NORWEGIAN UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF INFORMATION TECHNOLOGY, MATHEMATICS AND ELECTRICAL ENGINEERING
DEPARTMENT OF ELECTRONICS AND TELECOMMUNICATIONS

Deadline: 10/4 – 2015

TTT4115 Kommunikasjonsteori

Computer assignment 2

Practical details

Lab hours to be announced on It's learning.

Before you go to the lab

- Download all the the m-files to one catalog on your PC. You can get a description of the different functions by writing

  ```
  >> help syn
  >> help ana
  >> help synN
  >> help anaN
  >> help TFB
  >> help RFB
  ```

  in MATLAB.

- Try to work out the theoretical issues.

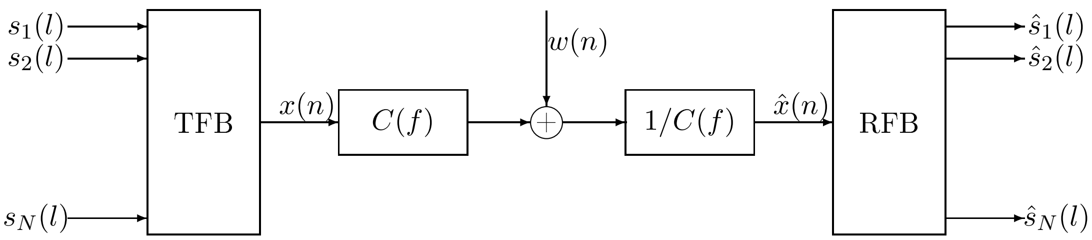
Reporting

The report should contain the names of the group members. The answers should be short and precise. It should contain:

- Derivations, numerical values and comments to all questions.
- Plots where that is required. Remember to indicate what the axes represent.

Introduction

In this computer assignment we shall analyze a multichannel transmission system with multilevel signaling in each channel. The overall system is depicted below.



The system is completely discrete time, which means that we have dropped the D/A conversion, analog lowpass filtering, and upsampling in the transmitter, and the reverse operations in the receiver assuming that these operations can be performed without errors (ideal filters and full synchronism).

The channel is frequency selective (with memory), characterized by its frequency response $C(f)$, and the noise is white.

The transmitter filter bank spreads the different input signals to different frequency bands, although there is significant overlap between the sub-channels.

The signal input to the channel can thus be written as

$$x(n) = \sum_{k=1}^{N} \sum_{l=0}^{\infty} s_k(l) g_k(n - lN).$$

where $s_k(l)$ is the symbol sequence input to channel k, and $g_k(n)$ is the impulse response (basis function) of that channel.

The input signal will be multilevel with integer values.

Problem 1 – Filter bank properties

The transmitter and receiver filterbanks are based on the tree-structured lattice filters with 8 channels, as presented in the lectures. The transmitter filterbank is called by

```
x=TFB(s1,s2,s3,s4,s5,s6,s7,s8,k);
```

where `s1`,`s2`,…,`s8` are the input vectors, and `k` is an array containing the 4 different filter coefficients. Suggested values: `k` $= \tan([0.39, 0.41, 0.45, 0.49] \times \pi.)$ The receiver filter bank is called by

```
[u1,u2,u3,u4,u5,u6,u7,u8]=RFB(x,k);
```

where `x` is the input signal from the channel and `u1`,`u2`,…,`u8` are the output signals. The two Matlab functions call `synN.m, syn.m, anaN.m, ana.m.`

a) Study the orthogonality properties of the different channel impulse responses obtained by generating `x` with unit inputs (`s=[1,0,0,...0]`) from one channel at a time.

b) Draw the frequency responses for each channel.

Problem 2 – Channel response and channel equalization

The channel is frequency selective with a frequency response given by $H(f) = e^{-\alpha|f|}$. To make life easier for you, we provide an impulse response which approximates the given function with $\alpha = 4.6$ in the frequency range up to half the sampling frequency:

$$\begin{aligned} h(n) \quad = \quad & [0.3336, 0.2975, 0.1328, 0.0729, 0.0389, 0.0274, 0.0172, 0.0140, 0.0098, 0.0087, \\ & 0.0064, 0.0061, 0.0047, 0.0048, 0.0037, 0.0042, 0.0029, 0.0046, 0.0010, 0.0086] \end{aligned}$$

This function will be used in the simulations.

As we understand, the different channels will have different attenuations. To counteract the loss in signal strength, we will use channel equalization in the receiver, as indicated by the inverse frequency response in the system diagram.

a) Use the function `y=filter(h,1,x)` to simulate the channel, and find its frequency response.

b) Combine the transmitter filter bank, the channel simulator and the receiver filter bank, but leave out the receiver filter. Check whether reconstruction of individual channels is possible.

c) The channel equalization can be performed with the same function, but now as `v=filter(1,h,u)`. Explain why, and show that the cascade of the two filtering operations give perfect reconstruction. Also explain why it is necessary that the channel is given by a minimum phase filter, and show that it is by using `zplane`.

d) Repeat the experiment in b) when adding the channel equalizer.

e) Keep the last configuration and add white, Gaussian noise to the channel. Observe the noise levels in the reconstructed signals.

f) Replace the Gaussian noise by a sinusoid with frequency centered in one of the channels, and observe the outputs.

Problem 3 – Optimal signaling and detection

As the sub-channels have different signal-to-noise ratios, their capacities are different. We will exploit this ability by using different number of levels in the various channels. Remember that a b-bit signal requires $L = 2^b$ levels for transmission. We will not limit the number of levels to be restricted to values where b is an integer, but rather let L be an integer in each channel.

The capacity in bits per sample is given by

$$C = \frac{1}{2} \sum_k \log_2 \left(\theta \frac{|C(f_k)|^2}{S_N(f_k)} \right),$$

where we have made the approximation that each channel has a flat frequency response and f_k is the center frequency in each band.

In the simulation the input signals are drawn from Gaussian sources and quantized to an appropriate number of levels. The quantization is easily obtained by `round(x)`when x has a correct variance. Assume that the range of signal components will be approximately $-4\sigma_X < x < 4\sigma_X$.

In the receiver the signal components will be approximated to the closest signal level by the same rounding operation.

a) Now set $S_N(f_k) = 1$ and compute the optimal number of levels in each channel as a function of θ.

b) Also compute the optimal power spectral density in each channel from the formula $S_X(f_k) = \theta - 1/|C(f_k)|^2$ (Equation 10.12 in the lecture notes).

c) In the following we only apply the first and last channels. Select the value of θ that would make 3 level signaling optimal in the upper channel. Scale the inputs to make the power levels correct. Remember to rescale the signal at the output of the receiver filter bank. Count erroneous detections by `sum((x-xa)>0.5)` when `x` is the information vector consisting of integer values, and `xa` is the received vector, and find the symbol error rate in each of the two channels. Comment on the relation between the two error rates.