Assignment 2

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(2) a)

$P(X_0) = 0.7$ $\quad P(X_{t+1} \mid X_t) = 0.8$

$P(X_{t+1} \mid \neg X_t) = 0.3$

e_{ta} = animal tracks $\quad e_{tf}$ = food gone

$P(e_{ta} \mid X_t) = 0.7$ $\quad P(e_{ta} \mid \neg X_t) = 0.2$

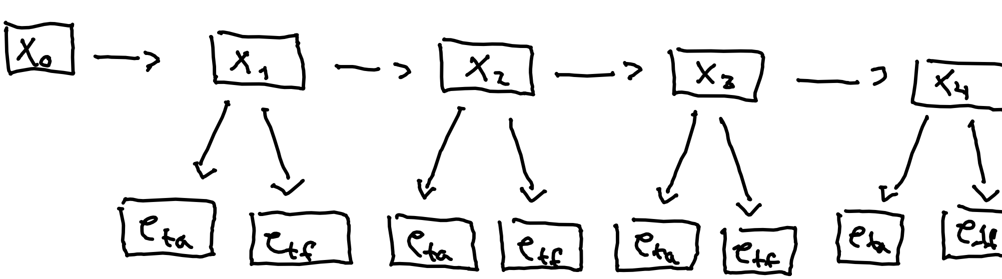
$P(e_{tf} \mid X_t) = 0.3$ $\quad P(e_{tf} \mid \neg X_t) = 0.1$

a_n = animals nearby

	$a_n\ X_{t-1}$	$\neg a_n\ X_{t-1}$
$a_n\ X_t$	0.8	0.3
$\neg a_n\ X_t$	0.2	0.7

	X_t	$\neg X_t$
e_{ta}	0.7	0.2
$\neg e_{ta}$	0.3	0.8

	X_t	$\neg X_t$
e_{tf}	0.3	0.1
$\neg e_{tf}$	0.7	0.9



b)

$P(X_t | e_{1:t})$ for $t = 1, 2, 3, 4$

$P(x_1 | e_1) = P(e_1 | x_1) \cdot P(x_1)$

$P(x_1) = \sum P(x_1 | x_0) \cdot P(x_0)$

$= \langle 0.8, 0.2 \rangle \cdot 0.7 + \langle 0.3, 0.7 \rangle \cdot 0.3$

$= \langle 0.56, 0.14 \rangle + \langle 0.09 + 0.21 \rangle = \langle 0.65, 0.35 \rangle$

$P(x_1 | e_1) = \langle 0.3, 0.1 \rangle \langle 0.7, 0.2 \rangle \langle 0.65, 0.35 \rangle \alpha$

$= \langle 0.1365, 0.007 \rangle \alpha \approx \underline{\langle 0.951, 0.049 \rangle}$

$P(x_2 | e_1) = \sum P(x_2 | x_1) P(x_1 | e_1)$

$\langle 0.8, 0.2 \rangle \cdot 0.951 + \langle 0.3, 0.7 \rangle \cdot 0.049$

$\langle 0.7608, 0.1902 \rangle + \langle 0.0147, 0.0343 \rangle$

$\langle 0.7755, 0.2245 \rangle$

$$P(X_2 \mid e_{1:2}) = \alpha P(e_2 \mid X_2) P(X_2 \mid e_1)$$

$$\boxed{e_2 = \{\text{no animal tracks, food gone}\}}$$

$$= \alpha \langle 0.3, 0.8 \rangle \langle 0.3, 0.1 \rangle \langle 0.7755, 0.2245 \rangle$$

$$= \alpha \langle 0.069795, 0.01796 \rangle$$

$$= \langle 0.7954, 0.2046 \rangle$$

$$P(X_3 \mid e_{1:2}) = \sum P(X_3 \mid X_2) P(X_2 \mid e_{1:2})$$

$$= \langle 0.8, 0.2 \rangle \cdot 0.7954 + \langle 0.3, 0.7 \rangle \cdot 0.2046$$

$$= \langle 0.6363, 0.159 \rangle + \langle 0.0614, 0.14322 \rangle$$

$$= \langle 0.6977, 0.3023 \rangle$$

$$P(X_3 \mid e_{1:3}) = \alpha P(e_3 \mid X_3) P(X_3 \mid e_{1:2})$$

$$\boxed{e_3 = \{\text{no animal tracks, food not gone}}$$

$$= \alpha \langle 0.3, 0.8 \rangle \langle 0.7, 0.9 \rangle \langle 0.6977, 0.3023 \rangle$$

$$= \alpha \langle 0.1465, 0.2176 \rangle$$

$$= \langle 0.4023, 0.5976 \rangle$$

$$P(X_4 \mid e_{1:3}) = \sum P(X_4 \mid X_3) \cdot P(X_3 \mid e_{1:3})$$

$$= \langle 0.8, 0.2 \rangle \cdot 0.4023 + \langle 0.3, 0.7 \rangle 0.5976$$

$$\approx \langle 0.501, 0.499 \rangle$$

$$P(X_4 \mid e_{1:4}) = \alpha P(e_4 \mid X_4) \cdot P(X_4 \mid e_{1:3})$$

$$\boxed{e_4 = \{\text{animal tracks, food not gone}\}}$$

$$= \alpha \langle 0.7, 0.2 \rangle \langle 0.7, 0.9 \rangle \langle 0.501, 0.499 \rangle$$

$$= \alpha \langle 0.2455, 0.0898 \rangle$$

$$= \underline{\langle 0.7322, 0.2678 \rangle}$$

c)

$P(X_t | e_{1:t}) \quad t = 5,6,7,8$

$P(X_5 | e_{1:4}) = \sum P(X_5 | X_4) P(X_4 | e_{1:4})$

$= \langle 0.8, 0.2 \rangle 0.7322 + \langle 0.3, 0.7 \rangle \cdot 0.2678$

$= \langle 0.6661, 0.3339 \rangle$

$P(X_6 | e_{1:4}) = \sum P(X_6 | X_5) P(X_5 | e_{1:4})$

$\langle 0.8, 0.2 \rangle 0.6661 + \langle 0.3, 0.7 \rangle \cdot 0.3339$

$\langle 0.629, 0.371 \rangle$

$P(X_7 | e_{1:4}) = \sum P(X_7 | X_6) P(X_6 | e_{1:4})$

$\langle 0.8, 0.2 \rangle \cdot 0.629 + \langle 0.3, 0.7 \rangle \cdot 0.371$

$\langle 0.6145, 0.3855 \rangle$

$P(X_8 | e_{1:4}) = \sum P(X_8 | X_7) P(X_7 | e_{1:4})$

$\langle 0.8, 0.2 \rangle 0.6145 + \langle 0.3, 0.7 \rangle 0.3855$

$\underline{\langle 0.6073, 0.3927 \rangle}$

d) Calculate a few more steps to verify that the probability converges towards $\langle 0.6, 0.4 \rangle$.

$$P(X_9 | e_{1:4}) = \sum P(X_9 | X_8) P(X_8 | e_{1:4})$$

$$\langle 0.8, 0.2 \rangle \cdot 0.6073 + \langle 0.3, 0.7 \rangle \cdot 0.3927$$

$$= \langle 0.6036, 0.3964 \rangle$$

$$P(X_{10} | e_{1:4}) = \sum P(X_{10} | X_9) P(X_9 | e_{1:4})$$

$$\langle 0.8, 0.2 \rangle \cdot 0.6036 + \langle 0.3, 0.7 \rangle \cdot 0.3964$$

$$= \langle 0.6018, 0.3982 \rangle$$

Observes that the probability converges towards $\langle 0.6, 0.4 \rangle$

The math to calculate the next X_6 is essentialy multiply the matrix

$\begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$ with the previous result

with $\langle 0.6, 0.4 \rangle$ as the previous result

$\begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$ => <u>Converges</u>

e)

$P(X_t | e_{1:4})$ for $t = 0, 1, 2, 3$

Smoothing:

$P(X_t | e_{1:4}) = \alpha f_{1:t} b$

$b_{t+1:4} = P(e_{t+1:4} | X_t)$

$P(e_4 | X_3) = \sum P(e_4 | X_4) \cdot P(e_{5:4} | X_4) P(X_4 | X_3)$

$= \langle 0.7 \cdot 0.7 \cdot 0.8 + 0.2 \cdot 0.9 \cdot 0.2,$

$0.7 \cdot 0.7 \cdot 0.3 + 0.2 \cdot 0.9 \cdot 0.7 \rangle$

$= \langle 0.428, 0.273 \rangle$

$P(e_{3:4} | X_2) = \sum P(e_3 | X_3) P(e_4 | X_3) P(X_3 | X_2)$

$= \langle 0.3 \cdot 0.7 \cdot 0.428 \cdot 0.8 + 0.8 \cdot 0.9 \cdot 0.273 \cdot 0.2,$

$0.3 \cdot 0.7 \cdot 0.428 \cdot 0.3 + 0.8 \cdot 0.9 \cdot 0.273 \cdot 0.7 \rangle$

$= \langle 0.111, 0.165 \rangle$

$$P(e_{2:4} \mid X_1) = \sum P(e_2 \mid X_2) P(e_{3:4} \mid X_2) \cdot P(X_2 \mid X_1)$$

$$= \langle 0.3 \cdot 0.3 \cdot 0.111 \cdot 0.8 + 0.8 \cdot 0.1 \cdot 0.165 \cdot 0.2$$
$$0.3 \cdot 0.3 \cdot 0.111 \cdot 0.3 + 0.8 \cdot 0.1 \cdot 0.165 \cdot 0.7 \rangle$$

$$= \langle 0.0106, 0.01224 \rangle$$

$$P(e_{1:4} \mid X_0) = \sum P(e_1 \mid X_1) P(e_{2:4} \mid X_1) \cdot P(X_1 \mid X_0)$$

$$\langle 0.7 \cdot 0.3 \cdot 0.0106 \cdot 0.8 + 0.2 \cdot 0.1 \cdot 0.01224 \cdot 0.2,$$
$$0.7 \cdot 0.3 \cdot 0.0106 \cdot 0.3 + 0.2 \cdot 0.1 \cdot 0.01224 \cdot 0.7 \rangle$$
$$\langle 0.00183, 0.00084 \rangle$$

$$P(X_0 \mid e_{1:4}) = \alpha P(X_0 \mid \quad) P(e_{1:4} \mid X_0)$$

$$\alpha \langle 0.7, 0.3 \rangle \langle 0.00183, 0.00084 \rangle$$

$$= \alpha \langle 0.00128, 0.000252 \rangle$$

$$= \langle 0.8355, 0.1644 \rangle$$

$$P(X_1 | e_{1:4}) = \alpha P(X_1 | e_1) P(e_{2:4} | X_1)$$

$$= \alpha \langle 0.951, 0.049 \rangle \langle 0.0106, 0.01224 \rangle$$

$$= \alpha \langle 0.01, 0.0006 \rangle$$

$$= \langle 0.943, 0.057 \rangle$$

$$P(X_2 | e_{1:4}) = \alpha P(X_2 | e_{1:2}) P(e_{3:4} | X_2)$$

$$= \alpha \langle 0.7954, 0.2046 \rangle \langle 0.111, 0.165 \rangle$$

$$= \alpha \langle 0.0883, 0.0338 \rangle$$

$$= \langle 0.7232, 0.2768 \rangle$$

$$P(X_3 | e_{1:4}) = \alpha P(X_3 | e_{1:3}) P(e_4 | X_3)$$

$$= \alpha \langle 0.4023, 0.5977 \rangle \langle 0.428, 0.273 \rangle$$

$$= \alpha \langle 0.1722, 0.1631 \rangle$$

$$= \langle 0.5136, 0.4864 \rangle$$