NORWEGIAN UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF INFORMATION TECHNOLOGY, MATHEMATICS AND ELECTRICAL ENGINEERING
DEPARTMENT OF ELECTRONICS AND TELECOMMUNICATIONS

Deadline: 26/2 – 2015

TTT4115 Kommunikasjonsteori

Computer assignment 1

Practical details

Work preferably in groups of two.

Lab sessions to be defined.

Deadline: **Friday February 26, 2015**.

Preparations before going to the lab

- This lab is closely related to Homework Assignment 2, Problem 2 and to the chapter on "Optimal linear baseband transmission". We will basically design filters, calculate theoretical rate-distortion ratios, and simulate to verify the results. Initially you should study the given references carefully.
- There are theoretical problems that should be worked out before you go to the lab.

Problem 1 – Generation of stochastic process

To simulate the system in Problem 2 we will in this problem generate the stochastic processes that will be used. The process will be Gaussian, stationary, and ergodic. Ergodicity is guaranteed as the process is generated from a white process, which by definition is ergodic. A process generated by passing this signal thorough an LTI filter will then also be ergodic if the initial transient is discarded. Ergodicity implies that ensemble averaging can be replaced by time averaging.

The input to our system will be a stationary, Gaussian AR(1) process. This can be obtained by inputting a Gaussian, white noise signal to a first order recursive filter described by:

$$X(n) = \rho X(n-1) + \sqrt{1-\rho^2}\ E(n),$$

where the input signal, $E(n)$, has unit variance, and $X(n)$ is our resulting signal. ρ is the nearest sample correlation. The given coefficients will render a unit variance signal.

a) Derive the autocorrelation function for the generated signal and plot it for the value $\rho = 0.9$.

b) Also derive the corresponding power spectral density and plot the result.

c) Write a Matlab function that generates the process $X(n)$ and produce the output by applying a signal using the MATLAB function 'randn' as inputs to the filter.

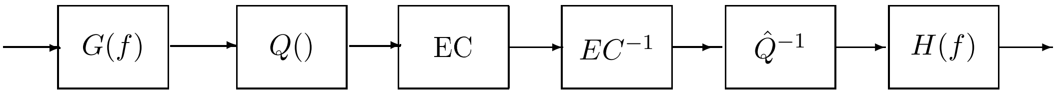
d) Check the obtained autocorrelation function by comparing to the theoretical plots. The autocorrelation function can be estimated from the process by using the function 'xcorr'.

e) Also find an estimate of the power spectral densities of $X(n)$ by taking the Fourier transform of the autocorrelation functions and compare to the true spectrum.

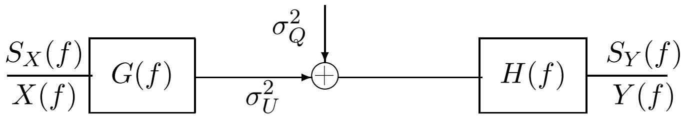
Problem 2 – Optimal, open-loop DPCM system

We shall study an open-loop DPCM system and confirm theoretical derivations by simulations.

The system in the figure shows an encoder-decoder structure. The encoder consists of a filter, $G(f)$, which partly decorrelates the input signal, a quantizer which discretizes the amplitudes, and an entropy coder which transforms the discrete amplitudes to a variable rate bit stream. On the receiver side the bit-stream is decoded, then the discretized amplitudes are reconstructed, and finally a filter, $H(f)$, reestablishes the correlation of the input as closely as possible while removing as much of the quantization noise as possible.



For analysis purposes the 4 blocks in the middle of the above figure can be replaced by an additive noise with zero mean and variance σ_Q^2 provided the quantization is fine enough and no transmission errors occur. The below figure can then be used for the analysis.



Assume further that a uniform quantizer is applied for which the quantization noise is uniformly distributed with variance approximated to $\sigma_Q^2 = \Delta^2/12$, and due to the entropy coding the bit-rate is upper bounded by the entropy

$$H = -\sum_{i=1}^{I} P_i \log_2(P_i),$$

where P_i is the probability finding an amplitude in quantization level i[1]. The summation must be taken over all intervals where $P_i \neq 0$. As the input signal is Gaussian, a good high-rate approximation for the rate is

$$H = \frac{1}{2}\log_2\left[2\pi e \frac{\sigma_U^2}{\Delta^2}\right],$$

where σ_U^2 is the variance of the input to the quantizer[2]. We require that $\sigma_U^2 = 1$, which can be obtained by scaling the input signal, $X(f)$, by the filter, $G(f)$. To obtain a certain bit-rate, the necessary quantization interval, Δ can be found from the above equation. This will, in turn, give us the Lagrange multiplier in Equation 3.15 in the lecture notes necessary for deriving the filters.

[1] Entropy expression to be used in the simulation to estimate the bit-rate.

[2] Entropy relation to be used in the optimization of the filters. Although this is an approximation, we apply it even for low bit-rates

As we are working with sampled signals all integrals over the frequency range will over the interval $0 \leq f < f_s/2$, and we set $f_s = 1$.

a) Derive the theoretical expressions for the optimal filters $H(f)$ and $G(f)$ and draw their frequency responses for the input sequence generated in Problem 1 for bit-rates 0.75, 2, and 5 bits per sample when $\rho = 0.9$[3]. Also draw the power spectral densities of the output noise and the signal component for the three combinations, and find the signal -to-noise ratios.

b) To simulate the system one might derive FIR filters (of adequate length) that approximate the optimal filters by using the *frequency sampling method*. Derive the filters[4], compare their frequency responses with the exact ones, and also estimate the frequency domain signal modification made by the cascade of the encoder and decoder filters.

c) Finally use the filters to simulate the system. Find the rates (in terms of the entropy of the quantized signals) and calculate the signal-to-noise ratio at the receiver output. Compare the simulation and the theoretical results and comment on any deviation. To find the distortion you have to take into account the filter delays!

Hand-in details

The report should answer all questions, but be neat and concise. Remember to write your names and group number on the solution.

[3] You will need to do a numeric integration to find a necessary constant in the derivation.

[4] The frequency sampling method requires that you sample uniformly over the interval $f = [0, f_s]$ while observing symmetry properties of the DFT. Further explanation will be given in the lectures.