Project 1 in FYS4150

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September 7, 2016

Abstract

This is a place to write something really deep. Like: The future is a really big place. Wow...

Introduction

Theory

$$\mathbf{A} \times \mathbf{v} = \mathbf{p}$$

$$\begin{pmatrix} b_1 & c_1 & 0 & 0 \\ a_1 & b_2 & c_2 & 0 \\ 0 & a_2 & b_3 & c_3 \\ 0 & 0 & a_3 & b_4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}$$

This gives us the following equations:

$$\begin{aligned} \text{I} &: v_1 b_1 + c_1 v_2 &&= p_1 \\ \text{II} &: a_1 v_1 + b_2 v_2 + c_2 v_3 &&= p_2 \\ \text{III} &: a_2 v_2 + b_3 v_3 + c_3 v_4 &&= p_3 \\ \text{IV} &: a_3 v_3 + b_4 v_4 &&= p4 \end{aligned}$$

We want only zeroes on the left side of the diagonal. Therefore, we do as follows:

$$\begin{aligned} p_2^* &= p_2 - p_1 \cdot \frac{a_1}{b_1} \\ &= a_1 v_1 + b_2 v_2 + c_2 v_3 - v_1 a_1 - v_2 \frac{c_1 a_1}{b_1} \\ &= v_2 (b_2 - \frac{c_1 a_1}{b_1}) + c_2 v_3 \\ &= v_2 b_2^* + c_2 v_3 \end{aligned}$$

Now we have the following matrix:

$$\hat{\mathbf{A}} = \begin{pmatrix} b_1 & c_1 & 0 & 0 \\ 0 & b_2^* & c_2 & 0 \\ 0 & a_2 & b_3 & c_3 \\ 0 & 0 & a_3 & b_4 \end{pmatrix}$$

As we can see, the a disappers from the second row. Just as we wanted. We do the same for the other rows

$$\begin{aligned} p_3^* &= p_3 - p_2 \cdot \frac{a_2}{b_2^*} \\ &= v_3(b_3 - \frac{c_2 a_2}{b_2^*}) + c_3 v_4 \\ &= v_3 b_3^* + c_3 v_3 \end{aligned}$$

General:

$$p_n^* = p_n - p_{n-1} \cdot \frac{a_{n-1}}{b_{n-1}^*} \tag{1}$$

$$b_n^* = b_n - \frac{c_{n-1} a_{n-1}}{b_{n-1}^*} \tag{2}$$

$$v_n = \frac{p_n^* - c_n v_{n+1}}{b_n^*} \tag{3}$$

Programs

Results

Discussion

Conclusion