TTK4190 Guidance and Control of Vehicles

Assignment 2

Written Fall 2020 By Magne Aune, Håkon Stenwig, Eirik Daltveit, Marius Udnæs

Problem 1: Rigid-Body Kinetics of a Rectangular Prism

Problem 1a

Using the fact that the center of gravity is at the center of volume, the formula for moment of inertia yields

$$I_z^{CG} = \frac{1}{12}m(L^2 + B^2) = 3.7544 * 10^{10} \tag{1}$$

Due to symmetry we also have

$$I_{xy}^{CG} = I_{yx}^{CG} = 0 \tag{2}$$

Problem 1b

To find I_z^{CO} we use $r_{bg}^b = [x_g = -3.7; y_g = 0; z_g = H/2]$ and Steiner's theorem (3.1 Fossen):

$$I_z^{CO} = I_z^{CB} + m(x_g^2 + y_g^2) = 3.7310e * 10^{10} \tag{3}$$

The ratio between the prism and the the real moment of inertia is then: $ratio = 1.7168$

Problem 1c

Using equation (3.49) and (3.62) in Fossen we get

$$M_{RB} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & mx_g \\ 0 & mx_g & I_z \end{bmatrix}, \quad C_{RB} = \begin{bmatrix} 0 & -mr & -mx_g r \\ mr & 0 & 0 \\ mx_g r & 0 & 0 \end{bmatrix} \tag{4}$$

Problem 1d

To show that C_{RB} is skew-symmetric we use the skew-symmetric property $C^\intercal = -C$. This property holds for C_{RB} and thus it is skew-symmetric. This is favourable because it makes it easier to simplify expression.
The skew-symmetric property is very useful when designing a nonlinear motion control system since the quadratic form $\nu^\intercal C_{RB}(\nu)\nu \equiv 0$. This is exploited in energybased designs where Lyapunov functions play a key role.

Problem 1e

$S(\nu_1)\nu_2 = -S(\nu_2)\nu$
Since $S(\nu_1)\nu_1 = 0$ this gives an expression for $C_{RB}(\nu)$that is independent of linear velocity. This is useful when irrotational ocean currents are present.

Problem 2: Hydrostatics

Problem 2a

$$\nabla = L \cdot B \cdot T = 161 \cdot 21.8 \cdot 8.9 = 3.123 \cdot 10^4 \tag{5}$$

Problem 2b

$$A_{wp} = B \cdot L = 3.5098 \cdot 10^3 \tag{6}$$

$$Z_{hs} \approx -\rho g A_{wp} z^n \tag{7}$$

Problem 2c

$$T_3 \approx 2\pi\sqrt{\frac{2T}{g}} = 8.464 \tag{8}$$

Problem 2d

We use the following to compute metacenter heights
$I_T = \frac{1}{12}B^3L, \quad I_L = \frac{1}{12}L^3B$
$BG = H/2 - T/2$
$BM_T = \frac{I_T}{\nabla}, \quad BM_L = \frac{I_T}{\nabla}$
$GM_T = BM_T - BG$
$GM_L = BM_L - BG$

Final values computed
$GM_T = 0.9998, \quad GM_L = 239.2560$

Problem 2e

Since both values are positive we can conclude that the ship is metacentrically stable. Additionally we can say that both have a large enough margin that this stability is valid in a large enough region.

Problem 3: Added Mass and Coriolis

Problem 3a

M_A consisting of surge, sway, and yaw components only:

$$M_A = -\begin{bmatrix} X_{\dot{u}} & X_{\dot{v}} & X_{\dot{r}} \\ Y_{\dot{u}} & Y_{\dot{v}} & Y_{\dot{r}} \\ N_{\dot{u}} & N_{\dot{v}} & N_{\dot{r}} \end{bmatrix} = -\begin{bmatrix} X_{\dot{u}} & 0 & 0 \\ 0 & Y_{\dot{v}} & N_{\dot{v}} \\ 0 & N_{\dot{v}} & N_{\dot{r}} \end{bmatrix} \tag{9}$$

Problem 3b

From property 6.2 in Fossen we can extract C_A consisting of surge, sway, and yaw components only:

$$C_A(\nu) = \begin{bmatrix} 0 & 0 & a_2 \\ 0 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{10}$$

where

$$a_1 = X_{\dot{u}}u \tag{11}$$
$$a_2 = Y_{\dot{v}}v + Y_{\dot{r}}r \tag{12}$$

Problem 4: Implementing the Maneuvering Model in Matlab

Problem 4a

It runs

Problem 4c

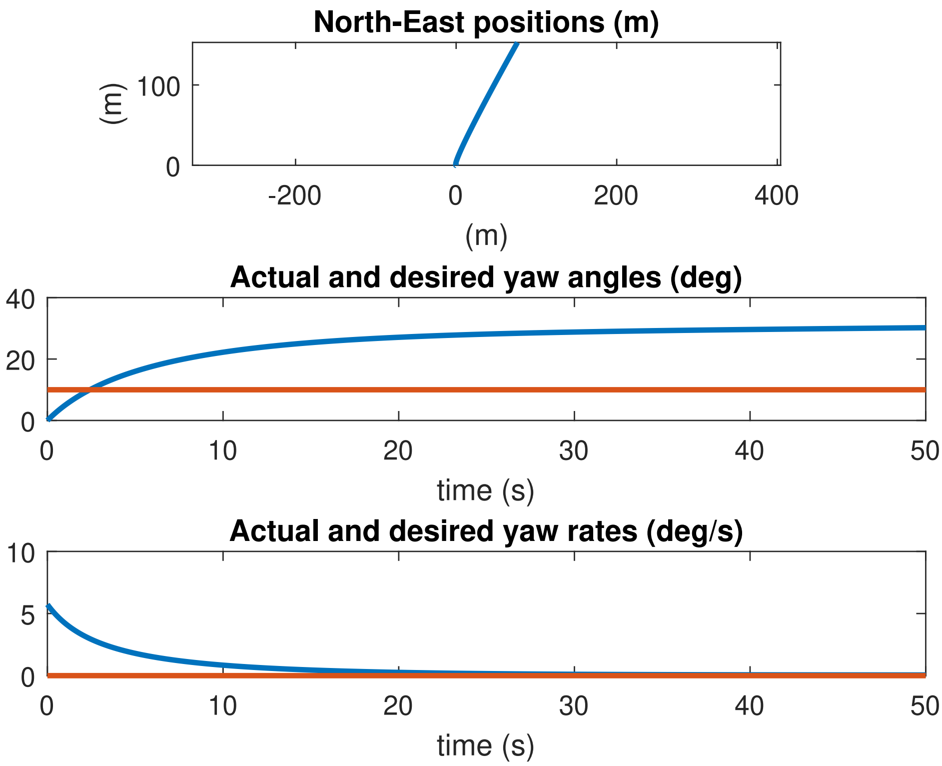
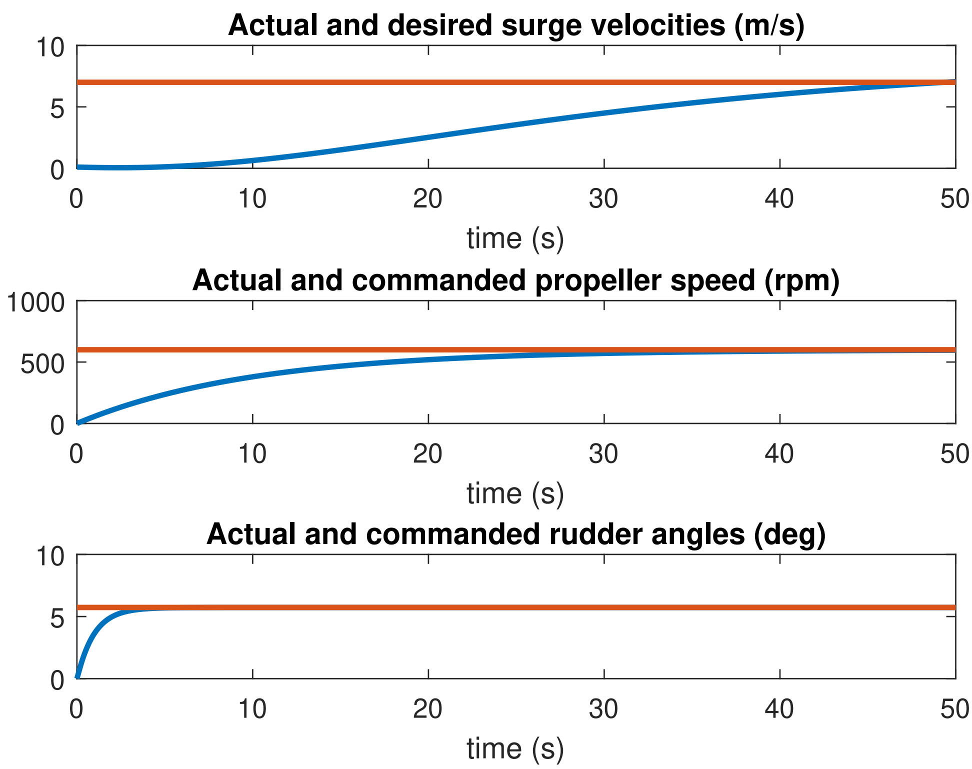

North-East positions (m)
(m)
100
0
-200
0
200
400
(m)
Actual and desired yaw angles (deg)
40
20
0
0
10
20
30
40
50
time (s)
Actual and desired yaw rates (deg/s)
10
5
0
0
10
20
30
40
50
time (s)

Figure 1



Actual and desired surge velocities (m/s)
10
5
0
0
10
20
30
40
50
time (s)
Actual and commanded propeller speed (rpm)
1000
500
0
0
10
20
30
40
50
time (s)
Actual and commanded rudder angles (deg)
10
5
0
0
10
20
30
40
50
time (s)

Figure 2

References