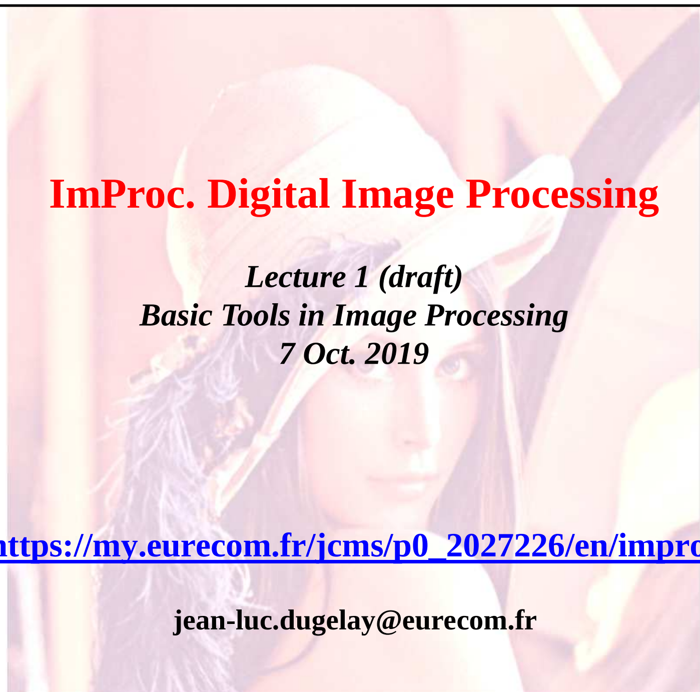

ImProc. Digital Image Processing

Lecture 1 (draft)
Basic Tools in Image Processing
7 Oct. 2019

https://my.eurecom.fr/jcms/p0_2027226/en/improc

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Typical Sequence

- **Pre-processing**
  - Noise reduction: low-pass Filtering
- **Processing**
  - Gradient maps + Threshold
  - Gradient based descriptor
- **Post-processing**
  - Edge thinning, closing
  - Outlier removal
- **Representation & Description**
  - Hough Transform

Spatial Masks

$$\begin{bmatrix} w_1 & w_2 & w_3 \\ (x-1,y-1) & (x-1,y) & (x-1,y+1) \\ w_4 & w_5 & w6 \\ (x,y-1) & (x,y) & (x,y+1) \\ w_7 & w_8 & w_9 \\ (x+1,y-1) & (x+1,y) & (x+1,y+1) \end{bmatrix}$$

$$T\left[f(x,y)\right] = w_1 f(x-1,y-1) + w_2 f(x-1,y)$$
$$+ w_3 f(x-1,y+1) + w_4 f(x,y-1)$$
$$+ w_5 f(x,y) + w_6 f(x,y+1) + w_7 f(x+1,y-1)$$
$$+ w_8 f(x+1,y) + w_9 f(x+1,y+1)$$

W1,…,W9: mask coefficents
8-neighbors of (x,y)

- ***size of the template***
- ***values of coefficients***

Direct Low pass filtering by averaging

averaging 2x2

$$\begin{bmatrix} \frac{\mathbf{1}}{\mathbf{4}} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{1} & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

averaging 3x3

Image smoothing
+ noise reduction
- edge smoothing

$$\sum_{p=0}^{k-1} \sum_{q=0}^{k-1} h(p,q) = 1$$

Low pass filtering by averaging

3x3

5x5 7x7

Median Filtering

Algorithm

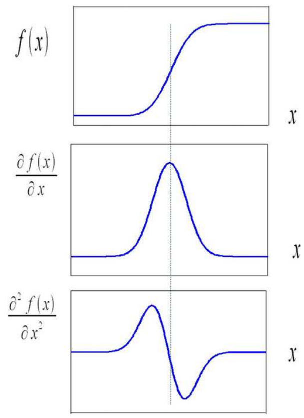
1. $Classify\ S = \{ f(x_j, y_j), (x_j, y_j) \in W \}$
2. $f'(x_i, y_i) = med(S)$

Non linear,

$$med(\alpha.I_1 + \beta.I_2) \neq \alpha.med(I_1) + \beta.med(I_2)$$

Useful in preserving edges while reducing noise (image smoothing)

Edges : Gradient & Laplacian



$f(x,y)$

$$\nabla f(x,y) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}$$

$$|\nabla f(x,y)| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

$$\nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

Image Sharpening

Gradient Pattern

- Gradient

$$G_x \begin{bmatrix} 0 & 0 & 0 \\ 1 & \mathbf{0} & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad G_y \begin{bmatrix} 0 & 1 & 0 \\ 0 & \mathbf{0} & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & \mathbf{0} & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Prewitt

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & \mathbf{0} & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Sobel

if $|\nabla f(x,y)| > Threshold \ at \ (x_0, y_0)$
then (x_0, y_0) is an edge point
else (x_0, y_0) is not an edge point

Gradient Pattern

$$\begin{bmatrix} 10 & 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 & 10 \\ 0 & 0 & 0 & 0 & 0 \\ -10 & -10 & -10 & -10 & -10 \\ -10 & -10 & -10 & -10 & -10 \end{bmatrix}$$

0°

$$\begin{bmatrix} 10 & 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 6 & 2 \\ 8 & 4 & 0 & -4 & -8 \\ -2 & -6 & -10 & -10 & -10 \\ -10 & -10 & -10 & -10 & -10 \end{bmatrix}$$

22.5°

$$\begin{bmatrix} 10 & 10 & 10 & 10 & 0 \\ 10 & 10 & 10 & 0 & -10 \\ 10 & 10 & 0 & -10 & -10 \\ 10 & 0 & -10 & -10 & -10 \\ 0 & -10 & -10 & -10 & -10 \end{bmatrix}$$

45°

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Questions

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{1} & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

If we apply it two times?

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & \mathbf{4} & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

2D filtering or 2 times 1D?

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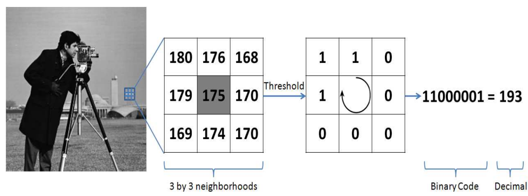
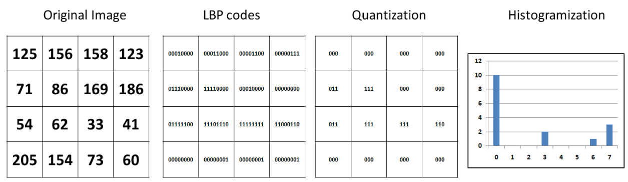
Gradient based descriptor: SURF, FAST, DAISY, SIFT, etc.
Local binary Pattern (LBP)



$$LBP = \sum_{p=0}^{p-1} s(g_p - g_c)2^p$$

$$s(x) = \begin{cases} 1\, if \;\; x \geq 0 \\ 0, otherwise \end{cases}$$



Widely used to compare two face images

$$\chi^2(S,M) = \sum_i^n \frac{(S_i - M_i)^2}{S_i + M_i}$$

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Laplacian Pattern

- Laplacian

$$\begin{bmatrix} 0 & +1 & 0 \\ +1 & \mathbf{-4} & +1 \\ 0 & +1 & 0 \end{bmatrix}$$

Edges: zero-crossing points

$$\nabla^2 f\left[(x,y)\right] \approx f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

$$\frac{\partial f}{\partial x}(x,y) \cong \frac{f(x+a,y) - f(x-b,y)}{a+b}$$

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Basic question on Filtering

- *What are the differences between averaging and median filters (e.g. of size 3x3) in terms of implementation and impact?*

Hough Transform

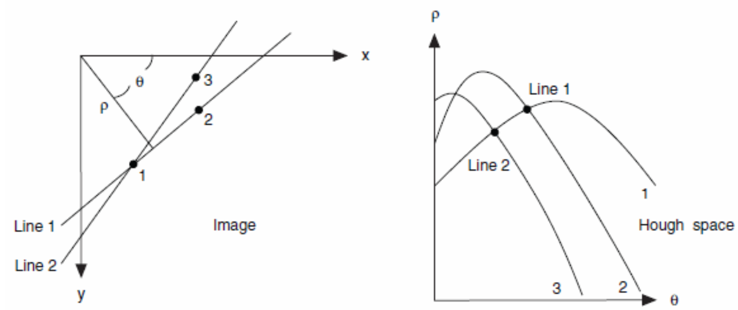
Originally designed for Line detection
but can also be used for any analytical curve (circle, ellipse, etc.)

IMAGE ***J*** $\rightleftarrows$ HOUGH ***H***

Image Space defined by $\{M_i\ (x_i, y_i)\}$
A set of pixels defines a curve described by a parametric equation,
$f(a_1, a_2, \ldots, a_n, x, y) = 0$
Ex.

Line $\{y = a.x + b$ or $\rho = x.\cos(\theta) + y.\sin(\theta)\}$: 2 parameters (a,b) or (ρ, θ)
Circle $\{(x - a)^2 + (y - b)^2 = c^2\}$: 3 parameters (center (a,b) and radius c, or (a, w, R)}

Hough Transform (Cont.)



A point in H corresponds to a straight line in I
A point in I corresponds to a sinusoid in H

Points on the same line in I give curves passing through a common point in H
Points on the same curve in H give lines passing through a common point in I

Hough Transform (Cont.)

Several kind of transformations:

- **From m to 1**
  Any "m-uplets" from the image space is associated to a parametric curve $\{a_i\}$ in the Hough space;
- **From 1 to n**
  Any pixel (x_i,y_i) from the image space is associated to m parametric curves $\{a_i\}$ in the Hough space;
- **From m to m'**
  Any "m-uplets" from the image space is associated to m' parametric curves $\{a_i\}$ in the Hough space;

Hough Transform (Cont.)

Line detection : from m to 1

- m=2

$M_i(x_i,y_i)$ and $M_j(x_j,y_j)$

$$\rho_k = \frac{|x_i y_j - x_j y_i|}{\sqrt{(y_j - y_i)^2 + (x_j - x_i)^2}} \qquad \theta_l = -\mathrm{Arctg}\,\frac{x_j - x_i}{y_j - y_i}$$

Hough Transform (Cont.)

Line detection : from 1 to m

Algo.

A- Partition H into cells ACC(ρ_k, θ_l) initialized to 0;
B- For each pixel $M_i(x_i,y_i)$ in I,
do ACC(ρ_k, θ_l) $\leftarrow$ ACC(ρ_k, θ_l) +1 if $f(\rho_k,\theta_l,x_i,y_i) \approx 0$ is verified
C- Hence, for a given cell, that is to say ACC(ρ_m, θ_n),
its value is incremented as many times as a pixel is on the straight line (ρ_m, θ_n) ;
D- If ACC(ρ_m, θ_n) > Threshold in H, then a straight line (ρ_m, θ_n) in I is detected.

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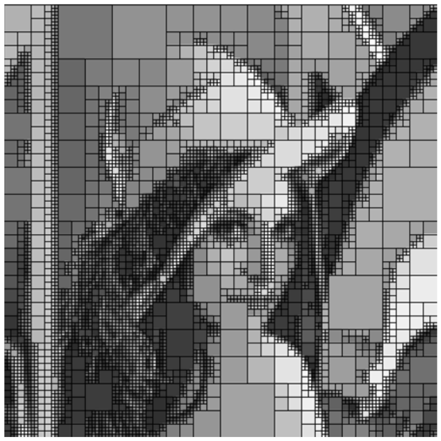
Quad tree



Bounding box

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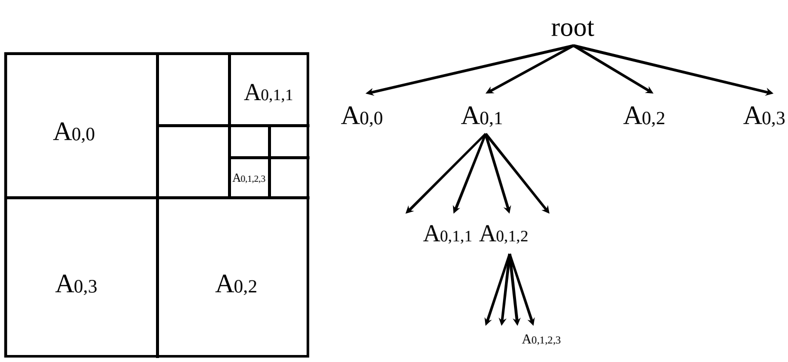
Quad tree



Split & Merge segmentation

- Split:

Split the picture into smaller and smaller areas until reaching a given uniformity criterion;

- Merge:

Merge neighboring areas according to a similarity criterion (mean, variance, etc.);

- Split & Merge:

Combination of both previous approaches

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Split

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	2
1	1	1	1	1	2	2	2
1	1	1	1	2	2	2	2
1	1	1	1	1	2	2	2
1	1	1	1	1	2	2	2
2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2

$$TEST \quad : E(R_l) = \frac{1}{card\ [R_l]} \sum_{x_i \in R_l} \left(g(x_i) - m(R_l) \right)^2 < Threshold \quad \_1$$

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Split

<table>
<tr><td colspan="2" rowspan="2">1.0</td><td>1.0</td><td>1.3</td></tr>
<tr><td>1.8</td><td>2.0</td></tr>
<tr><td>1.0</td><td>1.0</td><td colspan="2" rowspan="2">1.9</td></tr>
<tr><td>2.0</td><td>2.0</td></tr>
</table>

$$TEST \quad : E(R_l) = \frac{1}{card\ [R_l]} \sum_{x_i \in R_l} \left(g(x_i) - m(R_l) \right)^2 < Threshold \quad \_\ 1$$

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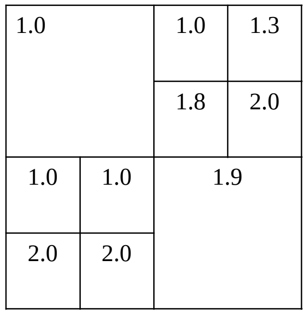
Split

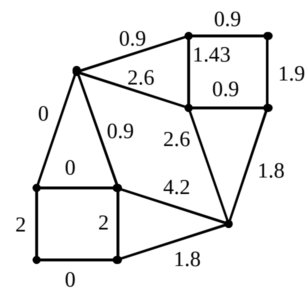
0	0	0	0	0	0	.3	.3
0	0	0	0	0	0	.3	.7
0	0	0	0	.8	.2	0	0
0	0	0	0	.2	.2	0	0
0	0	0	0	.9	.1	.1	.1
0	0	0	0	.9	.1	.1	.1
0	0	0	0	.1	.1	.1	.1
0	0	0	0	.1	.1	.1	.1

$$TEST \quad : E(R_l) = \frac{1}{card\ [R_l]} \sum_{x_i \in R_l} \left(g(x_i) - m(R_l) \right)^2 < Threshold \quad \_\ 1$$

Merge

Region adjacency graph





$$E(R_l,R_k)=\sum_{x_i\in R_l\cup R_k}\left(g(x_i)-\mu(R_l,R_k)\right)^2$$

$$\mu(R_l,R_k)=\frac{1}{\mathrm{card}\{R_l\cup R_k\}}\sum_{x_i\in R_l\cup R_k} g(x_i)$$

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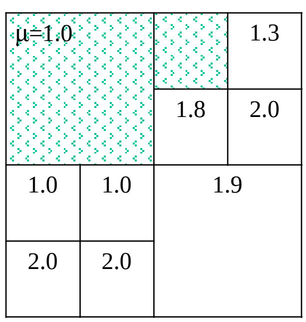
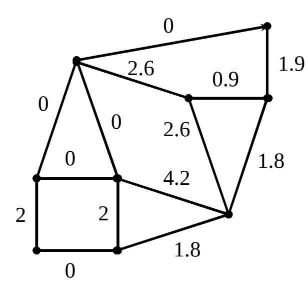
Merge

Region adjacency graph





$$E(R_l,R_k)=\sum_{x_i\in R_l\cup R_k}\left(g(x_i)-\mu(R_l,R_k)\right)^2$$

$$\mu(R_l,R_k)=\frac{1}{\mathrm{card}\{R_l\cup R_k\}}\sum_{x_i\in R_l\cup R_k} g(x_i)$$

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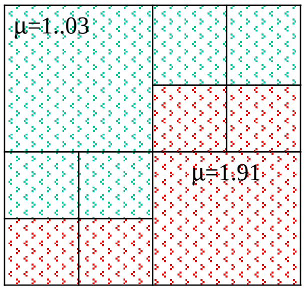
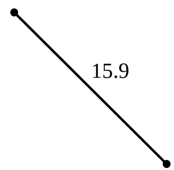
$$SEQ(L) = \sum_{l=1}^{L} \sum_{x_i \in R_l} (g(x_i) - \mu(R_l))^2 > Threshold\_2$$

Merge

Region adjacency graph





$$E(R_l, R_k) = \sum_{x_i \in R_l \cup R_k} (g(x_i) - \mu(R_l, R_k))^2$$

$$\mu(R_l, R_k) = \frac{1}{card\{R_l \cup R_k\}} \sum_{x_i \in R_l \cup R_k} g(x_i)$$

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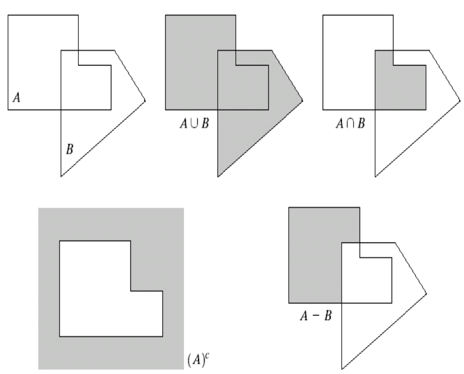
Math. Morpho.: Intro.



union

intersection

complementary

inclusion

difference

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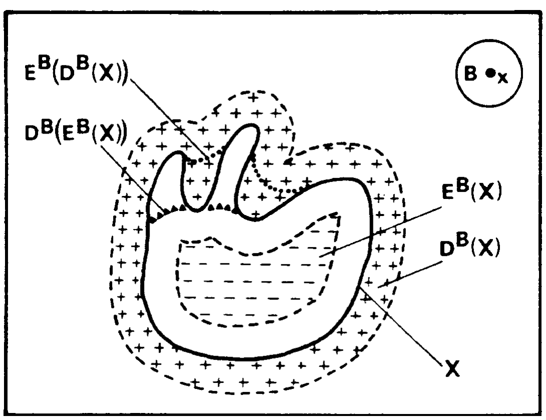
Math. Morpho.: Basic operations

- Object A
- Structuring element B_p
- EROSION $er(A, B_p) \equiv \{p | B_p \subset A\}$
- DILATATION $dil(A, B_p) \equiv \{p | B_p \cap A \neq \emptyset\}$
- OPENING dil(er(A,B_p),B_p)
- CLOSING er(dil(A,B_p),B_p)

Math. Morpho.: Some properties (Cont.)



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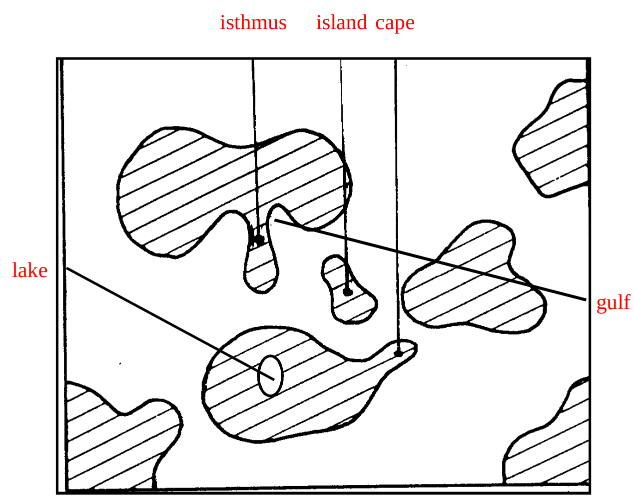
Math. Morpho.: Intro.(Cont.)



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Math. Morpho.: Some properties

- Translation invariant
- Erosion & dilatation are not inverses of each other

Duality: $dil(A, B_p) = [er(A^c, B_p)]^c$

- Increasing: $X \subset X' \Rightarrow T(X) \subset T(X')$
- ...

Opening: cuts narrow isthmus;
removes small islands and narrow capes.
Closing: fills narrow canals;
removes small lakes and narrow gulfs.

It is more severe to erode prior to dilate than the contrary.

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Erosion

Original image

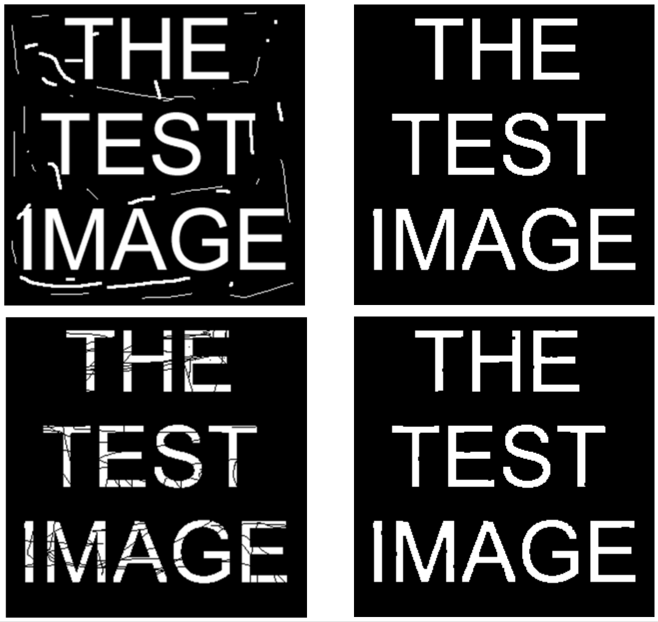
Eroded image

Erosion

Eroded once

Eroded twice

Opening and Closing



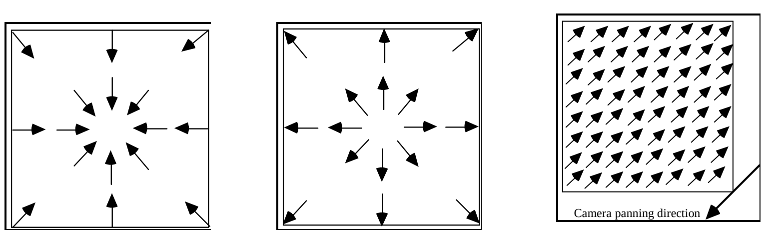
OPENING: The original image eroded twice and dilated twice (opened). Most noise is removed

CLOSING: The original image dilated and then eroded. Most holes are filled.

Optical Flow, or apparent motion field

relative motion between an observer (an eye or a camera) and the scene

Motion vector pattern
resulting from camera panning and zooming



zoom out | zoom in | Camera panning and tilting

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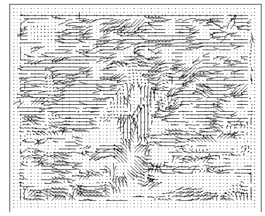
http://extra.cmis.csiro.au/IA/changs/motion/



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Apparent motion

Optical flow

between two consecutive image frames taken at t and t'(t + δt)

STEP 1. Local Estimation:

displacement: $d_i\left(d_{x_i}, d_{y_i}\right)$ ***for each pixel:*** $p_i(x_i, y_i)$

$$\begin{cases} d_{x_i} \equiv x'_i - x_i \\ d_{y_i} \equiv y'_i - y_i \end{cases}$$

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Apparent motion (Cont.)

STEP 2. Global Interpretation

$$\{z\_f, pan_x, pan_y\}:$$

$$\forall i, \begin{pmatrix} x'_i \\ y'_i \end{pmatrix} = z\_f. \begin{pmatrix} x_i \\ y_i \end{pmatrix} + \begin{pmatrix} pan_x \\ pan_y \end{pmatrix}$$

parameter	**name**	**interpretation**
Z_f	***zoom factor***	IF z_f is closed to 0 THEN no zoom ELSE Z_f>> 1 => backward zoom Z_f<< 1 => forward zoom
t_x	***vertical pan***	mean vertical displacement in pixels of the whole image
t_y	***horizontal pan***	mean horizontal displacement in pixels of the whole image

Estimation of the Optical Flow

Hypothesis: the luminance of a pixel is constant over time.

$$I(x,y,t) = I(x+\delta x, y+\delta y, t+\delta t)$$

Assuming that δx and δy are smalls

I(x,y,t) with Taylor series can be developed to get:

$$I(x+\delta x, y+\delta y, t+\delta t) = I(x,y,t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t + H.O.T.$$

$$\frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t = 0 \qquad \frac{\partial I}{\partial x}Vx + \frac{\partial I}{\partial y}Vy = -\frac{\partial I}{\partial t}$$

- Technique based on a relation between spatial and temporal gradients

(i.e. derivatives of the image at (x,y,t)

- 1 equation – 2 unknowns
- Compute V_x and V_y via an iterative process:

$$V^{(i)} = V^{(i-1)}(p,t) - \varepsilon.DFD(p,t,V^{(i-1)})\nabla I(x - V_x^{(i-1)}, y - V_y^{(i-1)}, t-1)$$

$d^{(i)} = d^{(i-1)}(p,t) - \varepsilon.\mathrm{sign}\{DFD(p,t,d^{(i-1)})\}\mathrm{sign}\{\nabla I(x - d_x^{(i-1)}, y - d_y^{(i-1)}, t-1)\}$

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Optical flow: basic illustration

(iteration #1)

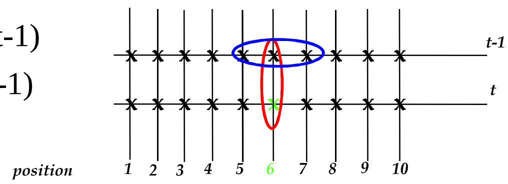
Pixel #6 at time t (from which position at t-1 does this pixel come from ?)

Assumption: $d^{(0)} = 0$

Gradients

Temporal I(6,t) - I(6,t-1)

Spatial I(7,t-1) - I(5,t-1)



Gain: $\varepsilon = 1$

Temporal > 0

Spatial > 0

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$d^{(i)} = d^{(i-1)}(p,t) - \varepsilon.\mathrm{sign}\{DFD(p,t,d^{(i-1)})\}\mathrm{sign}\{\nabla I(x - d_x^{(i-1)}, y - d_y^{(i-1)}, t-1)\}$

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Optical flow: basic illustration (Cont.)

(iteration #2)

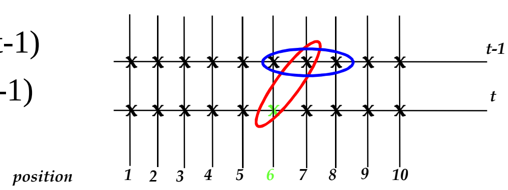
Pixel #6

$d^{(1)} = d^{(0)} - 1 = -1$

Gradients

Temporal I(6,t) - I(7,t-1)

Spatial I(8,t-1) - I(6,t-1)



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