Assignment 2

Eirik Olav

① $P(x_0) = 0.5$

$P(X_{t+1} \mid X_t) = 0.8$

$P(X_{t+1} \mid \neg X_t) = 0.3$

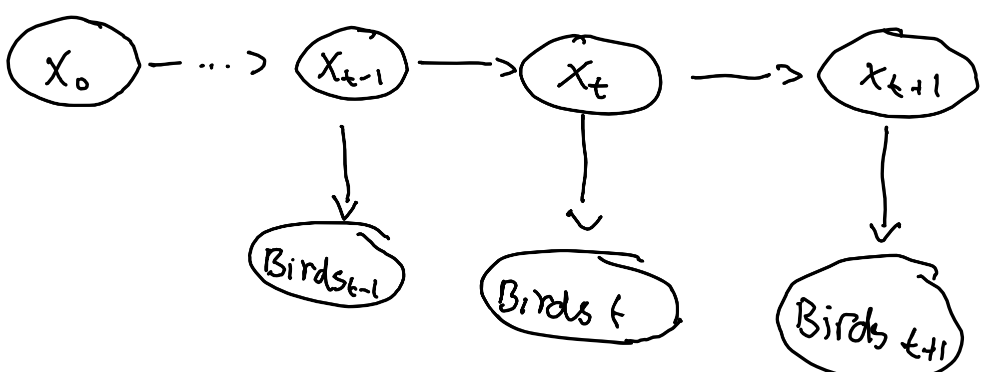
$P(e \mid X_t) = 0.75$

$P(e \mid \neg X_t) = 0.2$

F = Fish in nearby lake

	$F\ X_{t-1}$	$\neg F\ X_{t-1}$
$F\ X_t$	0.8	0.3
$\neg F\ X_t$	0.2	0.7

	X_t	$\neg X_t$
e	0.75	0.2
$\neg e$	0.25	0.8



Eirik Olav Aa – Assignment 2

Task 1
b)

Task 1b was a filtering operation. Computing $P(X_t|e_{1:t})$, for t = 1, ..., 6.

The output was

	P(Fish)	P(No Fish)
X0	[0.5	0.5]
X1	[0.82089552	0.17910448]
X2	[0.90197069	0.09802931]
X3	[0.48518523	0.51481477]
X4	[0.81645924	0.18354076]
X5	[0.43134895	0.56865105]
X6	[0.79970863	0.20029137]

This is a filtering operation which calculate the probability of being in one state at each step given the evidence that leads up to that step.

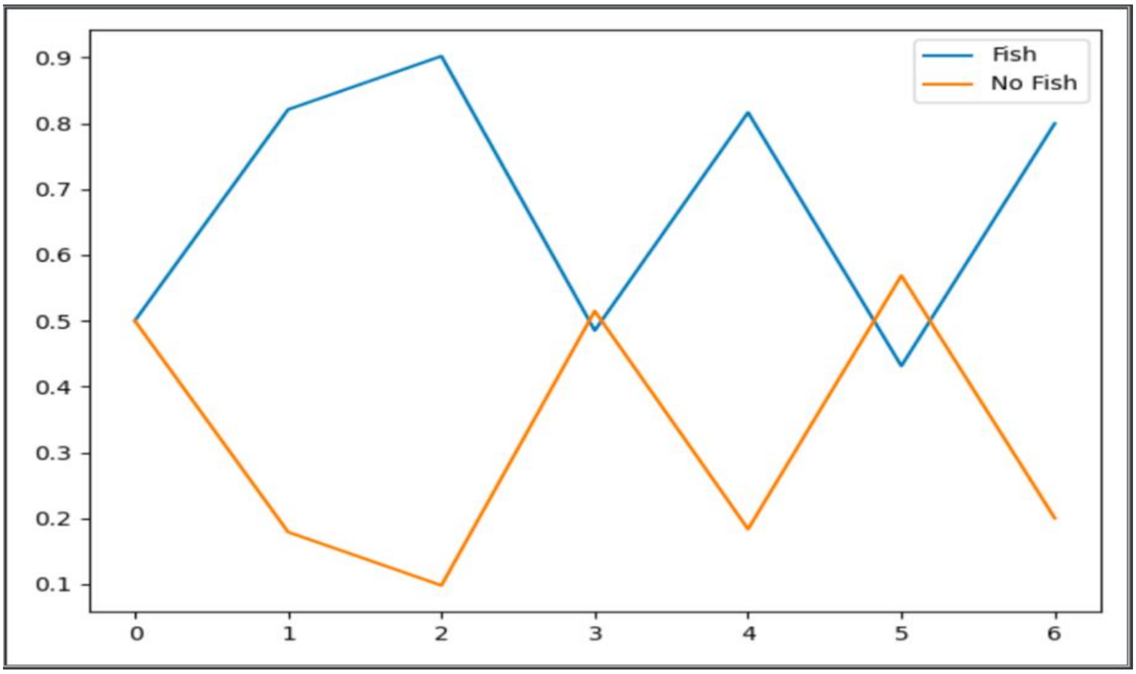


Figure 1: Graph representation of task 1b

c)
Task 1c was a prediction operation.

$P(X_t|e_{1:6})$, for t = 7, ..., 30.

	P(Fish)	P(No Fish)
X06	[0.64992716	0.35007284]
X07	[0.62496358	0.37503642]
X08	[0.61248179	0.38751821]
X09	[0.60624089	0.39375911]
X10	[0.60312045	0.39687955]
X11	[0.60156022	0.39843978]
X12	[0.60078011	0.39921989]
X13	[0.60039006	0.39960994]
X14	[0.60019503	0.39980497]
X15	[0.60009751	0.39990249]
X16	[0.60004876	0.39995124]
X17	[0.60002438	0.39997562]
X18	[0.60001219	0.39998781]
X19	[0.60000609	0.39999391]
X20	[0.60000305	0.39999695]
X21	[0.60000152	0.39999848]
X22	[0.60000076	0.39999924]
X23	[0.60000038	0.39999962]
X24	[0.60000019	0.39999981]
X25	[0.6000001	0.3999999]
X26	[0.60000005	0.39999995]
X27	[0.60000002	0.39999998]
X28	[0.60000001	0.39999999]
X29	[0.60000001	0.39999999]
X30	[0.6	0.4]

This is a prediction operation. Based on the evidence given to the current date, the operation calculates the probabilities for future events. The probability converges towards 60% chance of fish in a nearby lake.

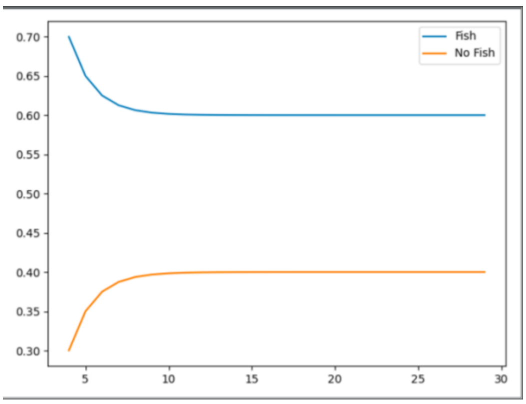


Figure 2: Graph representation of task 1c

d)

Task 1d was a smoothing operation.

$P(X_t|e_{1:6})$, for t = 0, ..., 5.

	P(Fish)	P(No Fish)
X0	[0.66485218	0.33514782]
X1	[0.87640731	0.12359269]
X2	[0.86578657	0.13421343]
X3	[0.59792735	0.40207265]
X4	[0.76663731	0.23336269]
X5	[0.57082582	0.42917418]

A smoothing is a process of computing the distribution of earlier states given the evidence up to present. This is achieved by computing the likelihood of the evidence.

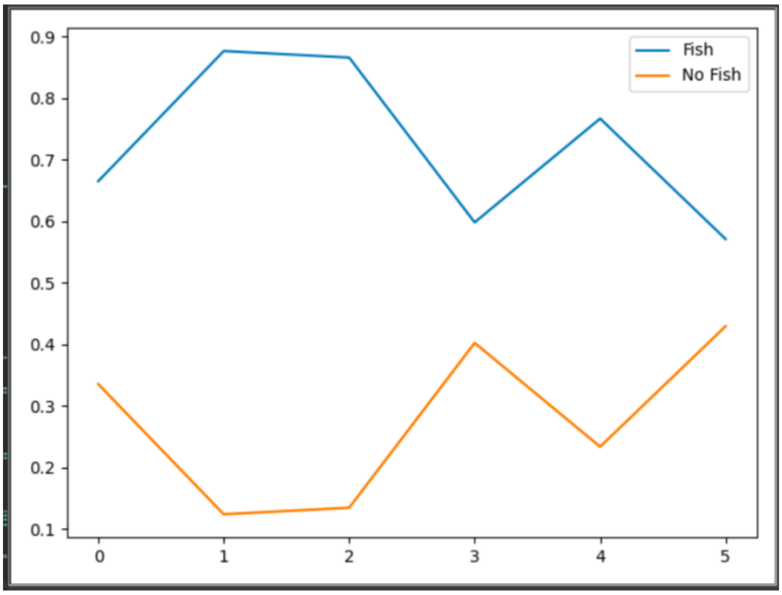


Figure 3: Graph representation of task 1d

e)

Task 1e was finding most likely path

arg max $_{x1,...,xt-1}$ $P(x_1, ..., x_{t-1}, X_t|e_{1:t})$, for t = 1, ..., 6.

Probability of fish/no fish

	P(Fish)	P(No Fish)	Parents
X1	[0.82089552	0.17910448]	[0. 0.]
X2	[0.49253731	0.03283582]	[0. 0.]
X3	[0.09850746	0.07880597]	[0. 0.]
X4	[0.05910448	0.01103284]	[0. 1.]
X5	[0.0118209	0.00945672]	[0. 0.]
X6	[0.00709254	0.00132394]	[0. 1.]

By observing the results, the most likely path ends in X6 in Fish. The parent of this node is Fish, and parent of this is Fish… All the parents are in the left column because every previous selected node has 0 as parent.

The most likely path is therefore:

Fish – Fish – Fish – Fish – Fish – Fish

The probability of this is 0.709%.

Assignment 2

Eirik Olav

② a)

$P(X_0) = 0.7$ $\quad P(X_{t+1} | X_t) = 0.8$

$P(X_{t+1} | \neg X_t) = 0.3$

e_{ta} = animal tracks $\quad e_{tf}$ = food gone

$P(e_{ta} | X_t) = 0.7 \quad P(e_{ta} | \neg X_t) = 0.2$

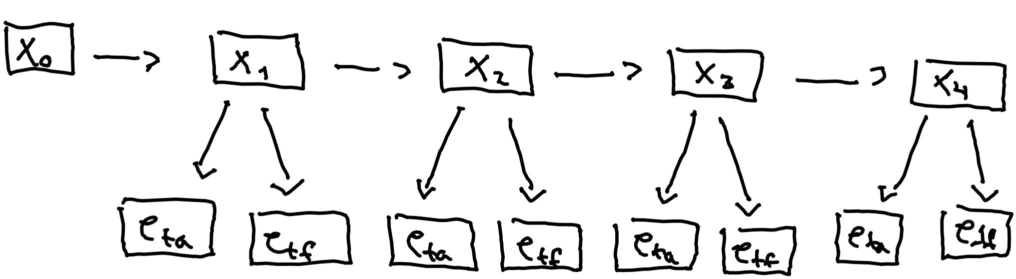
$P(e_{tf} | X_t) = 0.3 \quad P(e_{tf} | \neg X_t) = 0.1$

a_n = animals nearby

	$a_n \; X_{t-1}$	$\neg a_n \; X_{t-1}$
$a_n \; X_t$	0.8	0.3
$\neg a_n \; X_t$	0.2	0.7

	X_t	$\neg X_t$
e_{ta}	0.7	0.2
$\neg e_{ta}$	0.3	0.8

	X_t	$\neg X_t$
e_{tf}	0.3	0.1
$\neg e_{tf}$	0.7	0.9



b)

$P(X_t | e_{1:t})$ for $t = 1, 2, 3, 4$

$P(X_1 | e_1) = P(e_1 | X_1) \cdot P(X_1)$

$P(X_1) = \sum P(X_1 | X_0) \cdot P(X_0)$

$= \langle 0.8, 0.2 \rangle \cdot 0.7 + \langle 0.3, 0.7 \rangle \cdot 0.3$

$= \langle 0.56, 0.14 \rangle + \langle 0.09 + 0.21 \rangle = \langle 0.65, 0.35 \rangle$

$P(X_1 | e_1) = \langle 0.3, 0.1 \rangle \langle 0.7, 0.2 \rangle \langle 0.65, 0.35 \rangle \alpha$

$= \langle 0.1365, 0.007 \rangle \alpha \approx \underline{\langle 0.951, 0.049 \rangle}$

e_1

$P(X_2 | e_1) = \sum P(X_2 | X_1) P(X_1 | e_1)$

$\langle 0.8, 0.2 \rangle \cdot 0.951 + \langle 0.3, 0.7 \rangle \cdot 0.049$

$\langle 0.7608, 0.1902 \rangle + \langle 0.0147, 0.0343 \rangle$

$\langle 0.7755, 0.2245 \rangle$

$$P(X_2 \mid e_{1:2}) = \alpha P(e_2 \mid X_2) P(X_2 \mid e_1)$$

$$\boxed{e_2 = \{\text{no animal tracks, food gone}\}}$$

$$= \alpha \langle 0.3, 0.8 \rangle \langle 0.3, 0.1 \rangle \langle 0.7755, 0.2245 \rangle$$

$$= \alpha \langle 0.069795, 0.01796 \rangle$$

$$= \langle 0.7954, 0.2046 \rangle$$

$$P(X_3 \mid e_{1:2}) = \sum P(X_3 \mid X_2) P(X_2 \mid e_{1:2})$$

$$= \langle 0.8, 0.2 \rangle \cdot 0.7954 + \langle 0.3, 0.7 \rangle \cdot 0.2046$$

$$= \langle 0.6363, 0.159 \rangle + \langle 0.0614, 0.14322 \rangle$$

$$= \langle 0.6977, 0.3023 \rangle$$

$$P(X_3 \mid e_{1:3}) = \alpha P(e_3 \mid X_3) P(X_3 \mid e_{1:2})$$

$$\boxed{e_3 = \{\text{no animal tracks, food not gone}}$$

$$= \alpha \langle 0.3, 0.8 \rangle \langle 0.7, 0.9 \rangle \langle 0.6977, 0.3023 \rangle$$

$$= \alpha \langle 0.1465, 0.2176 \rangle$$

$$= \langle 0.4023, 0.5976 \rangle$$

$$P(X_4 | e_{1:3}) = \sum P(X_4 | X_3) \cdot P(X_3 | e_{1:3})$$

$$= \langle 0.8, 0.2 \rangle \cdot 0.4023 + \langle 0.3, 0.7 \rangle 0.5976$$

$$\approx \langle 0.501, 0.499 \rangle$$

$$P(X_4 | e_{1:4}) = \alpha P(e_4 | X_4) \cdot P(X_4 | e_{1:3})$$

$$\boxed{e_4 = \{\text{animal tracks, food not gone}\}}$$

$$= \alpha \langle 0.7, 0.2 \rangle \langle 0.7, 0.9 \rangle \langle 0.501, 0.499 \rangle$$

$$= \alpha \langle 0.2455, 0.0898 \rangle$$

$$= \underline{\langle 0.7322, 0.2678 \rangle}$$

c)

$P(x_t | e_{1:t})$ $t = 5,6,7,8$

$P(X_5 | e_{1:4}) = \sum P(X_5 | X_4) P(X_4 | e_{1:4})$

$= \langle 0.8, 0.2 \rangle 0.7322 + \langle 0.3, 0.7 \rangle \cdot 0.2678$

$= \langle 0.6661, 0.3339 \rangle$

$P(X_6 | e_{1:4}) = \sum P(X_6 | X_5) P(X_5 | e_{1:4})$

$\langle 0.8, 0.2 \rangle 0.6661 + \langle 0.3, 0.7 \rangle \cdot 0.3339$

$\langle 0.629, 0.371 \rangle$

$P(X_7 | e_{1:4}) = \sum P(X_7 | X_6) P(X_6 | e_{1:4})$

$\langle 0.8, 0.2 \rangle \cdot 0.629 + \langle 0.3, 0.7 \rangle \cdot 0.371$

$\langle 0.6145, 0.3855 \rangle$

$P(X_8 | e_{1:4}) = \sum P(X_8 | X_7) P(X_7 | e_{1:4})$

$\langle 0.8, 0.2 \rangle 0.6145 + \langle 0.3, 0.7 \rangle 0.3855$

$\underline{\langle 0.6073, 0.3927 \rangle}$

d) Calculate a few more steps to verify that the probability converges towards $\langle 0.6, 0.4 \rangle$.

$$P(X_9 | e_{1:4}) = \sum P(X_9 | X_8) P(X_8 | e_{1:4})$$

$$\langle 0.8, 0.2 \rangle \cdot 0.6073 + \langle 0.3, 0.7 \rangle \cdot 0.3927$$

$$= \langle 0.6036, 0.3964 \rangle$$

$$P(X_{10} | e_{1:4}) = \sum P(X_{10} | X_9) P(X_9 | e_{1:4})$$

$$\langle 0.8, 0.2 \rangle \cdot 0.6036 + \langle 0.3, 0.7 \rangle \cdot 0.3964$$

$$= \langle 0.6018, 0.3982 \rangle$$

Observes that the probability converges towards $\langle 0.6, 0.4 \rangle$

The math to calculate the next X_t is essentialy multiply the matrix

$\begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$ with the previous result

with $\langle 0.6, 0.4 \rangle$ as the previous result

$\begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$ => <u>Converges</u>

e)

$P(X_t \mid e_{1:4})$ for $t=0,1,2,3$

Smoothing:

$P(X_t \mid e_{1:4}) = \alpha\, f_{1:t}\, b$

$b_{t+1:4} = P(e_{t+1:4} \mid X_t)$

$P(e_4 \mid X_3) = \sum P(e_4 \mid X_4) \cdot P(e_{5:4} \mid X_4) P(X_4 \mid X_3)$

$= \langle 0.7 \cdot 0.7 \cdot 0.8 + 0.2 \cdot 0.9 \cdot 0.2,$

$0.7 \cdot 0.7 \cdot 0.3 + 0.2 \cdot 0.9 \cdot 0.7 \rangle$

$= \langle 0.428, 0.273 \rangle$

$P(e_{3:4} \mid X_2) = \sum P(e_3 \mid X_3) P(e_4 \mid X_3) P(X_3 \mid X_2)$

$= \langle 0.3 \cdot 0.7 \cdot 0.428 \cdot 0.8 + 0.8 \cdot 0.9 \cdot 0.273 \cdot 0.2,$

$0.3 \cdot 0.7 \cdot 0.428 \cdot 0.3 + 0.8 \cdot 0.9 \cdot 0.273 \cdot 0.7 \rangle$

$= \langle 0.111, 0.165 \rangle$

$$P(e_{2:4}|X_1) = \sum P(e_2|X_2)P(e_{3:4}|X_2)\cdot P(X_2|X_1)$$

$$= \langle 0.3\cdot 0.3\cdot 0.111\cdot 0.8 + 0.8\cdot 0.1\cdot 0.165\cdot 0.2$$
$$0.3\cdot 0.3\cdot 0.111\cdot 0.3 + 0.8\cdot 0.1\cdot 0.165\cdot 0.7\rangle$$

$$= \langle 0.0106, 0.01224\rangle$$

$$P(e_{1:4}|X_0) = \sum P(e_1|X_1)P(e_{2:4}|X_1)\cdot P(X_1|X_0)$$

$$\langle 0.7\cdot 0.3\cdot 0.0106\cdot 0.8 + 0.2\cdot 0.1\cdot 0.01224\cdot 0.2,$$
$$0.7\cdot 0.3\cdot 0.0106\cdot 0.3 + 0.2\cdot 0.1\cdot 0.01224\cdot 0.7\rangle$$

$$\langle 0.00183, 0.00084\rangle$$

$$P(X_0|e_{1:4}) = \alpha P(X_0| \quad)P(e_{1:4}|X_0)$$

$$\alpha\langle 0.7, 0.3\rangle\langle 0.00183, 0.00084\rangle$$

$$= \alpha\langle 0.00128, 0.000252\rangle$$

$$= \langle 0.8355, 0.1644\rangle$$

$$P(X_1 \mid e_{1:4}) = \alpha P(X_1 \mid e_1) P(e_{2:4} \mid X_1)$$

$$= \alpha \langle 0.951, 0.049 \rangle \langle 0.0106, 0.01224 \rangle$$

$$= \alpha \langle 0.01, 0.0006 \rangle$$

$$= \langle 0.943, 0.057 \rangle$$

$$P(X_2 \mid e_{1:4}) = \alpha P(X_2 \mid e_{1:2}) P(e_{3:4} \mid X_2)$$

$$= \alpha \langle 0.7954, 0.2046 \rangle \langle 0.111, 0.165 \rangle$$

$$= \alpha \langle 0.0883, 0.0338 \rangle$$

$$= \langle 0.7232, 0.2768 \rangle$$

$$P(X_3 \mid e_{1:4}) = \alpha P(X_3 \mid e_{1:3}) P(e_4 \mid X_3)$$

$$= \alpha \langle 0.4023, 0.5977 \rangle \langle 0.428, 0.273 \rangle$$

$$= \alpha \langle 0.1722, 0.1631 \rangle$$

$$= \langle 0.5136, 0.4864 \rangle$$