end{bmatrix}$$

b) $$z = [\underbrace{x_1^T \cdots x_{M-1}^T}_{M-1} \; x_M^T \; \underbrace{u_0^T \cdots u_{M-1}^T}_{M}]^T$$

M: time horizon

$$G = \begin{bmatrix} \underbrace{\begin{matrix} Q & & \\ & \ddots & \\ & & Q \end{matrix}}_{M-1} & & \\ & P & \\ & & \underbrace{\begin{matrix} R & & \\ & \ddots & \\ & & R \end{matrix}}_{M} \end{bmatrix}$$

Infinite horizon response is slightly faster, but reaches higher amplitude in x_2. The input is only active at the start.

Decreasing N has little effect on the infinite horizon, but affects the finite horizon more.