

# Assignments for the course Functional Programming

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## 1 Programming with lists

We consider the datatype  $\mathcal{LT}(B)$  of (finite) binary leaf-trees over type  $B$ , defined recursively as the smallest of all possible sets satisfying:

$$\begin{aligned} \langle b \rangle &\in \mathcal{LT}(B), \text{ for all } b \in B, \\ \langle s, t \rangle &\in \mathcal{LT}(B), \text{ for all } s, t \in \mathcal{LT}(B). \end{aligned}$$

9. Function  $L$ , of type  $\mathcal{LT}(B) \rightarrow \mathcal{L}_*(B)$ , is defined by, for all  $b \in B$  and  $s, t \in \mathcal{LT}(B)$ :

$$\begin{aligned} L \cdot \langle b \rangle &= [b] \\ L \cdot \langle s, t \rangle &= Ls + Lt \end{aligned}$$

...

This inspires us to consider the following generalization of function  $L$ , as a function  $F$  of type:  $\mathcal{L}_*(\mathcal{LT}(B)) \rightarrow \mathcal{L}_*(B) \rightarrow \mathcal{L}_*(B)$ , and with this specification, for all  $ss \in \mathcal{L}_*(\mathcal{LT}(B))$  and  $z \in \mathcal{L}_*(B)$ :

$$F \cdot ss \cdot z = flt(L \bullet rev \cdot ss) + z.$$

(a) Prove that, as stated above,  $L \cdot s ++ L \cdot t = flt \cdot (L \bullet [s, t])$ .

$$\begin{aligned} & flt \cdot (L \bullet [s, t]) \\ = \{def \bullet\} & flt \cdot (L \cdot s ++ L \cdot t) \\ = \{def flt\} & flt \cdot (L \cdot s) ++ flt \cdot (L \cdot t) \\ = \{def flt\} & L \cdot s ++ L \cdot t \end{aligned}$$

(b) Show that function  $F$  indeed is a generalization of  $L$ , by showing how  $L$  can be defined in terms of  $F$ .

(c) Derive an efficient recursive declaration for function  $F$  in which  $L$  does not occur anymore. (d) Explain (the usefulness of) the presence of function  $rev$  in  $F$ 's specification.

## References

[XXXX] Name, *Title*, vol. X. pp. XX–YY. Editor, Town, Year.